THE GROVND OF ARTS:

TEACHING THE PERfect worke and practise of Arithmeticke, both in whole Numbers and Fractions, after a more easie and exact forme then in former time hath beene set foorth: Made by

Mr. ROBERT RECORD
Dr. in Physicke.

Afterwards augmented by Mr. Iohn Dee.

And since enlarged with a Third part of Rules of Practise, abridged into a briefer method then hitherto hath beene published, with diuers necessary Rules, incident to the Trade of Merchandise.

By IOHN MELLIS.

Whereunto is added a *Compendium* of Interest, *with a Table of Board and Timber measure.* Calculated by *R. N.* Gent.

And now diligently perused, corrected, illustrated and inlarged: with an *Appendix* of Figurate Numbers, and the Extraction of their Rootes: by *Robert Hartwell, philomathematicus.*

Scientia non habet inimicum nisi ignorantem.

Fide ——— sed ——— Vide.

At London Printed by *I. B.* for *Roger Iackson.*
1618.

THat which my friend hath well begun,
For very loue to Common-weale,
Neede not all whole to be new done,
But new encreaſe I doe reueale.

Some thing herein I once redreſt,
And now againe for thy behooſe,
Of zeale I doe, and at requeſt,
Both mend and adde, fitte for all prooſe.

Of Numbers vſe, the endleſſe might,
No witte nor language can expreſſe,
Apply and try both day and night,
And then this truth thou wilt confeſſe.

I. D.

The Bookes Verdict.

TO pleaſe or diſpleaſe ſure I am,
But not of one ſort to euery man:
To pleaſe the beſt ſort would I faine,
The froward diſpleaſe ſhall I certaine.
Yet wiſh I will, though not with hope,
All eares or mouthes to pleaſe or ſtoppe.

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TO THE MOST mighty Prince *Edward* the sixt, by the grace of God, King of *England*, *France*, and *Ireland*, &c.

THe excellencie of mans nature being such, as it is by Gods diuine fauor (most mighty Prince) not onely created in highnes of degree far aboue all other corporal things, but by perfection, reason & search of wit, much approaching toward the Image of God, as not only the holy Scriptures doe testifie, but also those naturall Phylosophers, which exactly did consider the nature of man, and namely the far reach & infinit compasse of the works of the mind, were inforced to confes, that man scarcely was able to know himselfe. And if he would duely ponder the nature of himselfe, hee would finde it so strange, that it might seeme vnto him a very myracle. And thereof sprang that saying: *Magnum miraculum est homo: maximum miraculum sapiens homo.* For vndoubtedly as man is one of the greatest myracles that euer God wrought, so a wise man is plainely the greatest.

And therfore was it that some did account the heed of a man the greatest miracle in the worlde, because not onely of the strange workmanship that is in it, but much more of the efficacie of reason, witte, memorie, imagination, and such other powers, & works of the mind, which can more easily conceyue any thing in a manner, then vnderstand it selfe. And amongst all the creatures of God it findeth none more difficult to be perceiued then these same powers of it selfe, whereby it doth conceiue & iudge as it may be well coiectured

by the diuersitie of opinions, that the wisest Phylosophers did vtter, touching the spirit of man & the substance of it: wherof at this present I intend to make no rehearsall, but who so listeth to reade thereof, may find it largely set forth, not only in *Aristotle* his bookes *De anima*, but also in *Galene*, his booke called *Hystoria philosophica*, & againe, in *Plutarch* his worke, *De phylosophorum placitis*, whose wordes are also repeated of *Eusebius* in the xv. booke τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue desired to vnderstand the intricate difficultie of knowing our owne selues, as touching our best parte, and that part whereby we deserue to beare the name of men.

This matter seemed so obscure & difficult in knowledge, that *Galen*, who for his excellent wisdome and iudgement in naturall works, is called of many men a Miracle in nature, yet in searching the nature & substance of the spirite of man, hee not onely confesseth himselfe ignorant, but counteth it plaine temeritv to attempt to finde it. so farre aboue the hope of mans knowledge is that part, whereby man doth know and iudge of things. And although the ignorant sort (which hate all things that they know not) doe little esteeme the profoundnes of mans spiri. & of reason, the chiefe power & facultie of it: yet as there is a kind of feare and obedience of vnreasonable Beasts vnto man, by the working power of God, so is there in these small reasoned persons a certain kinde of reuerence toward wisedome & reasons which they do shew oftentimes, and by power of persuasion, are inforced to obey reason, will they, nill they.

And hereby came it to passe, that the rudenesse of the first age of Man was brought vnto some more ciuile trade, as it is well declared by *Cicero*, in the beginning of his first book *De inuentione Rhetorica*, where he saith thus: *Nam fuit quoddam tempus quum in agris homines passim bestiarum more vagabantur, & sibi victu feruo vi-*

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tam propagabant, nec ratione animi quicquam, sed pleraq; viribus corporis administrabãt. Nondum diuine religionis, non humani ratio colebatur. Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos, non ius aequabile quid utilitatis haberet, acceperat: ita propter errorem atq; insciam cæca ac temeraria dominatrix animi cupiditas, ad se explendam viribus corporis abutebatur, perniciosissimis satellitibus. Quo tempore quidam, magnus videlicet vir & sapiens, cognouit quæ materia esset, & quanta ad maximas res opportunitas in animis in esset hominũ, si quis eam possit elicere, & præcipiendo meliorem reddere. Qui dispersos homines in Agris, & in tectis syluestribus abditos ratione quædam compulit in vnum locum, & congregauit: & eos in vnamquamq; rem inducens vtilem atq; honestam, primo propter insolentiam reclamantes, deinde propter rationem atq; orationem studiosius audientes, ex feris & immanibus mites reddidit, & mansuetos.

This long repetition of *Tullyes* words will seeme tedious to them that loue but little, and care much lesse for the knowledge of reason, but vnto your Maiesty (I dare say) it is a delectable remembrance, and vnto me it seemed so pleasant, that I could scarce stay my pen from writing all that mine eyes did so greedily read.

This sentence of *Cicero* am I loath to translate into English, partly for that vnto your Maiesty it needeth no translation, but especially knowing how farre the grace of *Tullyes* Eloquence doth excell any Englishmans tõgue, & much more exceedeth the baseness of my barbarous stile, yet for the fruite of the sentence, I had rather vnto my meere English countreymen vtter the rudeness of my translation, then to defraude them the benefit of so good a lesson: trusting they will so learne to loue reason, that they will also gladly and greedily embrace all good Sciences, that may help to the iust furniture of the same, whẽ they consider that informed reason was the onely instrument, or at least the chiefest meanes to bring men vnto Ciuile regi-

ment, from barbarous manners, & beastly conditions?

For the time was (saith *Tully*) that men wandred abroade in the fields vp & downe like Beasts, & vsed no better order in feeding then they, so that by reasons rule they wrought nothing, but moste of theyr doings did they atchieue by force of strength. At this time there was no iust regard of Religion toward God, nor of dutie toward man. No man had seene right vse of mariage, neither did any man know their owne children from other, nor no man had felt the commodity of iust Lawes: so that through errour and ignorance, wilfull lust, like a blind and heady ruler, abused bodily strength as a most mortall minister for the satisfying of his desire. At that time was there one, which not onely in power, but also in wisedome was great, And he considered how that in the mindes of men was both apt instruments, and great occasion to the due accomplishment of most weighty affaires, if a man could apply them to vse, and by teaching of rules, frame them to better trade. This man with perswasion of reason gathered into one place the people that were wandring about the fields, and lay lurking in wilde cottages and woods: And bringing them into one common society, did trade them to al such things as eyther were profitable or honest, although not without repining as the first, by reason that they had not bin so accustomed before. Yet at length through reason and perswasion of words they obeyed him more diligently, and so of a wilde and cruell people, he made them curteous and gentle.

Thus hath *Tullie* set forth the efficacie of reason and perswasion, how it was able to conuert wild people to a mildnesse, and to change their furious cruelnesse into gentle curtesie: were it not now a great reproach in this out time (when knowledge raigneth so large) that men should shew themselues lesse obsequious to reason? Vnlesse it may be thought, that

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now euery man hauing ſufficient knowledge of himſelfe, needeth not to hearken to the perſwaſion of others.

Indeed he that thinketh himſelfe wiſe, will not eſteeme the reaſon of any other, be he neuer ſo wiſe, ſo that of ſuch a one it may well be ſaid: Hee that thinketh himſelfe wiſer then he is, may iuſtly be counted a double foole: wherefore ſuch men are not to be permitted in open audience to talke, but muſt be put to ſilence, & made to giue eare to reaſon, which reaſon conſiſteth not in a multitude of words heaped raſhly together, and applied for one purpoſe, but reaſon is the expreſſing of a iuſt matter with witty perſwaſions, furniſhed with learned knowledge. Such knowledge had *Moſes*, beeing expert in all learning of the Egiptians, as the Scriptures declare, and therefore was able to perſwade the ſtubborne people of the Iewes, although not without great paine. Such knowledge and ſuch reaſons did *Druys* ſhew, which was the firſt Law-maker of all the Weſt part of Europe. Like reaſon and wiſedome did *Xamolxis* vſe amongſt the Gothes: *Lycurgus* vnto the Lacedomonians, *Zeleucus* to the Locrians, *Solon* to the Atheniēſes, and *Dunwallo Molmutius* two thouſand yeares paſt amongſt the olde Britains of this Realme. And thereby came it to paſſe, that their Lawes continued long, till more perfect reaſon altered many of them, and wilfull power oppreſſed moſt of them.

Druis was ſon to King Sarron, & ſucceeded him in his kingdome.

At the beginning when theſe wiſe men perceiued how hard it was to bring the rude people to vnderſtand reaſon, they iudged the beſt meanes to attaine this honeſt purpoſe to depend of learning in euery kinde, for by learning (as *Ouid* ſaith): *Pectora molleſcunt, aſperitáſque fugit*: Stout ſtomackes doe waxe milde, and ſharpe fiercenēſſe is exilde. Therefore as *Beroſus* doth teſtifie, *Saron* that was the third King ouer all this Weſt part of Europe, for to bring the people

people from beastly rage to manly reason, did erect Schooles of liberall Arts, which tooke so good successe, that his name continued in that sort famous aboue two thousand yeares after: for *Diodorus Siculus* which was in the time of *Iulius Cæsar*, maketh mention of the learned men or Gothes of Celtes, and nameth them *Sarronides*, that is to say, *Sarron his Scholliers and followers*.

Among these Arts that then were taught, some did enforme the tongue, and make them able both to vtter aptly their minde, and also to perswade, as Grammer, Logike, and Rhetoricke, although not so curiously as in this time: some other did appertain to the iust order of partition of lands, the true vsing of weights, measures and reckonings in all sorts of bargaines, and for order of building and sundry other vses, those were Arithmeticke and Geometry. Againe, to incourage men to the honor of God, they taught Astronomy, whereby the wonderfull workes of God were so manifestly set forth, that no mans tongue nor penne can in like sort expresse his infinite power, his vnspeakeable wisedome, and his exceeding goodnesse toward man, whereby hee doth bountifully prouide for man all necessaries, not only to liue, but also to liue pleasantly. And so was their confidence in Gods prouidence strongly stayed, knowing his goodnesse to bee such, that hee would helpe man as he could, and his power to be so great, that hee could doe what hee would: and thirdly his wisedome to be so pure, that he would doe nothing, but that that was best. Beside these Sciences they taught also Musicke, which most commonly they did apply partly to Religious Seruices, to draw men to delight therein, and partly to songs made of the manners of men, in praise of Vertue, & discommendation of Vice, whereby it came to passe, that no man would displease them, nor doe any thing euill

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that might come to their hearing: for their songs did make euil men more abhorred in that time, than any excommunication doth in this time. The posteritie of these Musitians continue yet both in Wales and Ireland, called *Bardes* vnto this day, by the auncient name of *Bardus*, their first founder.

This Bardus Dyuidius the King of the Celtes raigned 69 yeare and died 1832 yeare before Christ.

And as these Sciences did encrease, so did Vertue encrease thereby. Againe, as those Sciences did decay, so Vertue lost her estimation, and consequently was little in vse: whereof to make a full declaration were a thing meet for a Prince to heare, but it would require a peculiar Treatise. Wherefore at this present I count it sufficient lightly to haue touched this matter in generall words, and to say no more of the particularity thereof, but onely touching one of those Sciences, that is, *Arithmetike*, by which not onely iust partition of lands was made, but also touching buying and selling, all Assises, Weights, and Measures were deuised, and all reckonings and accounts driuen, yea by proportion of it were the true orders of Iustice limited, as *Aristotle* in his Ethikes doth declare, and the degrees of estates in the common wealth established. Although that proportion be called Geometricall, & not Arithmetical, yet doth that proportion appertaine to the Art of Arithmeticke, and in Arithmeticke is taught the progression of such proportions, and all things thereto belonging. Wherefore I may well say, that seeing Arithmeticke is so many waies needfull vnto the first planting of a Common Wealth, it must needs be as much required to the preseruation of it also, for by the same meanes is any Common wealth continued, by which it was erected and stablished. And if I shall in small matters in appearance, but in deed very waighty, put one example or two, What shall we say for the Statutes of this Realme, which be the onely stay of good order in manner now? As touching the

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measuring of ground by length and breadth, there is a good and an auncient Statute made by Art of Arithmeticke, and now it shall be to little vse, if by the same Art it be not practised and tried. for the assise of Bread and Drinke, the two most common and most necessary things for sustentation of man, there was a goodly ordinace in the Law made, which by ignorance hath so growne out of knowledge and vse, that few men doe vnderstand it, and therefore the Statute bookes wonderfully corrupted, and the Commons cruelly oppressed: notwithstanding some men haue written, that it is too doubtfull a matter to execute those assises by those statutes, by reason they depend of the standerd of the coyne, which is much changed from the state of that time when those Statutes were made. Thus shall euery man reade (that listeth) in the Abridgement of Statutes, in the title of Weights and Measures, in the seauenth number of the English Booke, where he should haue translated a good ordinance which is set forth in the French booke: but no maruell if the Abridgement doth omitt it seeing thee great booke of Statutes doth omit the same statute, as it hath done diuers other very good Lawes. And this is the fruit of ignorance, to reiect and condemne all that it vnderstandeth not, although they vse some cloakes for it: but such cloakes as being allowed, might serue to repell all good Lawes, which God forbid.

Againe, there is an auncient order for Assise of fire-Wood and Coales, which was renued not many yeares past, and now how Auarice and Ignorance doth eanuse that Statute, it is too pitifull to talke of, and more miserable to feele.

Furthermore, for the Statute of Coinage, and the Standerd thereof, if the people vnderstood rightly the Statute, they should not, nor would not (as they often doe) gather an excuse for their folly thereby

thereby: but as I ſayd, theſe Statutes by wiſedome and good knowledge of Arithmetick were made, and by the ſame muſt they be continued. And let ignorance no more meddle with the vſe of them, than it did with the making of them. Oh in how miſerable caſe is that Realme, where the Miniſters and interpreters of the Law are deſtitute of all good Sciences, which be the Keyes of the Lawes. How can they eyther make good Lawes or maintaine them, that lacke that true knowledge whereby to iudge them. And happy may that Realme be accounted, where the Prince himſelfe is ſtudious of learning, and deſireth to vnderſtand equitie in all Lawes. Therefore moſt happy are we the louing Subiects of your Maieſtie, which may ſee in your highneſſe not onely ſuch towardneſſe, but alſo ſuch knowledge of diuers Arts, as ſeldome hath beene ſeene in any Prince of ſuch yeares, whereby we are enforced to conceiue this hope certainely, that he which in thoſe yeares ſeeketh knowledge when knowledge is leaſt eſteemed, and of ſuch an age can diſcerne them to be enemies both to his royall perſon, & to his Realmes, which labour to withdraw him from knowledge, to exceſſiue paſtime, and from reaſonable ſtudy, to idle or noyſome pleaſures, he muſt needes when he commeth to more mature yeares, bee a moſt prudent Prince, a moſt iuſt Gouernour, and a right Iudge, not onely of his Subiects commonly, but alſo of the miniſters of his Lawes, yea, and of the Lawes themſelues: and to be able to conceiue the true equitie and exact vnderſtanding of al his Lawes & Statutes, to the comfort of his good Subiects, and the confuſion and reproach of them which labour to obſcure or peruert the equity of the ſame Lawes and Statutes. How ſome of theſe Statutes may be applyed to vſe, as well in this our time, as in any other time, I haue peculiarly declared in this booke, and ſome other

ther I haue omitted for iuſt conſiderations, till I may offer them firſt vnto your Maieſtie, to wey them, as to your highneſſe ſhall ſeeme good: for many things in them are not to be publiſhed without your highneſſe knowledge and approbation, namely becauſe in them is declared all the rates of alloyes for all ſtanderds from one ounce vpward, with other myſteries of mint matters and alſo moſt part of the varieties of coynes that haue bin currant in this your Maieſties realme by the ſpace almoſt of ſixe hundred yeares laſt paſt, and many of them that were currant in the time that the Romanes ruled heere.

All which with the auncient deſcription of England and Ireland, & my ſimple cenſure of the ſame, I haue almoſt completed to be exhibited to your highneſſe. In the meane ſeaſon moſt humbly beſeeching your Maieſtie, to accept this ſimple Treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Maieſtie is, that this amongſt other of my bookes may paſſe vnder the protection of your highneſſe, whom I beſeech God moſt earneſtly, and daily, according to my duty, to aduance in all honour and Princely Regalties, and to encreaſe in all Knowledge, Iuſtice, and godly policie. Amen.

Your Maieſties moſt obedient

Subiect and ſeruant

ROBERT RECORD.

To

TO THE LOVING *Readers : the Preface of* Mr. Robert Record.

SOre oftentimes haue I lamented with my selfe the infortunate condition of England, seeing so many great Clearkes to arise in sundry other parts of the world, and so few to appeare in this our Nation: where as for pregnancie of naturall witte (I thinke) few Nations doe excell Englishmen. But I cannot impute the cause to any other thing, than to the contempt or misregard of learning. For as Englishmen are inferiour to no men in mother witte, so they passe all men in vaine pleasures, to which they may atteine with great paine or labour: and are as slacke to any neuer so great commoditie, if there hang of it any painefull study or trauelsome labour.

Howbeit yet all men are not of that sort, though the most part be, the more pitty it is: but of them that are so glad not onely with painfull studie and studious paine to atteine learning, but also with as great studie and paine to communicate their learning to other, and make all England, if it might be, partakers of the same, the most part are such, that vnneth they can support their owne necessary charges, so that they are not able to beare any charges in doing of that good, that else they desire to doe.

But a greater cause of lamentation is this: that when learned men haue taken paines to doe things for the aide of the vnlearned, scarce they shall be allowed for their well doing, but derided and scorned, & so vtterly discouraged to take in hand any like enterprise againe. So that if any be found (as there are some) that doe fauour learning, and learned wittes, and can be content to further knowledge, yea onely with their word, such persons though they be rare, yet shall they incourage learned men to enterprise some thing at the least, that England may reioyce of. And I haue good hope that England will (after she hath taken some sure taste of learning) not onely bring forth more fauourers of it, but also such learned men that shee shall be able to compare with any Realme in the world. But in the meane season, so few regarders of learning are, how greatly they are to bee esteemed that doe fauour and further it, my pen will not suffice at full to declare.

Therefore gentle Reader, where as I doe vpon most iust occasion iudge, yea and know assuredly, that there be some men in this Realme which both loue and also much desire to further good learning, and

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yet am not well able to write their condigne praise for the same, I thinke it better with silence to ouerpasse it, then either to say too little of it, or to prouoke against them the malice of such other, which doe nothing themselues that is praise-worthy, and therefore cannot abide to heare the praise of any other mans good deede.

And considering their great fauour vnto learning, though I my selfe be not worthy to be reckoned in the number of great learned men, yet am I bold to put my selfe in Presse with such abilitie as God hath lent me, though not with so great cunning as many men, yet with as great affection as any man, to helpe my Countrie-men, and will not cease daily, (as much as my small abilitie will suffer me) to endeuor some such thing, that shall be to the instruction, though not of learned men, yet at the least of the vulgar sort, whose argument alwaies shall be such as it shall delight all learned wits, though they doe not learne any great things out of it.

But to speake of this present booke of Arithmeticke, I dare not, nor will not set it forth with any words, but remit it to the iudgement of all gentle Readers, and namely such as loue good learning, beseeching them so to esteeme it, as it doth seeme worthy. And so either to accept the thing for it selfe, either at the least to allow my good endeuor. But I perceiue I needs not vse any perswasions vnto them, whose gentle nature and fauourable minde is ready to receiue thankefully, and interprete to the best all such enterprises attempted for so good an end, though the thing doe not alwaies satisfie mens expectation. This considered, did bolden me to publish abroad this little booke of the Art of numbring, which if you shall receiue fauourablie, you shall encourage me to gratifie you hereafter with some greater thing.

And as I iudge some men of so louing a minde to their native Countrey that they would much reioyce to see it to prosper in good learning, and wittie Arts, so I hope well of all the rest of Englishmen, that they will not bee vnmindfull of his due praise, by whose meanes they are holpen and furthered in any thing. Neyther ought they to esteeme this thing of so little value, as many men of little discretion oftentimes doe. For who so setteth small price by the wittie deuise and knowledge of numbering, he litle considereth it to be the chiefe point, (in manner) whereby men differ from all brute beasts: for as in all other things (almost) Beasts are partakers with vs, so in numbering we differ cleane from them, and in manner peculiarly, sith that in many things they excell vs against.

The Foxe in craftie witte exceedeth most men,
A Dogge in smelling hath no man his peere
To foresight of weather if you looke then,
Many beasts excell men this is cleere,
The wittinesse of Elephants doth letters attaine,
But what cunning doth there in the Bee remaine?

The Preface to the Reader.

The Emmet foreseeing the hardneſſe of Winter,
Prouideth victuals in time of Sommer.
The Nightingale, the Liner, the Thruſh the Larke,
In Muſicall harmonie paſſe many a Clarke.
The Hedgehogge of Aſtronomy ſcemeth to know,
And ſtoppeth his Caue where the winde will blowe.
The Spider in weauing ſuch Art doth ſhowe,
No man can him mend, nor follow I trowe.
When a houſe will fall, the Myſe right quicke,
Flee thence before, can man doe the like?

Many things elſe of the wittines of Beaſts and Birds might I here ſay, ſaue that another time of them I intend to write wherein they excell in manner all men, as it is daily ſeene: but in number was there neuer Beaſt found ſo cunning, that could know or diſcerne one thing from many: at by daily experience you may well conſider, when a Butcher hath many Whelps, or a Henne many Chickens: and likewiſe of other what ſoeuer they be, take from them all their young, ſauing one young one, and you ſhall perceiue plainely, that they miſſe none, though they will reſiſt you in taking them away, and will ſeeke them againe if they may know where they be, but elſe they will neuer miſſe them truely, but take away that one that is left, and then will they cry and complaine: and reſtore to them that one, then are they pleaſed againe: ſo that of number thus may I iuſtly ſay, it is the onely thing almoſt that ſeparateth man from beaſts. Hee therefore that ſhall contemne number, he declareth himſelfe as brutiſh as a beaſt, and vnworthy to be counted in the fellowſhip of men. But I truſt there is no man ſo ſoule ouerſeene, though many right ſmally doe it regard.

Therefore will I now ſtay to write againſt ſuch, and returne againe to this my Booke, which I haue written in maner of a Dialogue, becauſe I iudge that to be the eaſieſt way of inſtruction, when the Scholler may aske euery doubt orderly, and the Maſter may anſwere to his queſtion plainely. Why the Author wrote in Dialogue wiſe.

How beit I thinke not the contrarie, but as it is eaſier to blame another mans worke then to make the like, ſo there will be ſome that will find faut because I write in a Dialogue: but as I coniecture, thoſe ſhall be ſuch as doe not, cannot, either will not perceiue the reaſon of right teaching, and therefore are vnmeet to be anſwered vnto, for ſuch men with no reaſon will be ſatisfied.

And if any man obiect, that other bookes haue beene written of Arithmetike already ſo ſufficiently, that I needed not now to put pen to the booke, except I will condemne other mens writings: to them I anſwere: That as I condemne no mans diligence, ſo I know that no one man can ſatisfie euery man: and therefore like as many doe eſteeme greatly other bookes, ſo I doubt not but ſome will like this my booke aboue any other Engliſh Arithmeticke hitherto written, and namely ſuch as ſhall lacke inſtructers, for whoſe ſake I haue ſo

plainly set forth the examples as no booke that I haue seene hath done hitherto: which thing shall be great ease to the rude Readers.

Therefore (gentle Reader) though this booke can be but small and to the learned sort, yet vnto the simple ignorant (which needeth most helpe) it may be a good furtherance and meane vnto knowledge.

And though vnto the King his Maiestie priuately I doe it dedicate, yet I doubt not (such is his clemencie) but they bee can be content, yea and much desirous, that all his louing Subiects shall take the vse of it, and imploy the same to their most profit: Which thing if I perceiue that they thankefully doe, and receiue with as good will as it was written, then will I shortly with no lesse kindnesse set forth such introductions into Geometrie and Cosmographie, as I haue at times promised, and as hitherto in English hath not beene enterprised, wherewith I dare say all honest hearts will bee pleased, and all studious wittes delighted.

I will say no more, but let euery man iudge as he shall see cause. And thus for this time I will stay my penne, committing you all to that true fountaine of perfect number, which wrought the whole world by Number and measure: he is Trinitie in Vnitie, and Vnitie in Trinitie: To whom be all praise, honor and glorie. Amen.

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A Table of Board and Timber measure, exactly calculated, and the vse thereof.

A triple Table of Interest exactly calculated after 10 in the hundred.

Note that this ∆ Character representeth the Crowne by exchange.

Before

Before the Introduction of Arithmeticke, it were very good to haue some vnderstanding and knowledge of these Figures and Notes.

i	1	one.	rr.	20	twentie.
ij	2	two.	rl.	40	fortie.
iij	3	three.	l.	50	fiftie.
iiij	4	foure.	lr.	60	sixtie.
v	5	fiue.	lrr.	70	seuentie.
vi	6	sixe.	rc.	90	ninetie.
vii	7	seauen.	C.	100	a hundred.
viij	8	eight.	CC.	200	2 hundred.
ix	9	nine.	D.	500	5 hundred.
r	10	ten.	DC.	600	6 hundred.
ri	11	eleuen.	M.	1000	a thousand.
rij	12	twelue.	MD.	1500	a thou. 5. hun.
&c.	&c.	&c.	&c.	&c.	&c.

112 ℔ make a C. } weight.
56 halfe a C.
28 a quarter of a C.
16 ounces make a ℔ hauer de pois.

A pound Troy, wherby is wayed gold, siluer, & bread, is 12 ounces. For further knowledge, see in ye rule of Reduction.

12 inches make a foot.
3 foot make a yard.

The yard is parted into 4 quarters.

And each quarter into 4 nailes.

5 quarters of a yard make an elle. Which elle is also parted into 4 quarters.

And each quarter into 4 nailes.

A Dialogue betweene the Maſter and the Scholar: teaching the ART and vſe of Arithmeticke with Pen.

The Scholler ſpeaketh.

SIr, ſuch is your authority in mine eſtimation, that I am content to conſent to your ſaying, and to receiue it as truth, though I ſee none other reaſon that doth leade mee thereunto: whereas elſe in mine owne conceit it appeareth but vaine, to beſtow any time priuately in learning of that thing, that euery child may, and doth learne at all times and houres, when he doth any thing himſelfe alone, and much more when hee talketh or reaſoneth with other.

Maſter Loe, this is the fashion and chance of all them that ſeeke to defend their blinde ignorance: that when they thinke they haue made ſtrong reaſon for themſelues, then haue they proued quite contrarie. For if numbring be ſo common (as you grant it to be) that no man can doe any thing alone, and much leſſe talke or bargaine with other, but he

he shall still haue to doe with number: this proueth not number to be contemptible and vile, but rather right excellent and of high reputation, sith it is the ground of all mens affaires, so that without it, no tale can be told, no communication without it can be continued, no bargaining without it can duely be ended, or no businesse that man hath, iustly completed. These commodities if there were none other, are sufficient to approue the worthinesse of number. But there are other innumerable, farre passing all these, which declare number to exceed all praise. Wherefore in all great workes are *Clerkes* so much desired? Wherefore are *Auditors* so richly fed? what causeth *Geometricians* so highly to be enhaunced? why are *Astronomers* so greatlie aduanced? because that by number such things they finde, which else would farre excell mans minde.

Scholar. Verily sir, if it be so, that these men by numbring, their cunning do attaine, at whose great workes most men doe wonder, then I see well I was much deceiued, and numbring is a more cunning thing then I toke it to be.

Master. If number were so vile a thing as you did esteeme it, then need it not to be vsed so much in mens communication. Exclude *Number*, and answer to this question: how many yeares old are you?

Scholar.

Scholar. *Mum.*

Master. How many *daies* in a *weeke*? How many *weekes* in a *yeare*? What *lands* hath your *father*? How many *men* doth hee keepe? How long is it since you came from him to me?

Scholar. *Mum.*

Master. So that if number want, you answer all by *Mummes*: How many miles to *London*?

Scholar. A poake full of *Plummes*.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking, it maketh men dumbe, so that to most questions they must answer *Mum*.

Scholer. This is the cause (sir) that I iudged it so vile, because it is so common in talking euery while: For plenty is not daintie, as the common saying is.

Master. No, nor store is no sore: perceiue you this? The more common that the thing is being needfully required, the better is the thing, and the more to be desired. But in numbring, as some of it is light & plaine, so the most part is difficult, and not easie to attaine. The easier part serueth all men in common, and the other part requireth some learning. Wherefore as without numbring a man can doe almost nothing, so with the helpe of it you may attaine to all things.

Scholar. Yea sir: why: then it were best to learne

learne the *Art of numbring*, first of all other learning, and then a man nede learne no more, if all other come with it.

Master. Nay, not so: but if it be first learned, then shall a man be able (I meane) to learne, perceiue, and attaine to other *Sciences*, which without it he could neuer get.

Scholar. I perceiue by your former words that *Astronomy* and *Geometry* depend much on the helpe of numbring: but that other sciences, as *Musicke*, *Physicke*, *Law*, *Grammar*, and such like, haue any helpe of *Arithmetike*, I perceiue not.

Master. I may perceiue your great Clerkelinesse by the ordering of your *Sciences*: but I will let that passe now, because it toucheth not the matter that I intend, and I wil shew you how *Arithmetick* doth profit in all these, somewhat grossely, according to your small vnderstanding, omitting other reasons more substantiall.

Musicke. First (as you reckon them) *Musicke* hath not only great helpe of *Arithmeticke*, but is made, and hath his perfectnesse of it: for all *Musick* standeth by number and proportion.

Physicke. And in *Physicke*, beside the calculation of *criticall dayes*, with other things, which I omitte, how can any man iudge the pulse rightly, that is ignorant of the proportion of Numbers?

The Law. And as for the *Law*, it is plaine, that the man

man that is ignorant of *Arithmetike*, is neither meet to be a *Iudge*, neither an *Aduocate* nor yet a *Proctor*. ¶ or how can he well vnderstand another mans cause, appertaining to distribution of goods, or other debts, or of sums of money, if hee be ignorant of *Arithmetick*? This oftentimes causeth right to be hindered, when ỹ iudge eyther delighteth not to heare of a matter that he perceiueth not, or cannot iudge for lack of vnderstanding: this commeth by ignorance of *Arithmeticke*.

Grammer

Now, as for *Grammer*, mée thinketh you should not doubt in what it néedeth number, sith you haue learned that *Nounes* of all sorts, *Pronounes*, *Verbes* and *Participles* are distinct diuersly by numbers: besides the variety of *Nounes of Number*, & *Aduerbs* And if you take away number from *Grammer*, then is all the quantity of *Sillables* lost. And many other waies doth number helpe *Grammer*. Whereby were all kindes of méeters found and made? Was it not by Number?

Philosophie.

But how néedefull *Arithmeticke* is to all parts of *Philosophie*, they may soone sée, that do read eyther Aristotle, Plato, or any other *Philosophers* writings. For all their examples almost, and their probations, depend of *Arithmeticke*. It is the saying of Aristotle, that he that is ignorant of *Arithmeticke*, is méete for no *Science*. And Plato his Master wrote

wrote a little sentence ouer his Schol-house doze, Let none enter in hither (quoth he) that is ignorant of Geometrie. Seeing he would haue all his *Scholars* expert in *Geometrie*, much rather he would the same in *Arithmetick*, without which *Geometry* cannot stand.

Diuinity. And how needful *Arithmeticke* is to *Diuinitie*, it appeareth, seeing so many *Doctors* gather so great mysteries out of number, and so much doe write of it. And if I should goe about to write all the commodities of *Arithmetick* in ciuil acts, as in gouernance of com-

Armies. mon *weales in time of peace*, and in due prouision and order of *armies*, in time of *warre*. For numbring of the *hoste*, summing of their *wages*, prouisions of *victuals*, viewing of *artillerie*, with other *Armour*: Beside the cunningest point of all, for casting of *ground*, for *encamping of men*, with such other like. And how many waies also *Arithmeticke* is conducible for all priuate *weales*, of *Lords* & all possessioners, of *Merchants* and all other *occupiers*, and generally for all estates of men, besides *Auditors*, *Treasurers*, *Receiuers*, *Stewards*, *Bailiffes*, & such like, whose offices without *Arithmetick* are nothing: if I should (I say) particularly repeat al such commodities of the *noble Science* of *Arithmeticke*, it were enough to make a very great booke.

Scholar. No, no sir, you shal not need: For I doubt not, but this ye you haue said, were enough

enough to perſwade any man to thinke this Art to be right excellent and good, and ſo neceſſary for man, that (as I thinke now) ſo much as a man lacketh of it, ſo much he lacketh of his ſenſe and wit.

Maſter. What are you ſo farre changed ſince, by hearing theſe few commodities in generall? by likelihood you would be farre changed if you knew all the particular commodities.

Scholar. I beſeech you Sir, reſerue thoſe commodities that reſt yet behind vnto their place more conuenient: and if yée will be ſo good as to vtter at this time this excellent treaſure, ſo that I may be ſomewhat inriched thereby, if euer I ſhall be able, I will requite your paine.

Maſter. I am very glad of your requeſt, and will doe it ſpéedily, ſith that to learne it you be ſo ready,

Scholar. And I to your authority my wit doe ſubdue, whatſoeuer you ſay, I take it for true. The duty of a Scholar.

Maſter That is too much, and méete for no man to be beléeued in all things, without ſhewing of reaſon. Though I might of my Scholar ſome credence require, yet except I ſhew reaſon, I doe it not deſire. But now ſith you are ſo earneſtly ſet this Art to attaine, beſt it is to omit no time, leaſt ſome other paſſion cole this great heate, and then Perſeuerance in ſtudy.

you leaue off before you see the end.

Scholar. Though many there be so vnconstant of minde, that flitter and turne with euery winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indeed, and I trust you will increase rather then goe backe. For better it were neuer to assay, then to shrinke and fly in the midway: But I trust you will not so doe, therfore tell me briefly: What call you the Science that you desire so greatly?

Scholar. Why Sir? you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to haue studied a Science, and yet cannot tell how it is named.

Scholar. Some call it Arsemetike, and some Augrime.

Master. And what doe those names betoken?

Scholar That, if it please you, of you would I learne.

Arithmeticke. Master. Both names are corruptly written. Arsmetike for Arithmeticke as the Greekes call it, and Augrim for Algorisme, as the Arabians found it: which both betoken

ken the Science of Numbring: for Arithmos in Greeke is called Number: and of it commeth Arithmeticke, the Art of Numbring. So that Arithmeticke is a Science or Art teaching the maner and vse of Numbring. This Art may be wrought diuerslie, with Penne or with Counters. But I will first shew you the working with the penns, and then the other in order.

Scholar. This I will remember. But how many things are to be learned to attaine this Art fully?

Master. There are reckoned commonly seuen parts or workes of it. Numeration, Addition, Subtraction, Multiplication, Diuision, Progression, and Extraction of rootes: to these some men adde Duplation, Triplation, and Mediation. But as for these three last, they are contained vnder the other seuen. For Duplication and Triplation are contained vnder Multiplication, as it shall appeare in their place. And Mediation is contained vnder Diuision, as I will declare in his place also.

Scholar. Yet then there remaines the first seauen kindes of Numbring.

Master. So there doth: Howbeit if I shal speak exactly of the parts of numbring, I must make but fiue of them: for Progression is a compound operatiō of Addition, Multiplication, and Diuision. And so is the Ex-

traction of rootes. But it is no harme to name them as kinds seuerall, seeing they appeare to haue some seuerall working. For it forceth not so much to contend for the number of them, as for the due knowledge and practising of them.

Scholar. Then you will that I shall name them as seauen kinds distinct. But now I desire you to instruct me in the vse of each of them.

Master. So I will, but it must be done in order: for you may not learne the last as sone as the first, but you must learne them in that order, as I did rehearse them, if you will learne them spædily and well.

Scholar. Euen as you please. Then to begin; Numeration is the first in order: what shall I doe with it?

Master. First, you must know what the thing is, and then after learne the vse of the same.

Numeration.

NUmeration is that Arithmeticall skill, whereby we may duly value, expresse, and read any number or summe propounded: or else in apt figures and places set downe any number knowne or named.

Scholar

Scholar. Why then me thinketh you put a difference betwéene the value and the Figures.

Master. Yea, so doe I: For the value is one thing, and the Figures are another thing: and that commeth partly by the diuersity of Figures, but chiefly of the places wherein they be set.

Scholar. Then I must know here three things: the value, the figure, and the place.

Master. Euen so: but yet adde vnder to them as the fourth. And first marke, that there are but ten Figures that are vsed in Arithmeticke: and of those ten, one doth signifie nothing, which is made like an o, and is priuately called a Cipher, though all the other sometime be likewise named. The other nine are called signifying Figures, and be thus figured. A Cypher. Figures.

1. 2. 3. 4. 5. 6. 7. 8. 9.
And this is their value.
i. ij. iij. iiij. v. vi. vij. viij. ix.

But here you must marke, that euery Figure hath two values: One alwaies certaine, that it signifieth properly, which it hath of his forme: and the other vncertaine, which he taketh of his place:

A place is called the seate or rome that a Figure standeth in. And loke how many A place.

figures are written in one summe, so many places hath that whole number. And that must bee called the first place that is next to the right hand, and so reckoning by order towards the left hand, so that that place is last that is next to the left hand. As for example: If there stood before you six men in a row, side by side, and you should tel them as they stand in order, beginning with the man that were next to your right hand: then he that were next him should be called the second, and so forth to the furthest from your right hand, which is the sixth and the last.

Scholar. Sir, I perceiue you well: so might I reckon Letters or any other thing. As if I should write eight letters after this order, a,b,c,d,e,f,g,h, then must I say, h, is the first, g, the v.f. the iv. e. the iij. d. the v. c. the vi. b. the vij. and a. the eight.

Master. That is well done. And after the same sort vse hereafter, that what I declare by one example do you expresse by another: and so shall I perceiue whether you vnderstand it or no. And so passe ouer nothing, till you perceiue it well, and be expert therein.

Scholar. Sir, I pray you how many of these places be there in all?

Master. There is no certaine number of them, but they are sometimes more & sometimes fewer, according to the summe that is expressed. For so many as the figures are, so many

many are the places: and the laſt place is ſo called, not becauſe it is laſt of all other, but it is the laſt of that preſent ſumme, and it may be the middle place in another ſumme.

Scholar. Me ſeemeth I perceiue this very well, as touching the order of reckoning of the places: but as for the number of them, you ſay there is no certainty. Now there reſteth to declare the value of the figures by the diuerſity of places, which you called the value vncertaine. Value vncertaine.

Maſter. But firſt let mee heare whether you know perfectly the certaine value. Value certaine.

Scholar. Yes Sir, as you wrote them, ſo I marked them.

Maſter. How write you then fiue?

Scholar. By this figure 5.

Maſter. And how ſixe?

Scholar. Thus 6.

Maſter. Write theſe three numbers, each by it ſelfe, as I ſpeake them, by vij. iij.

Scholar. 7. 4. 3.

Maſter. How write you theſe foure other, ij. i. ix. viij?

Scholar. Thus (I trow) 2. 1. 6. 8.

Maſter. Nay, there you miſſe: looke on mine example againe.

Scholar. Sir, true it is, I was to blame, I tooke 6. for 9. but I will beware hereafter.

Maſter. Now then take héede, thoſe cer-

taine values euery Figure representeth when it is alone written without other Figures ioined to him. And also when it is in the first place, though many other doe follow: as for example: This Figure 9. is in standing now alone.

Scholar. How is he alone, and standeth in the middle of so many Letters?

Master. The Letters are none of his fellowes. For if you were in France in the middle of a thousand Frenchmen, if there were no Englishman with you, you would reckon your selfe to be alone.

Scholar. So it is. Then 9 without moe figures of Arithmetick, betokeneth it what. soeuer other Letters be about it.

Master. Euen so, and so doth it, if it be in the first place ioined with other, how many soeuer doe follow, as in this example, 3679. You see 9. in the first place, and doth betoken nine, as if it were alone.

Scholar. I perceiue that, and doth not 7. that standeth in the second place, betoken vij. and 6. in the third place, betoken vi. and so 3. in the fourth place betoken three?

Master. Their places be as you haue said, but their values are not so. For as in the first place euery figure betokeneth his own value certaine onely, so in the second place euery figure betokeneth his owne value certaine ten times: as in the example, 7. in

the

the second place is seauen times ten, and is, lxx. And in the third place euery figure betokeneth his owne value an hundred times, so that 6. in that place betokeneth vi, C. and in the fourth place euery figure betokeneth his owne value a M. times, as in the aforesaid number 3. in the 4. place standeth for 3. M. & in the 5 place, euery figure standeth for his own value x M times, & in the 6 place C M times, and in the 7 place a M M times, and in the eight place, x M M. so that euery place excedeth the former x. times.

Scholar. As thus: if I make this number at all aduentures, 91359684, here are eight places. In the first place is 4. & betokeneth but foure: in the second place is 8. & betokeneth ten times 8, that is 80. in the third place is 6, & betokeneth six hundred: in the fourth place 9. is nine thousand: and 5, in the fifth place is x M times 5. that is, fifty M. So 3. in the sixth place is a C M times 3. that is, CCC M. Then 1. in the seuenth place, a M M. and 9. in the eight place, ten thousand thousand times 9. that is, xc M M. But now I cannot easily nor quickly read it in order. A generall Rule.

Master. That shall you practice by this meanes. First, put a pricke ouer the fourth figure, and so ouer the seuenth. And (if you haue so many) ouer the tenth, thirteenth, sixteenth, and so forth, still leauing two figures betweene each two prickes. And those two

Ternaries

rowes betweene the prickes are called Ternaries.

Then begin at the last pricke, and see how many figures are betweene him and the end, which cannot passe three, reckoning himselfe for one: then pronounce them as if they were written alone from the rest, and at the end of their value, so many times thousands, as your numbers hath prickes.

After that, come to the next three figures, and sound them as if they were a part from the rest, and adde to their value so many times thousands, as there are prickes betweene them, e the first place of your whole number. And so doe by every other three Figures following, if you haue moe. As in example 91359684 this was your number.

Put a pricke ouer 9. in the fourth place, and ouer 1. in the seauenth place, and then no moze, (for your places come not to ten) as thus: 91359684.

Now goe to the last prick ouer 1. and take it and the Figure 9. that followeth it, and value them alone.

Scholar. 91. that is, xci.

Master. So it is, then adde for the number of your prickes twice M.

Scholar. That is, xci. thousand thousand.

Master. So it is. Then take the three other Figures from one to the next prick, and value them.

Scholar.

Scholar. 359. that is, CCC.lix.

Master. Now adde for the one pricke, that is betweene them and the first place, X.

Scholar. CCC.lix. thousand.

Master. Then come to the other three Figures that remaine.

Scholar. 684, that is, vjClxxxiiij.

Master. Now haue you valued all. And at the end of the last number you shall adde nothing, because there remaineth no pricke nor number after it: yet proue it in another number, as thus, 2308640891053 40.

Schol. 2308640891053 40. I haue pricked them as you taught me: but I am in doubt whether I haue done well or no, because of the Ciphers: for I remember you told me that they doe signifie nothing, and therefore I doubt whether I should reckon them for a figure in setting of the pricks: and againe, I know not wherefore they serue.

Master. That will I tell you now. Indeed they are of no value themselues, but they serue to make vp number of places, and so make the Figure following them to be in a further place, & therefore to signifie the more value, as in this example 90. the cipher is of no value, but yet he occupieth the first place & causeth 9. to be in the second place, & so to signifie ten times 9. that is 90. so that two ciphers thrusteth the Figure following them into the third place, and so forth. The vse of Ciphers.

Scholar.

Scholar. Then I perceiue in the example aboue, I haue pricked well enough: for though that Cypher that is pricked signifie nothing, yet must he haue the prick, because he came in the 13. place. Then will I proue to number that some. First, there 230. Ⓜ, Ⓜ, Ⓜ, Ⓜ, and then followeth 864. Ⓜ. Ⓜ. Ⓜ And what shall I now doe? There is a Cypher in the third place and no figure after him, but they that I haue reckoned.

Master. He did serue for them that you haue alreadie reckoned, to make them in a place further then they should be, if he were away: and therefore now ye shall let him goe. And so doe alwaies when he occupieth that place next before any pricke, which is the last of that Ternary, and a Cipher in the last place doth nothing.

Scholar. Then shall I say but 89. Ⓜ. Ⓜ

Master. So, but goe forth.

Scholar. 105. thousand. Now are all my prickes spent, and yet remaine 340. so that I must value them. CCC. xl. onely.

Master. Now can you recken after this sort: and remember, that euery such terme so parted, is called a Ternary or Trinity: for you haue numbred or valued the summe most truely and by the aide of the prickes each Denomination is declined most plainly

Trinity.

Denominations.

Scholar. What call you Denomination?

Master

Master. It is the last value or name added to any summe. As when I say, a hundred two & twenty pounds. Pounds is the Denomination. And likewise in saying 25. men. Men is the Denomination, and so of other: But in this place (that I spake of before) the last number of euery Ternary is the Denomination of it. As for the first ternary, the Denomination is vnites, and of the second Ternary, the Denomination is thousands: And of the third Ternary, thousand thousands, or Millions: of the fourth, thousand thousand thousands, or thousand Millions: and so forth.

Scholar. And what shall I call the value of the three Figures that may be pronounced before the Denominator as in saying: 203000000. that is, two hundred three millions. I perceiue by your words, that millions is the Denomination: but what shall I call the 000. ioyned before the Millions?

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value, or number. Numerator. Summe or value.

Scholar. Now is there any thing else to be learned in Numeration: or else haue I learned it fully?

Master. I might shew you here who were the first inuentors of this art, & the reason of all

all these things that I haue taught you, but that I will reserue till yée haue learned ouer all the practise of this Art, lest I should trouble you with ouer many things at the first.

Three kinds of numbers.

But yet this must you marke, that there are thrée kindes of Numbers, one called Digits: another Articles, & the third Mixt numbers,

Digits.

A Digit is any number vnder ten, as this: 1.2.3.4.5.6.7.8.9.

Articles.

And 10. with al other y may be diuided into ten parts iust, and nothing remaine, are called Articles, such as are 10.20.30.40.50. &c. 100.200. &c. 1000. &c.

Mixt.

And that number is called Mixt, that containeth Articles, or at the least one Article, and a Digit, as 12.16.19.21.38.107. 1005. and so forth: and for the more ease of vnderstanding and remembrance, marke this: The Digit number is neuer written with more than one figure, but the Article & the Mixt number are euer written with more then one figure. And thus they differ: that the Article hath euermore this Cypher 0 in the first place: and the Mixt number hath euer there some Digit.

Scholar. By these last words I perceiue it much better then I did before, and now (I thinke) I will neuer misse to know those thrée asunder.

Master. If you remember now all that I haue

haue said, you haue learned sufficiently this first kind of Arithmeticke, called Numeration. Howbeit, I will exhort you now to remember both this that I haue said, and all that I shall say, and to exercise your selfe in the practise of it: for rules without practise, are but a light knowledge, and practice it is, that maketh men perfect and prompt in all things.

And as you haue learned to gather and practise the value of a summe propounded, & set downe before you: so must you practice to marke, note, or write down with apt Figures and in due places, any number onely named or recited to you, or of your selfe imagined: as for a prose. How note you or write downe this summe, fiue thousand, two hundred, fifty, and seauen? Vse maketh mastery.

Scholar. This troubleth me now, whether I should begin at the first or at the last. For reason (me thinketh) should cause me to begin at the first: and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list: but the more ease for your hand is to begin with the last, that is to say, as I did speake them: yet for the more surety, a while you may begin with ỹ first, repeting my words backward thus: Seauen, fifty two hundred, fiue thousand: or els sounding them al by their digit or value,

value, as thus: seuen, fiue, two, fiue: for that way is eaſiest: But then muſt you looke wel whether there be any Cypher in your ſum, that hee may be ſet in his place. As if your laſt value of your ſum (as you call it) be aboue 9, then is there a Cipher in the firſt place. And if it be an hundred or aboue, then is there two Cyphers, one in the firſt place, and another in the ſecond, and ſo forth.

But becauſe this thing is ſuch that cannot be ſet forth without many words, I thinke beſt here now at the end of Numeration to adde a Table eaſie and ready for the firſt exerciſe of it.

Loe this is the Table.

The right side or hand.

The names of Digits, values, certaine, or vncertaine.

The denominations of the place or value vncertaine.	Vnites.	Tennes.	Hundreds.	Thousands	X. Thousands.	C. of Thousands.	Millions.	X. of Millions.	C. of Millions.	M. of Millions.	X. M. of Millions.
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eight.	8	8	8	8	8	8	8	8	8	8	8
Seuen.	7	7	7	7	7	7	7	7	7	7	7
Six.	6	6	6	6	6		6	6	6	6	6
Fiue.	5	5	5	5	5	5	5	5	5	5	5
Foure.	4	4	4	4	4	4	4	4	4	4	4
Three	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Cipher	0	0	0	0	0	0	0	0	0	0	0
The order of places.	Firſt.	Second.	Third.	Fourth.	Fift.	Sixt.	Seuenth.	Eight.	Ninth.	Tenth.	Eleuenth.

The left side or hand.

This Table (as we may ſee) hath eleuen places, and in each of them are ſet all the Digits, whoſe certaine value is written on the right hand of the Table, & the value vncertaine on the left hand, ſo that by this Table you may learne both how to expreſſe any number that you liſt, if that it exceede not

eleuen places) that is to say, ℔C. thousand millions, and so may you by helpe of it, value all summes proposed vnder the said number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred fifty and seauen. And if you wil expresse it, take the first number (as I speake it) which is fiue M. whose valuer or certaine value is v. and his vncertaine value or denomination is M. First, you shall séeke at the right hand of the valuer 5. Then séeke along vnder the title of denomination toward the left hand till you find thousands, & vnder it, right at the foot of the Table is the nũber of the place that is in the forth, wherin you must write your digit or valuer 5.

Afterward come to the second part of the number two hundred, whose value is 2. and his denomination C. Séeke two at the right hand of the Table, and goe along vnder the denomination toward the left hand, till you come vnder C: then looke to the Table, and there you shall see the number of the place, that is to say, three, wherein you must set your digit 2.

Then doe so by your other two numbers that remaine, and you shall find 5. in the second place for your fifty, & 7. in the first place for your seauen. And thus may you doe with other numbers.

Scholar,

Scholar. Master, I thanke you hartily. I perceiue you seek to instruct me most plainly & briefly, and not to hide your knowledge with subtle words as many doe. For this rule is so plaine that I can desire it no plainer. And though it seem somwhat long, yet I perceiue it to be a sure way.

Master. So it is, & though it be long, yet it is neither to long, neither to plaine for yong learners that lack practise: for this Table is instead of a teacher to them that lacke one. But now I trust I haue said enough of Numeration: which after you haue well practised, then may you learne forth,

Scholar, Yet I pray you in one thing to tell me your iudgment. Why doe men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Master. In that thing all men doe agrée, that the Chaldees, which first inuented this Art, did set these figures as they set all their Letters, for they write backeward, as you tearme it, & so doe they reade. And that may appeare in al Hebrew, Chaldee, & Arabicke Bookes, for they be not onely written from the right hand to the left, & so must be read, but also the right end of the booke is the beginning of it, whereas the Greeks, Latines, and all nations of Europe, doe write & read from the left hand toward the right: and all their Bookes begin at the left side.

Scholar. That reason doth satisfie me.

Master. It neither satisfieth me, neither liketh me well, because I see that the Chaldees & Hebrewes doe not se vse their owne numbers, as at another time I will declare. But this plaine reason may best satisfie you presently: that seeing in pronouncing of numbers we kepe the order of our owne reading, from the left hand to the right: and againe, we doe euer name the greater numbers before the smaller: it was reason that the lesser places, containing the lesser numbers, should be set on the right hand, & the greater places containing the greater numbers, to proceed toward the left hand.

Scholar. This reason is to me so plaine, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practice shall make me fully ready and expert in it. And in the meane season, I desire to learne the other kinds of Arithmeticke.

Master. That is wel said: but what should you next learne? can you tell?

Scholar. I remember you said that Addition was next.

Master. Euen so, and what that is must you first know.

Addition.

Addition.

Addition is the gathering together and bringing of two numbers or more into one summe. As if I haue 160. Bookes in the Latine tongue. & 136. in the Greeke tongue, and would know how many they be in all, I must write these two numbers one ouer another, writing the greatest number highest, so that the first Figure of the one being vnder the first figure of the other: and the second vnder the second, and so forth in order.

When you haue so done, draw vnder them a right line, then will they stand thus:

Now begin at the first places toward the right hand alwaies, and put together the two first figures of those two numbers, & looke what commeth of them, write vnder them vnder the line. As in saying 6. & 0. is 6. write 6. vnder 6. as thus:

	160
	136
	6

And then goe to the second figures, & doe likewise: as saying 3. and 6. is 9. write 9. vnder 6. and 3. as here you see.

	160
	136
	96

And likewise do you with the figures that be in the third place, saying, 1. & 1. be 2. write 2. vnder them, & then will your whole summe appeare thus:

	160
	136
	296

So that now you ſee, that 160. and 136: doe make in all 296.

Scholar. What: this is very eaſie to doe: me thinketh I can doe it euen ſince.

There came through Cheape ſide two droues of cattel: in the firſt was 848 ſheepe, and in the ſecond was 186 other beaſts.

Thoſe two ſummes I muſt write as you taught me, thus:

Then if I put the two firſt	484
figures together ſaying: 6. & 8.	186
they make 14. That muſt I	14
write vnder 6. and 8. thus:	

Maſter. Not ſo, and here are you twice deceiued. Firſt, in going about to adde together two ſummes of ſundry things, which you ought not to doe, except you ſeke onely the number of them, and care not for the things. For the ſumme that ſhould reſult of that addition ſhould be a ſumme neither of ſheepe, nor of other beaſts, but a confuſed ſumme of both. Howbeit ſometimes ye ſhal haue ſummes of diuers denominations to be added, of which I will tell you anon: but firſt I will ſhew you where you were deceiued in another point, and that was in writing 14. which came of 6: and 8. vnder 6. and 8. which is vnpoſſible: for how can two Figures of two places be written vnder one Figure and one place.

Scholar?

Scholar. Truth it is: but yet I did so vnderstand you.

Master. I said indéede, that you should write that vnder them that did result of them both together: which saying is alwaies true, if that sum do not exceed a digit. But if it be a mixt number, then must you write the digit of it vnder your Figures as you haue said before: but if it be an article then write o vnder them, & in both sorts you shall kepe the article in your mind, & therefore when you haue added your second Figures, which occupy the place of tens, you shall put that one thereto, which you kept in your mind: for though it were ten indéed, yet in that place it is but as one, because that euery one of that place is ten: for that it is the place of tens: and in like manner, if you haue in the second place so great a number that it amounteth aboue 9. then write the digit & reserue the article in your mind euer adding it to the nert place following, & so of all other places, how many soeuer you haue: And if you haue a mixt number when you haue added your last figures, thē write the digit vnder the last figures, & the article in the next place beyond them: so shall your number resulting of Additiō, haue one places more then the numbers which you shall adde together. A place.

Scholar. Now doe I perceiue you, & the reason of this, is (as I vnderstand) because

that no one place can contain aboue 9, which is the greatest figure that is, & then al tens or Articles must be put to the next place following: for euery place (as I may sæ) exceedeth the other place next before him by 10

Now (if it please you) I will returne to my example of cattel. But I remember you said I might not adde summes of sundry things together, & that I may sæ by reason.

Master. Truth it is, if you sæke the due summe of any thing, but if you onely sæke a bare súme, & haue no respect to the thing, then were it better to name the summe onely without any thing: as in saying, 848. without naming shæpe or any thing else. And likewise 186. naming nothing.

Now let mæ sæ how can you adde those two summes?

Scholar. I must first set them so, that the two first figures stand one ouer another, & the other each ons ouer his fellow of the same place, then shall I draw a line vnder them both. And so likewise of other figures, setting alwaies the greatest number highest, thus as followeth.

Then must I adde 6. to 8. which maketh 14. that is a mixt number, therefore must I take the Digit which is 4. and write it vnder 6. and 8. kæping the Article 1. in my mind thus:

848	
186	
4	

Next that, I doe come to the second figures,

figures, adding them together, saying, 8. & 4. make 12. to the which I put the one reserved in my mind, and that maketh 13. of which number I write the digit 3. vnder 8. and 4. and kæpe the Article in my minde, thus.

848
186
34

Then come I to the third figures, saying 1. and 8. make 9. and 1 in my mind maketh 10. Sir, shal I write the Cypher vnder 1. and 8?

Master. Yea.

Scholar. Then of 10. I write the cypher vnder 1. and 8. and kæpe the article in my minde.

Master. What nædeth that, seeing there follow no more Figures?

Scholar. Sir, I had forgotten, but I wil remember better hereafter. Then seeing I am come to the last figures, I must write the Cypher vnder them, and the Article in a further place after the Cypher, thus:

848
186
1034

Master. So now you see, that of 848. and 186. added together there amounteth 1034

Scholar. Now I thinke I am perfect in Addition.

Master. That wil I proue by this example. There are two armies of Souldiers: in the one are 106800. and in the other, 9400. How many are there in both armies, say you?

Scholar.

Schol. First, I set them one ouer another, beginning with the first numbers of the right hand thus:

	106800
But the nether number wil	9400

not match the ouer number.

Master. That forceth not.

Scholar. Then do I adde 0. to 0. and there amounteth 0. that must I write vnder the first place thus:

106800	
9400	
	0

Master. Well said.

Scholar. Then likewise in the second place I adde 0. to 0. and there ariseth 0. which I write vnder the second place thus:

106800	
9400	
	00

Then I come to the third place, saying, 4 & 8, make 12. of which I write the digit 2, and kéepe the article 1. in my mind, thus:

106800	
9400	
	200

Then I adde 9. to 6. which maketh 15. to that I adde the article 1, that was in my mind, and it is 16. I write 6. vnder 6. & 9. and kéepe one in mind thus:

106800	
9400	
	6200

Master. Why doe you not write both Figures, séeing you are come to the last couple of numbers?

Scholar. Nay, reason sheweth me, that I must adde that article that is in my mind vnto the next figure of the ouer sum, though there

there be no more in the nether summe.

Master. That is well considered: then doe so.

Scholar. Then say I, 0. in the ouer sum, and 1. in my mind, maketh 1. that write I vnder 0. Then followeth there yet one more in the ouer sum, which hath none to be added to it, for there is none in the nether sum, nor yet in my mind: therefore I thinke I must write that euen as it is.

Master. Yea.

Scholar. Then doth my whole summe appeare thus.

	106800
	9400
	116200

Master. If you marke this, you haue learned perfectly the commō Addition of al summes which are of one denomination: so that ye obserue this also, that in Addition you must haue two numbers at the least: or else how can you say that you doe adde? And euer let the greatest number be written highest: for that is the best way, though it be not necessary.

And forget not this, that (if you haue many numbers to adde together) you shal haue oftentimes an article of a greater value thē 10. sometimes 20. sometimes 30. somtimes more, yea (peraduenture) 100. Therefore as you did with the article 10. so do with them reserving them in your mind, and adding to the nūber next following so many, as their

value

valuer or value certaine is: that is to say 2. for 20. 3. for 30. 5. for 50, 10. for 100. 12. for 120. & so forth of other like. So that, if the article be 100. then must you set downe the 0. & kéepe 10. in mind, to be carried to the next row of figures or place, if any such happen to come. For your better vnderstanding take this example for all.

I would adde these xij. summes into one, which I set after this manner: then doe I begin and gather the summe of the first row of Figures, which commeth to 107. (for I take 9. there x. times, and that is 90.) then 9. and 8. is 17. that is in all 107. of which summe I write the 7. vnder the first row of figures, and then for that 100. is x. tens, I kéepe x. in minde, which ten I must adde vnto the next row of figures, which are in the second place:

4889	
4599	
2250	
3699	
2299	
4099	
1099	
3298	
299	
699	
499	
899	
389	

which second row of figures (when they are added together with that x. that I had in my mind) make in al 125. of which sum I write the digit 5. vnder the second row, & then (for that 120. containeth xij. tens,) I kéepe xij. in mind to be added to the third place or row of figures: which being added together, make in all 60. the cypher 0. I set downe vnder the row of figures in the third place.

And

And the figure 6. I kéepe in minde to be added to the rowe of figures in the fourth place: which (when they are added together) make 29. The figure or digit 9. I ſet downe vnder the fourth place. And, becauſe it is my laſt worke, I ſet downe the 2. Alſo that I haue in my mind to the 9. in the fift place: thoſe ſummes doe make in all 29057.

¶ But (for your more eaſe in worke) when you haue an addition of ſo many ſummes to bee added together, you were beſt part that ſumme into two or three parts, and worke them ſeuerall, and ſo put their additions together, and this were the beſt thing you could doe when ouer many ſummes fall to be added.

4889	
4559	
2290	
3699	
2299	
4099	
1099	
3298	
299	
699	
899	
499	
389	
29057	

Scholar. This ſéemeth ſomewhat hard by the reaſon of ſo many numbers together.

Howbeit, I thinke (if I doe often proue, euen with this ſmall example, either by working of it alone, or elſe by parting it as you ſaid euen now) that I ſhall be able to doe ſo ſhortly with any other ſumme.

Maſter. So ſhall you. For it is often practiſe that maketh a man quick and ripe in al things: but becauſe, aſwel in great ſummes as in ſmall, there may chance to be ſome

errour,

errour, I wil teach you how you shall proue whether you haue done well or no.

Scholar. That were a great help & ease.

The proofe of Addition.

Master. Begin first with the highest number, and then to all the other orderly, & adde them together, not hauing regard to their places, but as though they were all vnites: & still (as your number increaseth aboue 9) cast away 9. Then goe forth, euer casting away 9. as often as it amounteth thereto: and so do til you haue gone ouer al the numbers that you intended first to adde: & whatsoeuer remaineth after such additiõs & castings away of 9. write it in some void place by the end of a line, for the better remembrance, & thus is the first part of your worke proued. Then secõdly put together the figures that result of the addition vnder the line, still casting away 9 also. And then that that remaineth write at the other end of that line: and if those two figures be like, then haue you wel done: but if they be vnlike, thẽ haue you missed. As for example, in this present sum: The first figure of the ouer line is 9. let him goe: then 8. & 8 is 16. take away 9. there resteth 7. and adde that 7. to 4. that followeth, and it maketh 11. from which if you take 9. there resteth 2. Then come to the next row, whose first & second numbers are 9. therefore ouerpasse them both, & take the 5. to the 2. which did remaine in the first row, that maketh

maketh 7. put thereto the 4. following, and that maketh 11. thence take 9, and there remaineth 2. Next vnto that, goe to the third line, whose two first numbers you may let passe because they are nines: then take the two figures of 2. which (with the other two that remained in the second row) make 6. Then goe to the fourth row, whose two first numbers let goe, & take the 6. to the 6. that remained, and that maketh 12. take away 9. and there resteth 3. which with the 3. that is next, maketh 6. And so goe through al the other numbers, and you shal find that there remaineth 5. after you haue taken away 9, as often as you can find it: therefore write 5 at the end of the line in a void place, thus:

5 ———

Then gather all the figures of the totall summe, which is vnder the lowest line, and cast away 9. as often as you can find it: as thus: 7. and 5. make 12. take away 9. there resteth 3. to that if you adde the 2 that is last (for you may omit the 9) then doth it make 5. which 5. you must write at the other end of the line that you made in the void place, thus: 5 ——— 5

And then you se that those two figures be like: whereby you may know that you haue done well, & so you may proue in any other.

Scholar. (If it please you) I will proue in another summe.

Master.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First in the highest line 8. and 6. make 14. then 9. taken away, there remaine 5. to which I adde the 1 that followeth, & that maketh 6 Then come I to the second line, where I finde first 4. which with 6. maketh 10. from that I take 9. & there resteth 1. the next figure is 9. and therfore I let him alone, so find I 1 remaining which I set at the end of a line, thus:

106800
9400
110200

1——

Then I come to the totall sum, and there I find that al the figures put together, make ten, from which I take nine, and there resteth 1. also, which I put at the other end of the line thus. 1————1

And because they be like, I know that I haue well added.

Master. So, you know now both how to adde two sums or more together, and also how to proue whether you haue done well, or no: & now I will teach you how to add sums of diuers Denominations together: which thing can neuer be but when the one Denominatiō is such that it containeth the other certaine times, And yet you shal adde them to the other, not after this sort (as you did them that were of one denominatiō) but after

Addition of numbers of diuers denominations.

after such a sort as I will now shew you, that is to say:

If you haue a summe of diuers denominations, then take that you set euery denomination by himselfe, with some note or figure of his denomination, as they are wont to be written. Then write your other sums so vnder that first, that euery one be set vnder the other of the same denominations: As for example, if your denominations be pounds, shillings, and pence, write pounds vnder pounds, shilings vnder shillings, and pence vnder pence: and not shillings vnder pence, nor pence vnder pounds.

Scholar. Now that you haue spoken it, me thinketh, it néedeth not to warne me of it: for it were against reason so to confound sums: but yet, if you had not spokē of it, peraduenture I should haue bin deceiued in it.

Master. If you doe say it is so plaine, I wil speake no more of it, but with an example make the matter to appeare euidently.

First one man oweth me 22 l. 6 s. 8 d, another oweth me 45 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is altogether: Therefore must I first set downe my greatest sum, and then the other, euery one vnder his Denomination agréeing to the greatest summe, as here you sée with a line vnder them.

l	s	d
22	6	8
45	16	6
4	3	

Then muſt I begin at the ſmalleſt numbers (which muſt alwaies be ſet next to the right hand) and adde them together: and (if the ſum of them wil make 1.02,03.of the next denomination) then muſt I kéepe it in my mind til I come to that place, and vnder that firſt place muſt I note the reſidue (if there remaine any of the ſame denomination:) but, if there remaine none, then néed I to write vnder it nothing. And this is all ỹ you muſt marke in this Addition: for all other things are like to the manner of Addition before mention. *Therefore, the cheefeſt point of this Addition is, to know the values of common Coines & rated ſums.* As how many ſhillings be in a pound? how many pence in a ſhilling? of which (& of other like things) I will inſtruct you hereafter in teaching of Reduction. But now I may not diſturbe your wit from the thing that we are about.

Therefore let vs returne to that former example which I propoſed of three Debtors: which ſummes when I had ſet orderly, they ſtood thus with a line vnder them.

	£	s	d
	22	6	8
	45	16	6
	4		3

Then to adde them into one ſumme, I muſt begin at the right hand where ỹ ſmalleſt denominatio is, & adde them together, firſt ſaying, 6. and 8. make 14. Now, ſéeing theſe 14. are pence, which containe one ſhil-

ling

ling and 2. pence: the 2. pence I set downe vnder the like of pence: and the one shillinge I kæpe in my minde to carry to the next row being the place of shillings.

	£	s	d
	22	6	8
	45	16	6
	4	3	
		6	2

Then doe I adde the shillings together, saying, 1. in my mind & 3. make 4. & 6. make 10. & 6. make 16. and 1. in the second place, which standeth for 10. make 26. which is 1. pound 6. s. The 6 s I set downe vnder the place of shillings, as appeareth in the example. And the 1: pound I kæpe to carry to the pounds.

	£	s	d
	22	6	8
	45	16	6
	4	3	
	2	6	2

Then come I to the pounds, adding them all together, saying, 1. that I kæpe & 4. make 5 and 5. make 10. & 2. make 12. The figure or digit 2. I set down right vnder that place or row of pounds where I gather them: and the article 1. I kæpe to carry to the next place, saying, 1. in minde and 4. is 5. and 2 is 7. which 7. I set downe directly vnder that row also. And then appeareth my whole summe thus.

	£	s	d
	22	6	8
	45	16	6
	4	3	
	72	6	2

And thus must you doe with any such like sums whatsoeuer, whether they be money,

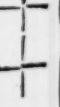
weight or measure, which (if you practise your selfe well therein, by setting downe of diuers sums) you shall be well aquainted with the feat of Addition.

But now, can you tell how to proue this Addition, or such other like of diuers Denominations, and to trie whether you haue well done or no?

Scholar. I would I could.

Master. That shall you doe by this meanes. You must make a Crosse which shall haue so many lines as you haue sundry Denominations in your Addition: As if you haue but two Denominations, then you may make it thus: that the ouer part and nether part may serue for one Denomination. And if you haue three Denominations (as pounds, shillings, and pence) then must you make three lines, thus: The vpright line may serue for pounds, and the highest thwart line for shillings, and the lowest for pence: as for example be the sum which we last wrought.

Proofe of Addition of diuers denominations.



£	s	d
22	6	8
45	16	6
4	3	0
72	6	2

For

For the proofe of the which, because it containeth three denominations, I must make a crosse of 3. lines, as in the example before. Then I reckon first at the right hand, the pence: 6. and 8. make 14. from which I take 12. for the next Denominatiō, that is to say, a shilling, and there resteth 2. which I must write at one end of the nether thwart line.

After that, I gather the summe of the shillings, 3. 16. 6. which maketh 25. to whom I put 1. that I took of the pence, & that maketh 26. from those I take 20. the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6. which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds 4. 5. and 2. which make 11. to them I adde the one that came of the shillings, & they make 12. from whence I cast 9. and there resteth 3. That 3. I ioine to the 4. in the next place, and they make 7. then the two more make 9. which I cast away, and so there is left 0. which 0. I set at the vpper end of the Crosse also: (And so should there be set also any number that were left vnder 9.) And thus is my first part of my worke proued.

That done, I come to the total summe vnder the line, & examine it, beginning at the pence, where I find but 2. and cannot take 9. from him: therefore, I set him at the other end of the nether thwart line: then I

come to the shillings, where I find onely 6. which (because it is lesse then nine, I set it at the other end of the line of shillings, that is, the ouermost thwart line.

Last of all, of the 27. l. I take eight times 9. which is 72. and there remaineth 0. which I write vnder the vpright line: either else I may reckon them simply without any respect of their valuation or place: in saying, 2. & 7 make 9. which I take away, & so resteth 0. Then I consider euery number, comparing it to the number that is against it: and, because I find them to be euery one like his match, I know that I haue well done.

Scholar. This cross I perceiue doth serue for these three denominations, pounds, shillings, pence: but what if I had l. s̄. d̄. ob. and q?

Master These lines (as I haue said) doe serue for three denominations, such as they be, as here three doe serue for pounds, shillings, & pence: but if you haue no pounds in your sum, then may they serue for shillings, pence & halfe pennies: yea, for d. ob. and q. or in weight, for C. q; and l. or, in measure, for Elles, Quarters and Nailes, if you haue no greater Denomination: so that you remember that the vpright line serueth for the greatest Denomination, and the highest thwart line for the next, and the lowest for the least.

And so, if you haue foure Denominations,

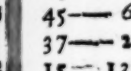
you

you muſt make your Croſſe with ſo many lines. And (if that your ſumme be of more denominations) make ſo many lines in your croſſe. And thus will I make an end of Addition, ſauing that here (for the better vnderſtanding of this rule) I haue ſet you down certain examples both of mony, waight & meaſures, with their workes and proofes.

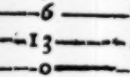
Examples of Addition.

£	s	d
23	10	4
45	6	8
37	2	9
15	13	6
121	13	3

£	s	d
130	17	10
28	6	8
13	13	4
120	0	0
192	17	10

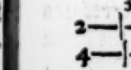


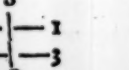
The proofes.



C.	q;	£
24	1	3
12	2	2
7	3	4
13	0	13
57	2	22

yards,	q.	nailes.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3





Subtraction.

Scholar.

Hen haue I learned the two firſt kinds of Arithmeticke: now (as I remember) doth follow Subtraction, whoſe name (mee thinketh,) doth ſound contrary to Addition.

Maſter. So it is indeed: for, as Addition increaſeth one groſſe ſum, by bringing many into one: ſo contrariwiſe Subtraction diminiſheth a groſſe ſumme by withdrawing of other from it. *So that Subtraction or Rebating, is nothing elſe but an Art to withdraw and abate one ſumme from another, that the remainer may appeare.*

ubtracti-
n.

Scholar. What doe you cal the remainer?

Maſter. That you may perceiue by the name.

Scholar. So me thinketh: but yet it is good to aſke the truth of all ſuch things, leaſt in truſting to mine owne coniecture, I be deceiued.

Maſter. So it is the ſureſt way. And (as I ſée cauſe) I will all declare things vnto you ſo plainely, that you ſhall not need to doubt. Howbeit, if I doe ouerpaſſe it ſometimes (as the manner of men is to forget the ſmall knowledge of them to whom they ſpeake:) then

then doe you put me in remembrance your-selfe, and that way is surest.

And, as for this word that you last asked me, take you this discription: The remainer is a sum left after due Subtraction made, which declareth the excesse or difference of the other two numbers: as if I would abate or subtract 14. out of 18. there should remaine 4. which is called the Remainer, and is the difference betweene those two numbers 14. and 18. Remainer.

Scholar. I perceiue then what Subtraction is: Now resteth to know the order to worke it.

Master. That shal you doe by this meanes First, you must consider, that if you should goe about to rebate, you must haue two sundry sums proposed: the first, which is your grosse sum (or sum totall:) and it must be set highest: and then the rebatement (or sum to be withdrawne) which must be set vnder the first, (whether it be in one parcell or in many) and that in such sort, that the first figures be one iust ouer another, and so the second, and third, and all other following as you did in Addition: then shall you draw vnder them a line, & so are your sums duely set to begin your working.

Then begin you at the right hand (as you did in Addition) and withdraw the neather number out of the higher, & if there remaine any

any thing, write that right vnder them beneath the line: and if there remaine nothing (by reaſō that the two figures were equal) then write vnder them a Cypher of nought. And ſo doe you with all the other Figures, euermore abating the lower out of the higher, and write vnder them the Remainer ſtil, till you come to the end. And ſo will there appeare vnder the line what remaineth of your groſſe ſum, after you haue deducted the other ſum from it, as in this example.

I receiued of your Father 48 ſ. of which I haue laid out for you 36 ſ, now would I know what doth remaine? And therefore I ſet my nūbers thus in order. Firſt, I write the greateſt ſumme, and vnder him the leſſer, ſo that the figures at the right ſide be euen one vnder another, and ſo the other, thus.

ſ
48
36

Then doe I rebate 6. out of 8. and there reſteth two, which I write vnder them right beneath the line, thus:

48
36
2

Then I goe to the ſecond figures, and do rebate 3. out of 4. where there remaineth 1. which I write vnder them right, and then the whole ſumme and operation appeareth thus.

48
36
12

Whereby

Whereby it appeareth, that if I withdraw 36. out of 48. there remaineth 12.

Scholar. Now will I proue in a greater summe, and I will subtract 2367924. out of 3468946. those sums I set in order thus.

3468946	
2367924	

Then doe I begin at the right side, and deduct 4. out of 6. & there resteth 2. which I write vnder them. Then goe I to the second figures, and withdraw 2. out of 4. and there remaineth 2. which I set vnder them also: then I take 9. out of 9. and there resteth 0. which I write vnder them (for you say that if the Figures be equall, so that nothing doe remaine, I must write this Cypher 0. vnder them.)

Master. It was well remembred: now goe forth.

Scholar. Then I come to the fourth place, and draw 7. out of 8. and there remaineth 1. which I write vnder them also: Then in the fift place I take 6. out of 6. and there resteth 0. (for if I write vnder them the Cipher 0) Then in the first place 3. rebated frō 4. there remaineth 1. which I write vnder them, & likewise in the seauenth and the last place 2. taken from 3. there is left 1.

Which I write vnder them: so haue I done my whole working, and my summes doe appeare thus. Whereby

3468946	
2367924	
1101022	

I

I ſée, that (if I doe rebate 2367924. out of 3468946.) there remaineth 1101022.

Maſter. This is well done. And that you may bée ſure to perceiue fully the Art of Subtraction, let me ſée how you can ſubtract 52984732. out of 8250003456.

Scholar. Firſt, I ſet downe the greateſt ſumme, and after that, I write vnder it the leſſer number, beginning at the right ſide: and then my figures will ſtand thus.

8250003456	
52984732	

Then take I 2. from 6. and the reſt is 4. which I write vnder them: then doe I withdraw 3. from 5. and there remaines 2. which I write vnder them. Then take I 7. out of 4. but that I cannot, what ſhall I now doe?

Note. Maſter. Marke well what I ſhall tel you now, how you ſhall doe in this caſe, and in all other the like: if any figure of the nether ſumme bee greater then the figure of the ſumme that is ouer him (ſo that it cannot be taken out of the figure ouer him) then muſt you put 10. to the ouer figure, and then conſider how much it is, and out of that whole ſumme withdraw the nether figure, and write the reſt vnder them. Can you remember this?

Scholar. Yes, that I truſt I ſhall. Now then in mine example where I ſhould haue taken 7. out of 4. and could not, I put 10. to that

that 4. which maketh 14. from it I take away 7. and there resteth 7. also, which I write vnder them.

Master. So haue you done wel, but now must you marke another thing also: that (whensoeuer you doe so put ten to any figure of the ouer number) you must adde one still to the figure or place that followeth next in the nether line: as in this example there followeth 4. to which you must put 1. and make him 5. and then goe on as I haue taught you.

Scholar. Then shall I say, 4. & 1. (which I must put to him for the 10. that I added to 4. before) make 5. which I should take out of 3. but that cannot be: therefore I must put to it also 10. and then it wil be 13. from which I take 5. and there resteth 8. to be written vnder them: and because of that 10. added to the 3. I must adde 1. to 8. that followeth in the nether line. & that maketh 9. which I should take out of 0. and cannot: therefore, I put thereto 10. and that maketh 10. from 10 I take 9. and there remaineth 1. which I write vnder them.

Then doe I adde one likewise to the next figure beneath, which is 9. and that maketh 80. that 10. should I take out of the figure aboue, but I cannot: for it is 0. therefore I put 10. to it, & so take I 10 out of 10. & there resteth 0. to be written vnder them.

Then

Then come I to the next figure which is 2. and to him I doe adde 1. which maketh 3 that 3: I cannot take out of nought: therefore of that nought I make 10. and thence doe take 3. so there remaineth 7. to be written vnder them: likewise doe I put 1. to 5. that followeth, and then it is 6. that would I take not of 5. I cannot, therfore I adde 10 to that 5. and make it 15. from which I rebate 6. and there remaineth 9 which I write vnder them. Now haue I spent all the nether figures, and what shall I doe more?

Master. You should haue added one to the nert figure following (if there had bene any) because you added 10. to the last figure before the ouer line: but seeing there is no figure following, you must adde that one to the place following, & then deduct that one from the number aboue.

Scholar. Then shal I say, because I borrowed 10. to the ouer 5. I must put 1. in the next place beneath, that is vnder 2, then must I subtract that 1. from 2. & there resteth 1. to be written vnder that in the ninth place. Now I haue no more to subtract, for there is not any figure remaining beneath, neither yet any Vnite to be added, because I borrowed not 10. to the figure last before: & yet is there 8. remaining in the ouer line, which I thinke (by reason) should be set at the end of the figures in the lowest row,

which

which is vnder the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indeed.

Scholar. But Sir, I beseech you, shall I alwaies when any number so remaineth alone as this 8. did) write him vnder the line straight against his owne place?

Master. Yea, what else? Whether they be one or many: and this wel remembred, you haue sufficiently learned Subtraction: howbeit, because of certaine things that might deceiue you, if you did not take good héed to your working, I will propose to you another example of many numbers to be subtracted as thus.

I receiued of a friend of mine to kéep 2869. crownes, of which at one time I deliuered him again 500. at another time 368. at another time 440. at another time 80. and another time 64 now would I know how many doe rest behind? Therefore first I set downe my grosse sum and a line vnder it: 2869 *crowns receiued*

and vnderneath it I set al the parcels thus, & vnder them a double line. 500 / 368 / 440 / 80 / 64 *Deliuered.*

Then first I begin at the first place, & gather together the sum—

of

of all those lines (saue the ouermost) in their first figures: & so I doe with all the figures of the second place, and so forth, as I did in Addition, saue that I leaue out the highest row of numbers (as the line warneth mée) & that sum so gathered betwéen the double line, is the sum deliuered in all: which sum I doe afterwards subtract out of the highest row of numbers, & the remainer do I set vnder the nethermost line: as for example.

Note.

I set the summes as before: then doe I gather the first Figures of al the places deliuered, together: where I find but 4. and 8. that make 12. (for thée Cyphers increase no summe in Addition, as you learned before:) of the 12. therefore, doe I write the Digit 2. betwéene the double line, and kéepe the Article in my minde, till I come to the second place, where I find 6.8.4.6. that maketh 24 to them I put the Article in my mind, & it is 25. of which I write 5. vnder the secõd place & kéepe the Digit 2. in my mind for the third place, where I find 4.3.5. that makes 12. to the which I adde the 2. in my minde: and it maketh 14. thereof I write the 4. vnder the third

2869 *Crowns receiued*		
500	}	*Deliuered.*
368		
440		
80		
64		
1452 *Deliuered in all.*		
1417 *Rest behind.*		

third place: because there remaines no more figures to be added, I write the Digit in the fourth place, as you see in the example: and so it appeareth, I haue deliuered in all, a thousand foure hundred fifty two Crownes.

Then come I to the subtracting of this summe betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceede to subtract it from the ouermost summe, saying 2. from 9. remaine 7. to bee written vnder them beneath the lowest line. Then in the second place I take 5. from 6. and there resteth 1. to bee written vnder them. Then in the third place 4. from 8, resteth 4. Last of all, in the fourth place 1. from 2. remaineth 1. And thus I see that after those 5. summes are subtracted from 2869. the Remainer is 1417.

Master. This is a very sure and easie way for a learner: therefore I counsel you to practise it well.

Scholar. This I perceiue: but is there no shorter way and more speedy?

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest sum. But, if in quantity those numbers added together exceede the highest summe or vpper number, then shall you (as before hath been taught you) imagin to borrow 10. 20. or 30. more, as neede shall

An abridgment of the former manner of Subtracting.

require, & put them to the vpper number, to helpe to further the abatement, restruing or restoring the Articles that you borrowed to the next place again: & so still goe forward till you haue ended your worke: as for example. In the last sum proposed, I gather first in the first place 4 & 8. that maketh 12. which 12. I should deduct or take out of 9. in the vpper number aboue the line, but I cannot: therfore I adde vnto 9. an article of 10 and that maketh the vpper number 19. from whence I take 12. then there resteth 7. then for the Article 10. I adde to the next place of mony delinered: saying, 1. that I bring & 6 make 7. & 8. make 15. & 4. make 19 & 6 make 25. which 25. I should take out of 6. in the vpper number, but I cannot. Therfore I adde 2. tens or 20. vnto 6. in the vpper number, & that maketh 26. then 25. out of 26. resteth 1. then the 2 tens which I borrowed, or haue in mind, I adde to the next row or sum delinered, saying: 2. that I bring, & 4. make 6. & 3 make 9. & 5. make 14. then 14. out of 8. I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to be added, the one that I borrowed or haue in mind, I rebate from 2. in the vpper line, & there remaineth 1. which I set downe in the remainer line, and so my sum appeareth (as before) to be 1417. Crownes.

Loe thus haue you now a shorter way.

Scholar.

Scholar. I like both waies wel, and I perceiue both well: yet, as in the one the working seemeth somewhat long, so in the other it leaueth very much (me seemeth) to remembrance, and therfore may cause errour quickly, except a man haue a quicke and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will giue me leaue) I will try what I can doe in such a like sum, to worke it the shortest way: whereupon I would subtract out of 40301964 these three parcels.

	40 01964
20003428	20003428 |
10002432	10002432 |
10101461	10101461 |

Therfore I set them first in due order: then I gather the parcels of the first place, which are 8. 2. 1. that is a 11. which I should take or deduct out of 4. which is ouer him, but I cannot: therfore I adde an article or one ten to 4. which maketh 14. then 11 out of 14. therefore resteth 3. to be written vnder the 1. place betweene the 2 lines.

Then come I to the second place, saying, 1. that I borrowed to haue in mind, and 6. make 7. and 3. make 10. and 2. make 12. which I cannot take from 6. therefore I adde 10. to 6. which maketh 16. and then 12. from 16. resteth 4. which I write vnder

the second place betwéne two lines.

Then come I to the third place, saying: I that I borrowed or haue in mind, & 4. make 5. & 4. is 9. and 4. make 13. which I should take out of 9. that is ouer them, but I cannot. therefore I adde 10. to 9. which make 19. then 13. out of 19. rest 6.

Then come I to the 4 place, saying: 1. in minde & 1. is 2. and 2. is 4. and 3. make 7. which because it cannot be taken from 1. I take it from 11. and there rests 4.

After that, I come to the fift place, where are onely thre Ciphers, which make nothing vnto which I adde 1. in mind, then should I take that (that is to say) 1. from the figure ouer them, which is also a Cipher: therfore I say thus: I cannot take 1. from 0. but 1. from 10 remaiuete 9. so must I write 9. vnder them. Then in the first place I find but 1 which I take out of 3. ouer him, & the remainer is 2. that must be written betwéne the two lines in the 6 place. So I goe to the 7, where I find only Ciphers, & in the grosse sum ouer them a Cipher also: therfore must I write the remainer (wch is nothing) with a Cipher also. Then in the eight & last place I gather 1, 1, 2. that maketh 4. which if I take out of that 4. that is ouer them, there will nothing remaine. And that must be noted with a Cirher betwén ỹ 2 lines (as I haue often said) & so haue I ended my worke, and

the figures ſtand thus.

	40301964
	20003428
	10002432
	10101461
	00194643

But Sir, I remember you taught me that Cyphers ſhould not come in the laſt place, for becauſe they ſerue onely to increaſe the value of other figures which follow them, & ſerue not thoſe figures that goe before them: and now in my example I haue ſet two Cyphers in the two laſt places.

Maſter. I commend you for your remembrance. And truth it is, you ſhould not haue ſet thē here, but onely becauſe that I would make you plainly to perceiue the art of Subtraction. Therefore ſeing that you do now perceiue it, whenſoeuer you would write downe a Cipher, looke whether any other figures be yet behind: and if not; then let goe the 0. alſo, for it needeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now ſuffer you) to teach the vſe of Subſtraction the plainer.

Therefore your figures muſt ſtand thus whē the worke is ended.

40301964.	*Charge.*
20003428	*Diſchar.*
10002432	
10101461	
194643	

Scholar. Sir, I doe think with that that you taught me before, and by theſe two ſummes that you

taught me last also, that now I could subtract any summe.

Master. So may you, if you haue marked what I haue taught you. But, because this thing (as all other) must bee learned surely by often practise, I will propound here two examples to you: wherein if you often exercise your selfe, you shall bee ripe and perfect to subtract any other summe lightly, for in them is contained all the obseruances of whole numbers. And because you shall perceiue somewhat both how to doe it, and also whether it be wel done when you haue proued to doe it, therefore haue I written vnder them both the Remainers.

30696. *Lent.*		30606. *Debt.*	
10249	*Repaid.*	10354	*Paid.*
10249		10249	
163		163	
20661. *Paid in all.*		9840. *Rest.*	

9840 *Rest to pay.*

Scholar. Sir, I thanke you: but I thinke I might the better doe it, if you did shew me the working of it.

Master. Yes, but you must proue your selfe to doe some things without any aid, or else you shal not be able to do any more then you are taught: And that were rather to

learne

learne by rote (as they call it) then by reason. And againe, there is nothing in this example, or any other of whole number, but I haue taught you the rules of them already, sauing I haue not yet shewed you how you shall proue this worke.

Sch. Sir, I pray you then shew it me also.

Master. For the performance whereof if you marke well what I said in Addition, you may easily perceiue what may be done for the proofe of Subtraction, and that is only performed by the aid of Addition, thus: Draw vnder the lowest number, which is your Remainer a line, and then adde the summe paid in all (which before was made by the particulars) and the remainer together. And if these twaine being added together in one summe doe make the contents of the vpper number aboue the line, then is this Subtraction well wrought, or else not. Proofe of Subtraction.

For the triall whereof I will propone one of our examples which are done, which was of 2869. Crownes receiued to keepe of a friend.

Of which I deliuered these particular summes. 500, 368, 440, 80, 64

Which by Additiõ make. 1452 *deliuerd in al.*

So resteth to my friend 1417 *rest to pay.*

2869 *Proofe.*

Now put or adde this remainer, and the summe deliuered together, saying, 7. and 2. make 9. and 1. and 5. make 6. and 4. and 4. is 8. and 1. and 1 is 2. which in one sum amounteth to 2869. and is equall with the summe 2869. aboue the line: whereby I know the subtraction is well wrought: and this is the order and proofe hereof, and of all such like workes of Subtraction.

Scholar. Then I trust by practise to attaine the vse of it. And this is all that I shall learne of Subtraction?

Master. Yea, sauing that (as you haue seene in Additiou) there are numbers of diuers Denominations, in which the working is not much vnlike: yet (without some instructions be giuen of it) it might seeme to a learner more difficult than indeede it is. Therefore I will briefly shew you the vse of it onely by one example or two.

A certaine man owed to me 14 *l*, 2 *s*, 8 *d*, *of which he paid me at one time* 4 *l*, 6 *s*, 8 *d*: *at another time* 3 *l*: *at another* 2 *l*, 3 *s*, 4 *d*: *and last of all*, 6 *s*. 8 *d*.

Now would I know what remaineth vnpaid yet? therefore I set my summes thus, euery one in their due place: As pounds vnder pounds, shillings vnder shillings, pence vnder pence.

l	*s*	*d*
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholar.

Scholar. Sir, I pray you why doe you write 2. l. for the common speech vseth rather to say, 40 s.

Master. We must here vse the Denomination that is greatest in any sum, so that wee may not write according as wee vse to speake, saying, 16 d. 18 d. or likewise 7. groats, 8. groats, 24 s. 40 s. 48 s. and such other: but we must write euery Denomination that is in any summe by it selfe.

Note how the Penne differeth from the common order of Counters.

Namely, shillings and pounds. So must wee write for the last summes now named, 1 s. 4 d. 1 s. 6 d. 2 s. 4 d 2 s. 8 d. 1 l. 4 s. 2 l. 8 s. and so forth of other like:

Scholar. So that we may not write in Arithmeticke, pence, when the sum amounteth to shillings, nor shillings, when the summe maketh pounds. Now, (if it please you) end your example.

Master. When my summes are so set as I shewed, then (according to the rules of Addition) I gather all the particular summes which be paide mee into one totall summe, directly to be set vnder them betweene the two lines, not medling with the 14 l. 12 s. 8 d, as the line warneth mee: therefore must I first begin with the smallest Denomination, saying, 8, 4. 8 is 20. pence, which maketh one shilling & 8. pence.

the

the 8 d. I set downe under the place of pence, and the one shilling I kepe in mind to carry to the next denominations of shillings. Then come I to the shillings, and say, one that I bring or have in minde, and 6. is 7. and 3. is 10. and 6. makes 16. which, because it containeth not one pound, I set directly under the place of shillings. Then come I to the pounds, whose parcels are 2, 3. 4. that is in all 9. that 9. doe I set downe directly under the pounds: And so the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

	l	s	d
	14	12	8
	4	6	8
	3	—	—
	2	3	4
		6	8
	9	16	8
	4	16	0 *Rest.*

Now, for the worke of Subtraction, I must rebate that totall summe of Addition out of the highest number, that is to say, from the 14 l. 12 s. 8 d.

Therefore to performe the worke, I say, 8 d. out of 8 d. remaineth or resteth nothing, therefore, in the place of the rest or remaine, right under the denomination, I set downe 0. Then comming to the shillings where I find 16. which should be taken out of 12. but I cannot: therefore I imagine to borrow 1. of the next Denomination, that is,

is, of the 9 l. and put that one pound so borrowed vnto 12 s. that maketh 32 s.

Now 16 s. out of 32 s resteth 16 s. which 16 s. I set downe directly vnder the place of the rest.

Lastly, comming to the pounds, saying one pound in mind that I borrowed, and 9. make 10. then 10. out of 14. there resteth 4. So doth my whole rest or remaine appeare to be 4 l. 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as they are set in order, beginning with the pence, saying, 8. 4. 8. make 20 d. which 20 d. you should take out of the 8 d. above the line, but you cannot, therefore that you borrow 1. of the next denominati-

l	s	d
14:	12	—8
4	—6	—8
3	—0	—0
2	—3	—4
0	—6	—8
9	—16	—8

on, that is to say 1. of the shillings, & put it to the 8 d. that maketh 20 d. now 20 d. out of 20 d. resteth 0. which Cipher I set downe directly vnder them.

Then one shilling that I borrowed or had

in

in mind, & 6. make 7. and 3. make 10. and 6. make 16. then 16. out of 12. I cannot take, therefore of the next Denomination I doe borrow one ℔ and put it to 12 s. which maketh 32 s then 16 s. out of 32 s. resteth 16 s.

Lastly, I come to the pounds, saying: 1 ℔. in mind, or that I borrowed, and 2. make 3. and 3. is 6, & 4 is 10. then 10. out of 14. there resteth 4.

So doth my remainer or rest appeare as before to the 4 ℔. 16 s. 0 d.

Scholar. This doe I perceiue very well, and if there bee none other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckoned to be next in order.

A proofe of Subtraction in numbers of one denomination.

Master. We haue done indeed with the art of Subtraction, as touching the working. But yet before we goe to Multiplication I wil instruct you how to examine your worke, whether it be well done or not. For the performance wherof, if you marke what I said right now in the last manner of proofe, you may easily perceiue what is to be done for this proofe, which is onely made (as before was taught you) by the aid of Additiō thus

Draw vnder the lowest number (which is your Remainer) a line, and then adde the summe paid in all, and the Remainer together. And if these twaine added together in one summe make the contents of the vpper

number

number aboue the line, which in this example is 14 l. 12 s. 8 d. then is the subtraction well wrought, or else not.

As for example, in our first summe, which stood thus.

	l	s	d
	14	12	8
	4	6	8
	3	0	0
	2	3	0
		6	8
Paid in all.	9	16	8
Rest.	4	16	0
	14	12	8

Where, in the title of pence, I find 8. and 0. the 8. I set downe directly vnder him in that place of pence. Then in the place of shillings I find 16. & 16. which make 32. shillings, wherein is contained 1 l. and 12 s. the 12 s. I set down directly vnder them in their due place of shillings, and one pound I kéepe.

Then comming to the pounds, saying, 1. that I kéepe, & 4. is 5. and 9. is 14. which 14. in due order I set downe directly vnder the as this figure sheweth. Which is also 14 l. 12 s. 8 d. agréeing with the vpper number aboue. So I find the worke is good, and the Subtraction well wrought.

Now for the proofe of the latter Subtraction, wherein we reckoned for the shorter worke: Draw vnder the Remainer also a line, then adde that remainer & all the other numbers that ye did subtract before together, & write that that amounteth vnder the lowest line, and

Proofe in Subtraction of diuers denominations.

and if the ſumme that commeth thereof be equall to the higheſt number aboue, then is the Subtraction well wrought, or elſe not.

As for example alſo in the laſt ſummes which ſtood thus.

	℔	s̄	d̄
	14	12	8
	4	6	8
	3	0	0
	2	3	4
	0	6	8
	4	16	0
	14	12	8

Firſt, in the title of pence I adde 8. 4. 8. that maketh 20 d̄. which containeth one ſhilling and 8. pence. The 8. I ſet downe vnder the loweſt line in the row or title of pence, and that one ſhilling I kéep to carry to the next Denomination or place of ſhillings.

Then returning to the ſhillings, ſaying: one in minde, or that I kéepe, and 16. make 17. and 6. make 23. and 3. make 26. and 6. make 32 ſhillings, which amounteth to one pound, 12 s̄. the 12 s̄. I ſet downe vnder the title of ſhillings, and 1 pound I kéep or haue in mind to carry to the nert Denomination or place of pounds. Then come I to the pounds, ſaying, 1. that I bring and 4. make 5. and 2, make 7. & 3. is 10. and 4. make 14. then doe I write 14 vnder the pounds, & ſo haue I ended the Addition. and I ſée that the loweſt line is like vnto the vppermoſt line in number, wherefore I know that I haue well done.

And thus new haue I taught you the Art of

of Subiraction, and the meanes to proue whether it be well wrought or not.

Many other worke and proofes might be shewed of Subtraction, but one of these, whether of them you please, are most apteſt, and best allowed of any other worke or proofe, whether it be of l, s, d, or any other grosse summe whatsoeuer.

Scholar. Sir, I thanke you most hartily, for now I vnderstand well both the feate of Subtraction and his proofe.

Master. Therefore now will I make an end of Subtraction, and wil instruct you in Multiplication.

Multiplication.

Multiplication is an operation that two summes produce the third, which third ſuame ſo many times ſhall containe the firſt, as there are Vnites in the ſecond. And it ſerueth in ſteade of many Additions. As for example. When I would know many are 30. times 48: if I ſhould adde 48. thirty times, it would be a long worke. Therefore was this worke of Multiplication deuiſed, which ſhall doe that at once that Addition ſhould doe at many times.

Muliplication what it is.

Scholar.

Scholar. I perceiue the commodity of it partly, but I shall not see the full profit of it till I know the whole vse of it. Therefore Sir, I beseech you, teach mee the working of it.

Multiplycation of Digits.

Master. So I iudge it best, not because that great sums cannot be multiplied, but by the Multiplication of Digits, therefore I thinke it best to shew you first the way of multiplying them: As when I say, 8. times 8 or 8. times 9. &c. And as for the small digits vnder 5. it were but folly to teach any rule, seeing they are so easy that euery childe can doe it. but for the Multipication of the greater Digits, thus shall you doe,

First, set your Digits one ouer the other right then from the vppermost downeward, & from the nethermost vpward, draw straight lines, so that they make a crosse, commonly called Saint Andrewes crosse, as you see here. Then loke how many each of them lacketh of 10. and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7. times 8. I must write those Digits thus.

Digit difference

8 7

The difference.

Then doe I loke how much 8 doth differ from 10 and I find it to be 2, that I doe

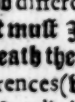
doe I write at the right hand of 8, at the end of the line thus.

After that I take the difference of 7 likewise from 10. that is 3, and I write that at the right side of 7. as you see in this example.

Then do I draw a line vnder them, as in Addition thus.

Last of al, I multiply the two differences, saying 2, times 3, make 6. that must I euer set vnder the differences, beneath the line: then must I take 1. of the differences (which I wil, for al is like) from the other digit (not from his owne) as the lines of the Crosse warne me, and that that is left, must I write vnder the digits. As in this example if I take 2, from 7, or 3, from 8, there remaineth 5. that 5, must I write vnder the digits, & then there appeareth the multiplication of 7. times 8. to be 56. And so likewise of any other Digits, if they be aboue 5. for if they be vnder 5 then will their differences be greater then themselues, so that they cannot be taken out of them. And againe, such little summes euery child can multiply, as to say: 2, times 3,

Digit differēce.

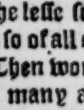


or 4 times 5. and such like.

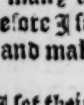
Scholar. Truth it is. And seeing me seemeth that I understand the multiplying of the greater digits, I will proue by an example how I can do it. I would know how many are nine times 6.

Master. It is all one in value to say nine times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and so of all other summes.

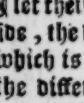
Scholar. Then would I know how many are 6 times 9, therefore I set the digits thus, and make the crosse thus.



Then do I set their differences from 10. at the right side, the difference of 9, which is 1, against it, & the difference of 6, which is 4, against it also, as in this example.



And vnder them I draw a line. Then doe I multiply the digits together, saying: one time 4 maketh 4, that 4 doe I write vnder the differences thus.



Then take I one of the differences from the other digit, as one from 6, or else 4 from 9, & each waies there resteth 5, which I do write vn-

der ẏ digits thus. And so appeareth the multiplication of 6 times 9 to be 54. Thus I see the feat of this manner of multiplication of digits.

9	1
6	4
5	4

Master. Now might you goe straight to the multiplication of great numbers, saue that both for your ease and suretie in working I will draw you here a Table, whereby shall appeare the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	5[illegible]	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seeke your first or last Digit in the greater figures, and from it go right forth toward the right hand, till you come vnder the number of your second Digit, which is in the highest rowe, and then the number that is in the meeting of the rowes of little squares (which

come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this table the multiplication of 7. times 9. séeke first 7. in the greater figures, & then goe right forth toward the right hand, till you come vnder 9. of the highest row, in which place where you so come vnder the other digit (as here for example you come vnder 9.) is alwaies contained the ofcome or product, which you séeke: and that place we terme to bee in the common angle, in respect of the two numbers so taken on the outsides, as here in that common angle, where the rowes of little squares (directly proceeding from 7 & 9. do méet, you haue 63, which 63. is the summe of the multiplication of 9 by 7.

To multiply greater sums.

Scholar. This is very good & ready. And so may I find the multiplicatiō of any digits: but now how shal I do in greater summes?

Master. *When you would multiply any sum by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number vnder it. for that is the place of the Multiplier or Multiplicator, that is to say the number by which Multiplication is made, and is in English alwaies put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times, is*

Multiplier.

Times.

that which must be multiplied.

Therefore when I would multiply one number by another, I must write the greatest highest, & the lesser vnder it, as in Addition. And vnder them must I draw a line. As for example: If I would multipy 264. by 29, I must set them thus.

264
29

¶ Of which numbers thus set downe to be multiplied, may be formed a question, as thus. There are 2. men, and each man hath 264 Lambes. The question is, how many Lambs they haue in all.

To the performance wherof, I must multiply euery figure of the higher row, by euery figure of the nether row: and that that amounteth, I must set vnder the line. as thus

First I doe multiply 4 by 9, saying: 9 times 4 (or 4 times 9 which is al one) & that maketh 36, as the Table breefe of digits doth declare, of that 36 I must write the 6. that is the Digit, vnder the 9, and the Article 3. I keepe in mind to carry to the next place.

264
29
6

Then come I to the second figure of the higher row, which is 6, and say: 9. times 6. make 54, and with the 3. in me mind make 57 the 7 I set downe vnder the 2, and 5 I keepe in minde.

264
29
76

after that I come to the next figure, which is 2, and multiply it by , and that maketh 18, and with 5 that I haue in mind maketh 23: wherfore because it is the last worke of the Multiplier, I set downe in order as you see:

264
29
2376

And so haue I ended the first figure of the Multiplier. Wherfore I giue it now a fine dash with my pen.

Then begin I with the next figure, and multiply it into all the higher figures as thus:

264
29
2370
8

First, 2 times 4 make 8, that 8 doe I write vnder the second place: for euermore the Digit or first figure of the Multiplication that amounteth of the first figure of the higher number, must be set vnder the Multiplier of it, and the other in their order toward the left hand.

Scholer. I vnderstand you thus, that the Digit of the sum amounting of the Multiplication of the first figure of the higher row, by the first figure of the lower row, or Multiplier, must be set vnder the first place: and that that amounteth of the same first figure by the second Multiplier must be set vnder the second place, and so of the other, if there be more Multipliers.

Master. So meane I indeed: and if there amount but a Digit, then must it be set vnder the Multipler.

And

And now to goe forth: I multiply by the same 2, the second figure of the higher row, which is 6, saying: two times 6, make 12: whereof I write the digit 2 vnder the third place, and the Article 1, I kéepe in minde.

Then doe I multiply the last figure of the higher summe by that same 2, saying: two times 2 is 4, and with the 1 that I haue in mind maketh 5, which 5 I write vnder the fourth place. And so haue I ended the whole Multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as soone as I haue dispatched any Digit, by which I multiply: and the summes stand thus.

Then must I draw a line vnder all those summes that mount of the multiplication, and must adde all them into one summe, as in the example you may see.

264	
29	
2376	
528	
7656	

Where in the first place I find but 6, and therefore write I it vnder the line. Then in the second place 8 and 7 make 15, whereof I write 5, and kéepe one in my mind, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amounteth of the Multiplication of 264.

by 29. & that is the iust number of Lambes the 29. men had.

Scholar. If there be no more to be obserued in it, then can I doe it, I suppose, as by this example I shall proue.

¶ There is a peece of ground which containeth 1365. yards in length: and 236 yards in breadth, I would know how many yards square there is in all this peece of ground, which numbers I set downe with the greater aboue, and the lesser vnder, as you see.

	1365
	236

Then doe I multiply 5 by 6. saying: 6. times 5, make 30. of which I write the cipher in the first place and the Article 3, I doe kepe in minde to carry to the next place.

	1365
	236
	0

Then doe I by the same 6, multiply the second figure of the higher sum, which is 6. saying, 6 times 6, make 36, and 3, in my mind make 39, of which I write the 9, vnder the second place, & the Article 3, I kepe in minde.

	1365
	236
	90

Then doe I multiply the third figure, which is 3, by the same 6, and that maketh 18, and with 3, in my minde make 21. Then 1. I set downe, and kepe 2, in mind.

	1365
	236
	190

Then come I to the last figure of the higher

summe,

ſumme, & multiply it by 6, ſaying, 6 times 1, make 6, and 2, in my mind make 8, that 8, doe I write vnder the fourth place. And ſo haue I ended the firſt Multiplier and waſh him ſleightly with my pen.

1365
236
8190

Then beginne I with the ſecond multiplier, and ſay firſt 3, times 5, that maketh 15, of which I ſet the 5, vnder the ſecond place, becauſe that the Multiplier is there, and the Article 1, I kæpe in minde.

1365
236
8190

Then come I to the ſecond Figure, that is 6, and multiply it by 3, which maketh 18, and with one in minde maketh 19, the 9, I ſet down vnder the third place, and 1 I kæpe in minde.

1365
236
8190
95

Then come I to the third Figure, which is 3, and multiply it by 3, ſaying: three times three make 9, and with 1, in my minde make 10, the Cypher I ſet vnder the fourth place, and the Article I kæpe in minde.

1365
236
8190
095

And then comming to the laſt figure 1, I multiply it by 3: and it maketh 3, and with the one in mind it maketh 4, which 4, I ſet in the fift place, and then haue I ended

1365
236
8190
4095

ended two of the Multipliers, & the summes stand as you may see in the latter end of the page going before: & then I giue 3 his dash.

1365
236
8190
4095
0

Then come I to the third Multiplier, and multiply it into every figure of the higher sum, and first I say: 2 times 5 make 10, of which I set the Cypher vnder the Multiplier in the third place, and the Article 1 I keepe in minde.

And so multiplying the second figure 6, by that same 2, there amounteth 12: & with one in my minde make 13. whereof I write the Digit 3 vnder the fourth place, and the Article 1 I keepe in mind.

1365
246
8190
4095
30

Then doe I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6. and with the one in minde make 7. which 7, I set downe vnder the fift place, as appeareth by the example.

1365
263
8190
4095
730

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the first place, and then doth the summe stand thus.

1365
236
8190
4095
2750

And so haue I ended the whole

whole Multiplication.

But now (as you taught me) to know what this whole sum is, I must adde all those parcels together, and then vnder the line will appeare, as you may see, the grosse or totall summe is, 322140. Where-by I know there is so many yards square in that peece of ground.

	1365
	236
	8190
	4095
	2730
	322140

Master. This is well done.

Scholar. Then, me thinketh I could call it well done, when I know, whether I had well done or no.

Master. It is to be proued by 9, as Addition was, but the surest proofe is by diuision, and therefore I will reserue that proofe by diuision, till you haue learned the Art of Diuision. And anone I will shew you how it is commonly proued.

¶ *But first, for your further instruction in this exercise of Multiplication, I will with one example more try your cunning and so make an end: And the question is this. I would know how many daies it is since the Natiuitie of our Lord and Sauiour Iesus Christ vnto the yeere 1590. Which to performe, you must multiply by the daies in one whole yeere, which are 365.*

Scholar. Now, for that you haue giuen me so

so much light into the question, you shall see I will handsomely finish the worke, for according to your former instructions I set them downe with a line vnder them, thus.

1590
365

Then say I, 5 times 0. is 0. the 0. I set downe vnder the first place, as here appeareth. Then say I 5, times 9, make 45, the digit 5, I set downe in the second place vnder 9, & the article 4. I kéepe in mind to be added to the next multiplication. Then saying fiue times 5, make 25, & 4, in mind makes 29. the 9, I set downe in the third place, & 2, I kéepe in minde. Then comming to the last figure, I say once fiue is 5, or fiue times 1, is 5, & 2, in mind make 7, that 7, doe I set downe vnder the fourth place: and thus haue I ended my first Multiplier, and therefore I giue it a dash with my pen:

1590
365
7950

Then come I to the second Multiplier, which is 6, and doe likewise multiply it into the vpper number, saying, 6, times 0, is 0. the 0. I set downe in the second place right vnder his Multiplier: then say I, 6, times 9, make 54, the 4 I set downe vnder the third place, and 5 I kéepe in mind. Then say I 6, times 5, make 30. and 5. I kéepe in mind make 35, the digit 5, I set downe in the fourth place, & 3. I kéepe

1590
365
7950
9540

kæpe in minde. Then say I, 6 times 1 is 6: or once 6 is 6, and 3 in mind make 9, which I set downe next, and so haue I ended two Multipliers: wherfore I dash the 6 with my penne.

Then I begin to multiply the third Multiplier into the ouer number, saying, 3 times 0. is 0.: the 0. I set downe in the 3 place right vnder his Multiplier. Then say I, 3 times 9 make 27, the digit 7 I set downe in order next, & 2 I kæpe in minde, then say, I 3 times 5 is 15, & 2 in mind is 17. the 7 I set downe, & 1 I kæpe. Lastly I say, once 3 is 3. & 1 I kæpe is 4, which I set downe orderly next: And so haue I ended ỹ Multiplication, and my figures stand thus.

	1590
	365
	7950
	9540
	4770

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one sum. it will be 580350, which is the grosse or totall summe of that Multiplication, and declareth the number of dates since our Lord and Sauiour his incarnation. vnto the end of 1590 yæres, beside 397 dates and twelue houres for leape yæres.

Scholar. This is maruelous me thinketh, that such great matters may so easily be atchieued by this Art, which heretofore I euer thought had bæne impossible, as infi-

nite

nite sorts of people are of that minde.

Master. Truth it is, that knowledge hath no greater enemy than ignorance, for this is one of the least of tenne thousand things that may be done by this Art, as hereafter you shall be able to iustifie.

Scholar. This manner of Multiplication I perceiue, if there be no more in it.

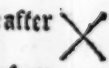
Master. *Yes, there are other formes and helpes for ease and shorter labour of the worke of Multiplication: but I will remit them till you haue a little tasted Diuision, where also the like helpe into Diuision may be vsed: and so therefore vnder one example for both, will I shew you both ease in Multiplication, and also in Diuision.*

But sith the other formes and workings do nothing differ from these workes in effect, but onely in setting of the numbers, I will ouerpasse them till a more meet place and time. And now will I instruct you in Diuision, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholar. Yes, Sir, I thanke you, but I do not perceiue how to examine my worke, to trie whether I haue well done, or no: therefore as you promised me ere-while, I pray you first shew me how I shall proue it.

Master. *That is commonly vsed by the proofe of 9. as you learned before in Addition, sauing that it differeth from that forme in diuers*

diuers respects: as for example.

First, you must make a crosse after this manner. 

Then must you examine your sum that should be multiplied, & loke what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoeuer remaineth in it after casting away 9 so oftē as you can, write that at ỳ other side of the crosse: thē must you multiply those two numbers together, & loke what amounteth thereof, if it be vnder 9, write at the higher part of the crosse: but, if it be aboue 9, then take then 9 as often as ye can, and write the rest at the head of the crosse: as first for example, we will proue the example you put forth of the pece of ground that contained 1365 yards in length, and 236 yards in breadth. Proofe of multiplication.

Therefore first, I cast away the nines from the summe to be multiplied, saying: 5 and 6 make 11, cast away 9, rest 2: then 3 and 2 makes 5, and 1 is 6: that 6 I write at the one side of the crosse thus. 6

Then do I examine the Multiplier, which is 236, wherein, when the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the crosse. 2 6

Then doe I multiply 6 by 2, and it maketh

maketh 12, from which 12, I withdraw 9, then resteth 3, that 3, doe I set at the head of the crosse. Then doe I examine the grosse summe amounting of the multiplication, which is 322140, where I find 9, once, & 3, remaining; that 3. I set at the foote of the crosse, & then I see it to agree with the other 3, at the top of the crosse, & so know I that I haue done well for if they two did differ, then were my worke vaine, and the Multiplication false.

This is the common proofe, but the most certaine proofe is by Diuision, of which I will anone instruct you.

A sure proofe of Multiplication.

Scholar. Sir, what is the chiefe vse of Multiplication?

Master. The vse of it is greater then you can yet vnderstand: howbeit, these plaine commodities it hath, that if you would resolue any great and whole value into many smal & lesse portions, as if you would change pounds into shillings, pence, or any other greater or smaller parcels by Multiplication, ye shal doe it spedily & easily. Also if you should need to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more spedily, readily, easily, and surely, then by often and sundry Additions. Take you these commodities grossely shewed for an answer at this time,

and

and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Scholar.

WEll Sir, then in Diuision I pray you to instruct me. But me thinketh by the name of it, that it should be all one with Multiplication: for I call that Diuision, when any thing is parted into diuers and manie parts.

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceiue that it is quite contrary to Multiplication, and doth not part any thing or few things into many, but contrary wayes, it bringeth many parcels into few, but yet so, that these few taken together, are equall in value to the other many: for by Diuision, pence are turned into shillings and shillings into pounds: As for example, of 120 shillings it maketh 6 pounds, so are 120 turned into 6. which is a smaller number: but then if you consider the Denominators, you shall see that they are such that

one of the latter is equall to 20. of the first, and so in value the summes are one, though in number they doe farre differ, and the latter summe is the lesser, and so it is alwaies in Diuision: howbeit, yet in the working, the summe is parted by another, and therefore doth it take the name.

Scholar. I thinke I shall better vnderstand the reason of the name when I know the vse of the worke, therefore now would I gladly learne that.

Diuision what it is

Master. *Diuision is distributing of a greater summe by the vnites of a lesser. Or Diuision is an Arithmeticall producing of a third number, in respect of two propounded numbers, which third number shall so often containe an vnite, as the greater of the two propounded numbers can containe the lesser.* So that euen as Multiplication did séeme to serue instead of many Additions, so Diuision may séeme to be in place of many Subtractions: Because that third number briefly expresseth how many times the lesser of your two propounded numbers may be subtracted from the greater: as in practise will more plainely appeare. Therefore (as you may perceiue) vnto Diuision are required three numbers: the first, which should be diuided, and that must (generally) be the greater: and the second, by which the other must be diuided, & that is (generally) the lesser, and is called the

the Diuisor. And the third, which answereth to the question (How many times?) and therefore is called the Quotient.

The first must be first written, and the second so set vnder it, that the last figure of the lower number be right vnder the last of the higher, contrariwise to the worke of the other kinds of Arithmeticke: for in them the two first figures were set euer mét one vnder the other: but in Diuision the last figures must be set mét, except it chance so that the last figure of the Diuisor be greater then the last of the higher number, for then you shall set the last of the Diuisor vnder the last, saue one of the higher number, as for example. A generall rule for placing the figure. An exception.

If you should diuide 365 (which are the summe of the daies of a yéere) by 28, which are the daies of a common moneth, then should you set them thus.

365	
28	

But if you would diuide those 365 daies, by 52, which is the number of wéekes in one yéere, then should you set them thus,

365	
52	

Likewise, if I would diuide the same 365 by 4, which is the summe of the quarters of the yéere, then must I set them thus.

365	
4	

Scholar. Sir, this doe I vnderstand, but how now should I doe to diuide the one by the other?

Master. You must begin with the last figure next the left hand, and see how many times the last figure of the Diuisor may be taken out of the last figure of the other number, and that shall you note within a croked line toward your right hand. As for example. I would diuide 365 by 28, then set I those two sums thus.

365(
28

And I loke how many times I may find 2. (which is the last figure of the Diuisor) in 3, (which is the last of the number to be diuided) & considering that I can take 2 out of 3 but once, I make a croked line at the right hand of the numbers, and within it I set 1, & that is called ỳ Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 ouer 3, and deface or cancell the 3 and the 2, then will the figures stand thus:

Quotient number.

365(1
28

Then come I to the next figure of the Diuisor, & take it likewise so many times out of the figures that be ouer it, and loke what doth remaine, that I must write ouer them, and cancell them, as in this example.

Therefore now doe I take once 8 out of 16, and there remaineth 8, which I must set ouer the 6, and cancell or crosse out the 16 and the 8 of the Diuisor: and then will

will the figures ſtand thus.

And ſo haue I once wrought. 18

Scholar. So I perceiue that 365 (1
you take the nether figure, not 28
onely out of the other that is right ouer him, but out of that with the other alſo that remaineth before, and are written toward the left hand.

Maſter. So muſt you doe: for you muſt ſo take the Diuiſor out of the ouer number, that there remaine not ouer it ſo great a ſumme as it ſelfe is, for then were your worke in vaine.

But yet againe here muſt you marke, that when you ſeeke how many times the laſt figure of the Diuiſor may bee found in the number ouer him, that you looke alſo whether you may as often find al the figures following in thoſe that are aboue them (conſidering all the remainers, if there be any) if not, take your Quotient leſſe by one, and then proue againe, and ſo ſtill till you find a meet quotient: and by that meete Quotient muſt you alwaies multiply your Diuiſor, & the product ſet vnder your Diuiſor, ſo that the firſt figure ſtand vnder the firſt figure of your Diuiſor, and the ſecond vnder the ſecond, & ſo forth: and then ſubtract that product frō the number to be diuided that ſtandeth directly ouer it, as you haue ſeen me do.

When you haue thus wrought once, then

must you begin again, & write your Diuisor a new, neerer toward the right hand by one place, as in this example, you shal set 2 vnder 8, & 8 vnder 5 thus.

```
1 8
3 6 5 (1
2 8 8
  2
```

Then (as before) seeke how many times you may take your diuisor out of the number ouer him now.

Scholar. That may I do here 4 times.

Master. Truth it is, that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

Mark how to consider this kinde of Remainer.

Master. Well, then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following make 25, in which sum I find 8 three times also: & therefore I take 3 as a true quotient, and write it within the crooked line of the quotient before ẏ 1 thus.

```
  2
1 8
3 6 5 (13
2 8 8
  2
```

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therfore I cancel the 8, and write ouer it the 2 that doth remaine, thus:

Then

Then do I take 8 as many times out of 25, saying: 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1, so then I cancell 25 and 8, and ouer the 5 I set 1, thus.

Or, you might (after you sould 3 to be a fit quotient) straightway haue multiplied the whole Diuisor 28, by that 3 at once: which giueth 84, which being set vnder 28, and duely subtracted from 85, of the number diuided, giueth 1, the remainer of the whole Diuison, as before you had. Work which way you list, here you see also the forme.

x	
x 8 1	
3 6 5	(13
2 8 8	
x	

And now haue I done with the diuiding, for I cannot finde my Diuisor 28 no more in the ouer summe.

Scholar. No, except you worke part the 1 that remaineth into 28 parts.

Master. That is wel said, and so must we do in such cases, when there remaineth any thing: but I will let that passe now, and wil make you perfect in diuision of whole numbers, and will hereafter teach you particularly of broken numbers, called Fractions.

Now if you doe perceiue the order of Diuision, then do you diuide this sum 136280 by 452.

Scholar. First, I set downe the number

that should be diuided, then doe I set the Diuisor vnder it, so that the last figure of it be right vnder the last figure of the ouer number. Then will it bee thus.

	136280
	452

Master. Can you take the last of your Diuisor (which is 4) out of 1, which is the last of the ouer number?

Scholar. I had forgotten because the last of the Diuisor cannot be taken out of the last of the ouer number, in so much as it is the greater, therfore must I set the Diuisor one place more forward toward the right hand thus.

	136280
	452

And then must I loke how often I may find the last figure of the Diuisor (that is 4) in 13. which thing I may doe 3 times; therfore doe I say: 3 times 4 is 12: which I take out of 13, and there remaineth 1. Then do I make at the right hand of my summes a crooked line, & write before it my Quotient 3, and I cancell 13 and 4, and ouer the 3 I set the 1 that remaineth, and then the figures stand thus:

1	
x36280(3	
452	

Then doe I multiply the same Quotient into euery figure of the Diuisor, & withdraw the summe that amounteth out of the numbers

bers ouer them, as first I say: 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth thus.

```
 ~~1~~1
~~136~~280(3
 ~~45~~2
```

Then doe I say likewise, 3 times 2 make 6, which I take out of 12, and there resteth 6, therefore I cancel the 12 and the 2, and ouer the 2 I write the 6 that remaineth thus.

```
 ~~11~~6
~~236~~280(3
 ~~45~~2
```

Then should I set forward the Deuisor into the next place toward the right hand thus.

```
 ~~11~~6
~~136~~280(3
 ~~45~~22
  45
```

Master. But you may see that ouer the 4 is no figure, therefore I must set the Deuisor yet forwarder by another place.

And marke, whensoeuer it chanceth so, that you should set forward the Diuisor, and that it cannot stand there, because there is no number ouer the last place, or if there be any, it is lesser then the last figure of the Diuisor, then must you remoue the diuisor yet once again: and because that his first place of removing serued not to subtract him so much as once, therfore you shal write in the quotient a ciphero: & if you should by chance need to do so oft times, for euery time write

a cipher

cipher in the Quotient. The reason of this will I shew you hereafter.

Scholar. Then must I set my summes thus.

x 1 6
x 3 6 2 8 0 (30
4 5 2 2
4 5

And, because I remoued the Diuisor, so that I ouerskipped one place, I must write a Cypher in the quotient: and then must I take a new quotient, as in this example I must say; How many times 4. is there in 6? (and sith it can bee but once) therefore doe I write 1 in the quotient; and then say I, 1 times 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 ouer them thus.

x
x 1 6 3
x 3 6 2 8 0 (301
4 5 2 2 2
4 5 5
4

Then say I againe once 5 out of 28, remaineth 23. I let the 2 stand as it did, and ouer that 8 I set 3, cancelling the 8, and the 5, vnder it, thus.

x
x 1 6 3
x 3 6 2 8 0 (301
4 5 2 2 2
4 5 5
4

Master. You might as well haue said, once 5 out of 8, and so remaineth 3, but now goe forward.

Scholar

Scholar. Then once 2. out of 0. cannot be: What shall I now doe?

Master. Borrow of the next number that is behind (for there is 230) and doe as you learned in Subtraction in a like case.

Scholar. Then must I borrow 1, of the 3. comming behind next, and make that 0. to be 10. and then take I 2. out of 10. and there resteth 8. And because I borrowed one of the 3. I must cancell the 3. and write 2. ouer it: then doth the Figure stand thus:

```
      2
  11638
136280(301
  45222
   455
     4
```

Master. Now haue you done, and yet remaineth 228. and your Quotient sheweth you that if you diuide 136280. by 452. you shal find your Diuisor in your greater number 301. that is CCC times and once, and 228. remaining.

And in the other example (where I diuided 3695. by 28. the Quotient was 13. and 1. remained, whereby I knew that in a yéere (which containeth 365 daies) there are 13. moneths, reckoning 28 daies (or 4 wéekes) iust to a moneth, and 1 day more.

Scholar. Why then doe wee call a yéere but twelue moneths?

Master. Of that at a more conuenient time will I fully instruct you: but now it is not conuenient to intangle your minde with

other

ther things then doe directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short manner of Diuision, which I would haue you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other propper points touching it.

Scholar. Then I pray you tell mee how I shal examine and try my worke, whether I haue done well or no, that though no man be by me to tell me, yet I may perceiue it my selfe.

Proofe of Diuision.] Master. Some men (yea and commonly most) doe trie it by the rule of 9 as in all the other kinds, saue that their order is: First, they cast away 9 as often as they can out of the Diuisor, and that remaineth they set at one side of a crosse, as in our first example the Diuisor was 28 from which you may take 9, three times, and one remaineth: which they set by a Crosse thus:

Then doe they likewise examin the Quotient (which in our example is 13) and from thence they cast away 9. as often as they can, and the remainder they set at the other side of the crosse, and then multiply they together those two remainers: and to it that amounteth they adde the remainer of the Diuision, if there were any: from that whole sum they withdraw 9. as often as they can,

and

and the reſt they ſet at the head of the croſſe, as in our example the quotient is 13, from which take 9. & there remaineth onely 4. and therefore muſt you ſet 4. at the other ſide of the Croſſe thus:

Then multiply 4 by 1, and it yéeldeth but 4: thereto adde the remainer of the Diuiſion (which was 1) & it wil be 5. which ſum doth not amount to 9, and therefore muſt be ſet wholly at the head of the croſſe, as you ſée heere.

And this number on the head of the croſſe is the firſt proofe, to which if you find another like in the number that was diuided, then haue you done well.

Therefore now ſhall you likewiſe examine the whole ſum that was diuided, and take away 9 as often as you can, & that that remaineth, ſet at the foot of the croſſe: and if it be equall to that in the head of the croſſe, then haue you done well, elſe not:

As in our example the whole ſumme was 365 which maketh 14: from that take 9, and there reſteth 5, which ſet at the foote of the croſſe, thus:

And you ſhall ſée that they agrée: therefore haue you well done.

Now will I likewiſe examine our ſecond example,

er am.ple, where the Diuiſor was 452, which maketh 113, from thence I take 9, and the 2 that remaineth I ſet at the right ſide of the croſſe thus:

Then eramin I the quotient, which was 301, where I find but onely 4, that doe I ſet at the other ſide of the croſſe, thus:

Then doe I multiply 4 by 2, and it maketh 8: to that doe I adde the remainer of the Diuiſion, (which was 228, & maketh 12) and they two make 20: wherin I find twice 9, and 2 remaining: that 2 muſt I ſet at the head of the croſſe, thus:

Then doe I eramine the whole number to be diuided, which was 136280, where I find twice 9 and 2 remaining, which I ſet at the foote of the croſſe, thus:

And becauſe that it doth agree with the figure at the head of the croſſe, I know that the Diuiſion was well wrought.

Againe of the Proofe of Diuiſion

Maſter. This the common proofe. Howbeit, the more certaine working is by the contrary kind: as, to proue Diuiſion by Muitiplication, thus:

Multply the quotient by the Diuiſor, and if

if the sum that amounteth be equall to the sum that should be diuided, then haue you well diuided: else not.

How be it, this must you marke, that if there remained any thing after the Diuisiō, that must you adde to the sum that amounteth of the Multiplication. As in our first example our quotient was 13, and the Diuisor was 28: Now multiply the one by the other and the sum will be 464: to that if you adde the 1 that remained after the Diuision, then will it be 365, which was the summe that should be diuided: and therfore I know that I haue well done.

Scholar. Now will I proue the same in the second example, whose Diuisor was 452 and the quotient 301: these doe I multiply together, and there amounteth 136052: to which if I addethe 228 that remained, then will it be 136280, which was the whole summe to be diuided: and therefore I perceiue that I haue well done.

Master. This is the surest way, to examine Diuision by Multiplication: and contrariwise the surest proofe of Multiplication is by Diuision.

And therfore (according to my promise) now will I shew you how you may proue Multiplication by Diuision. A proofe of Multiplication by Diuision.

When you haue ended Multiplication, & would know whether you haue well done

or not, set the grosse sum that amounteth of the Multiplication ouermost, and diuide it by the multiplier: and if the Qotient be the same number that should be multiplied, then haue you well wrought, else not, as in that example where wee multiplied 264 by 29, the grosse sum was 7656.

Now if you will know whether that Multiplication be true, you shall diuide that 7656 by the multiplier 29, and you shal perceiue that the Quotient will be 264, and that is a token that you haue well wrought.

Scholar. By your patience I will proue that, and first set downe the grosse sum and the multiplier, not after the rule of Multiplication, but after the rule of Diuision, for now that number is become

the Diuisor, that was before	7656
the Multiplier, I shall set	29

them therefore thus:

Then shall I seeke how many times 2 in 7: that may be 3 times, and 1 remaineth: but then may not 9 be found so often in 16, therfore must I take a lesser Quotient, that is to say 2: then say I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then doe I cancell 7 and 2, and ouer 7 I write 3, and in the Quotient I set 2: so the figures stand thus:

3	
7656	(2
29	

Then say I forth, two times 9 make 18, which

which I abate out of 36, and there resteth 18: then cancell I 3, and ouer him set 1, and likeise I cancell 6 and 9, and ouer them I set 8. so that thus stand the Figures:

	1
	38
	7656(2
	29

Then doe I set forward the Diuisor by one place, & seeke a new Quotient, that is to say, how many times 2, are in 18, which I find to be 9 times: but then can I not find 9 so many times in 5, therefore I take a lesser quotient, as to say 8: but yet that is too great: for if I take 8 times 2 out of 18 there remaineth but 2, and I cannot find 8 times 9 in 25: therefore yet I take a lesser quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 25, therefore yet I seeke a lesser quotient, as to say 6: then say I, 6 times 2, make 12, that I take out of 18, and there remaineth 6. so I cancel 18, and the 2, and write 6 ouer 8 thus:

	x6
	38
	7656(26
	299
	2

Then say I forth, 6 times 9 maketh 54, that take I out of 65, and there remaineth 11 and the Figures stand thus:

	1
	x6
	381

Then must I set forth the diuisor againe, & seeke a new quotient, which will be 4: for

	7656(26
	299
	2

I though

though I may find 2 in 11, 5 times, & 1 remaine, yet I cannot find 9 so often in 6. therefore I set the figures thus:

And the 4 in the Quotient I multiply into the figures of the Diuisor, saying, foure times 2 maketh 8, which I take out of 11, & there resteth 3: therefore I cancel the 11, & the 2, and set 3 ouer the first place of 11, thus:

And then doe I say forth: 4 times 9 maketh 36, which I take from 36, & there remaineth nothing: so that the Quotient of this Diuision also (where 7656 is diuided by 29) is 264. Which doth declare, that if 264 be multiplied by 29, the sum will be 7656. And thus I perceiue now how both Multiplication is proued by Diuision, and Diuision also by Multiplication.

Master. Now haue I ended the fiue common kinds of Arithmeticke. For (as touching Mediation, Duplation, Triplation, & such other) they are no seuerall kinds of Arithmeticke, but are contained vnder the other. For Mediation is contained vnder Diuision, & is nothing else but diuiding by 2, & so are Duplation & Triplation contained vnder Multiplication: for Duplation is nothing els but multiplying by 2, & Triplation

is multiplying by 3, of which I will onely propose an example for the rules you haue heard already.

If you would mediate or diuide into 2, this sum 4531010. you shal set 2 for the Diuisor, and worke as you learned before, as thus:

4531010 (2265505

2

An example of Mediation.

Then I find 2 in 4 two times: therefore my Quotient must be 2, so I cancel 4 and 2 and remoue the Diuisor forward thus, as the worke requireth, and as before in Diuision hath beene declared.

4531010 (2265505

xxxxxxx

Which mediation or diuision by 2 being finished, you shall haue for your Quotient 2265505, which is the halfe of 4531010, as you may trie by Duplation, for, double that Quotient, or multiply it by 2, and the same number will amount.

Duplation

I will no longer tarry about these, seeing they are but members of the other kinds. But here now (according to my promise) I will teach you certaine easie formes both of Multiplication and of Diuision. And first of Multiplication.

If you would therefore multiply any sum by 10, you shall need to do no more but adde a cipher before his first place: as for example, 36 multiplied by 10 make 360,

Easie formes of of Multiplication.

Likewise if you would multiply any sum by 100, put two ciphers at his beginning So if you would multiply any sum by 1000,

adde thꝛée Cyphers to the beginning of it.

Scholar. This doe I well perceiue, and also the reason of it.

Master. I will omit all reasons till our nert mæting, when I shall tell you the reason of all other parts of Arithmetick also: & as to our matter now, loke, as I haue told you, that you both remember it, and also often pꝛactise it.

And now you haue learned how to multiply easily by 10, 100, 1000: & of like manner may you doe with any other of that sort.

But now if you will multiply by 20, 30, 40, & so forth, oꝛ by 200, 300, and such like, where there is one Cipher in the first place, oꝛ many oꝛderly in the first places, you shall take away those Cyphers, and multiply the sum onely by the other figure, oꝛ figures (if they be many) and then at the begining of the sum that amounteth you shall set so many Cyphers as you toke away.

Example of 2873, which I would multiply by 300. First I omit the 2 Cyphers from the Multiplier, and I multiply the sum by onely thꝛée that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted oꝛ toke away, & then is it 861900. And that is the sum that amounteth when 2873 is multiplied 300.

Scholar. And if there were two oꝛ more figures beside the cyphers, I must only take away

away the Cyphers, & multiply by the other figures, as I learned before: as if I would multiply 93648 by 25000, I should take away the three Diphers, and multiply the same by 25, and then at the begining of that total sum should I add the 3 Ciphers againe.

Master. Euen so: but if it chance the number that should be multiplied, or both the summes, as wel the number that should be multiplied, as ẏ Multiplier, to haue Ciphers en their first places, euermore omit the Ciphers, & worke by the rest. But remember to restore as many cyphers to ẏ amounting sum as you bated before, as in this example: 30200 shall be multiplied by 206, I shall onely take away the two Ciphers from the greater number, and then multiply 302 by 206, and afterward adde the two Ciphers againe. But if I would multiply the same 30200 by 2060, I shall not onely take away the two cipbers from the number that should be multiplied, but also I may take away the one cipher from the Multiplrer and then must I adde 3 Ciphers to the sum that amounteth: but take hēd ẏ āno take away no Cipher that cometh after any signifing figure, as in the last example, you may not take away that in the fourth place of the higher number, neither ẏ in the third place of ẏ Multiplier: howbe it, yet thus you may doe: If one Ciper or more come in the midst

of your summes, you may multiply by the other Figures, and ouerskip them: but so, that you giue euery Figure his due places as thus: I will multiply 3026 by 2004 therefore I set them thus:

3026	
2004	
12104	
52	

And thus I doe multiply them: First 4 times 6 make 24. I set the 4 vnder the first place and kéepe the 2 still in my mind. Then say I againe, 4 times 2 maketh 8, and the 2 that is in my mind maketh 10. I set downe the cipher 0, & kéep the article 1 in my mind Then 4 times 0 is 0, and the 1 in my mind maketh 1. I set downe the figure 1, & say againe, 4 times 3 is 12. I set downe 2, & kéeping the 1 still in my mind (hauing no more places of the vpper number to multiply it withal) I put it down next 2 in the 5 place.

But now when I come to the next place (being a cipher 0) I let it go, because it multiplieth nothing, & likewise the second cipher.

But then, when I doe come to the 2, and multiply into the 6 of the ouer number, you must take héed (according as I taught you in Multiplication) that the first number amounting of the Multiplication be set right vnder the Multiplier, & the other orderly toward the left hand, according as you may see in this example

3026	
2004	
12104	
6052	
6064104	

Which

which being finished, with the addition thereof gathered together, will stand as this example sheweth.

Which is indeed wrought so much the sooner and shorter by ouerskipping of the 2 ciphers: which otherwise (if the same example were wrought at length) it would haue had 2 woorkings more, as by the same example here also set downe both appeare.	3026 2004 12104 0000 0000 6052 6064104

Scholar. Sir I thanke you; for I see great ease in this way of Multiplication: and if you can shew me such like in Diuision) you shall greatly further me.

Easy formes of Diuision

Master. Yes, I will teach you some easie waies in Diuision also, and first this: If you would diuide any summe by 10, you shall onely with your pen make a square line betweene the first figure of your sum and the second, & then haue you done: for the whole number that followeth the line standeth for the Quotient, and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, 364|8

Where 364 is the Quotient, and betokeneth that so many times are 10 in 3648, & the 8 after the line is the remainer, which cannot be diuided into 10, but by breaking it

into fractions, wherewith I wil not meddle yet.

And so likewise if you would diuide any summe by 100 with your pen, you shall cut away the two first figures, & if you would diuide by 1000, you must cut away the three first figures, & so of any other Diuisor, whose last figure is 1, & the other ciphers, loke how many ciphers the diuisor hath, and so many figures at the beginning shall you cut away with the square line, & they stand alwaies for the remainer, because they are lesse then the Diuisor, and cannot be diuided by it, and the other Figures that are behind the line stand for the quotient.

But now if your Diuisor haue any other figure in his last place then 1, and in all his other places haue cyphers, loke how many ciphers they be, cut away so many of the first figures of the number that should bee diuided, and diuide the rest that followeth the line by that Figure that is in the last place, as if it were the whole Diuisor.

Example of 64284, which I would diuide by 300, here must I cut away the two first figures (for so many Cyphers my diuisor hath) and must diuide the rest by 3, which is the figure in the last place of the Diuisor: First therefore I part away the two first figures, & the summe standeth thus:

642 (48

Then

Then doe I diuide 642 by 3, & the quotient is 214: for in 6 I find twice 3, and in 4 once, & 1 remaining, which 1 with the 2 next before doth make 12: wherin I find 3 foure times: and this is a ready way to turne shillings into pound: for sith one pound doth containe 20 shillings, I must diuide the whole number of shillings by 20. Therfore easily to do it, I see it that my diuisor hath one cipher, & therfore I cut away one figure from the beginning of the whole summe of shillings, & then I do mediate or diuide by 2 the other figures or sum that followeth.

Scholar. I will put an example.

If you would diuide 64287 shillings by 20: that is to say, if I would turne so many shillings into pounds, I must cut away the first figure, that is 7, & diuide the rest, that is 6428 by 2, so shall the quotient be 3214, wherby I know that 64287 shillings make 3214 pounds, and 7 shillings remaining.

Master. Now proue by Multiplication whether you haue done well or no.

Scholar. The quotient is 3214, which I doe multiply by the Diuisor 2, and it doth amount to 6428.

Master. Hereby you may perceiue not onely that you haue well done, but also how by Diuision you may turne shillings easily into pounds: and conerariwise by Multiplication you may turne pounds into shillings.

But

But here shall you see amongst diuersmen diuers formes of such diuision: but if you marke what I haue told you, you shall perceiue easily all the waies. For some men do not cut away so many of the first figures of the summe that they would diuide as there are ciphers in the first places of of the Diuisor: but they set all their ciphers orderly vnder the first places of the number that they would diuide; & then with the other figure or figures (if there be many) they diuide the rest of their sum. Example If they would diuide 725931 by 3400, they doe set their summes thus:

Another maner of the abridgement.

725931	725931
3400,	34 00

And then do they diuide orderly till they come to the ciphers: for there they stay and end their worke, as in this example.

They seeke how often 3 may be found in 7, which is two times, & 1 remaining: therfore they set 2 in the quotient, and cancell 3 and 7, and 3 ouer 7 they set the 1 that remained thus:

1	
725931 (2	
34 00	

Then doe I goe forth saying, two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus:

14	
725931 (2	
34 00	

Then renew they the diuisor forward, & seeke how often 3 may be found in 4, which is

is but once, and 1 remaineth, then set they 1 in the Quotient, and cancel 3 and 4, and ouer them they set that 1, thus.

```
  1
 1 4
7 2 5 9 3 1 (21
3 4 4  0 0
  3
```

Then take they once 4 out of 15, and ther resteth 11. Or else more easily: Take once 4 of 5 and there resteth 1, so they cancell the 4 and 5, and set 1 ouer them, thus.

```
  1
 1 4 1
7 2 5 9 3 1 (21
3 4 4  0 0
  3
```

Then set they forth the Diuisor againe, & seeke how many times three, are in 11, which they finde three times, & 2 remaining: so they set 3 in the Quotient, and cancell 11 and 3, & ouer them set 2, thus.

```
  1 2
 1 4 1
7 2 5 9 3 1 (213
3 4 4 4 0 0
  3 3
```

Then doe they multiply 4 by 3, which maketh 12, that withdraw they out of 29, and there resteth 17, of which the 7 must be set ouer the 9, and the 1 ouer the 2, thus.

```
  1
 1 2
1 4 1 7
7 2 5 9 3 1 (213
3 4 4 4 0 0
  3 3
```

And now are ye two ciphers next ensuing, so that the Diuisor can no more be set forward, and therefore is the Diuision ended,

and

and the remainer is 1731.

Now the quotient, which is 213, doth declare, that if you diuide 725931, by 3400, you shall finde it therein 213 times, & there remaineth 1731: so shall you finde it, if you worke as I taught you, by cutting away the 2 first figures, because of the 2 ciphers.

But this must you marke (as you may perceiue by this last example,) that if there be left any other Remainer in the sum that was behinde the squire line, that the Remainer must be set to the latter end of the first Remainer, which was cut away with the squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, then would your summes appeare thus.

Note.

	1	
	xx	
x4x	7	
72 39	21	(213
3444		
33		

So that 17 which remaineth after the line, must be set to the 31 (ỹ was cut away with the line) in higher places, as you see here: where that 17 with the 31, doe make 1731.

¶ And heere will I make an end of Diuision, (sauing that I do request you to exercise your selfe wel herein by many summes, till you haue attained some expertnesse therin.) *For the reasons and conclusions thereof are so many, and so availeable for all sorts of men whatsoeuer, that if I should speake of the in-*

finite

finite vses thereof, I should rather lack words than matter. And therefore recommending it to your iudgement hereafter, vpon your further trauell into the Art, I will here end this Treatise, representing vnto you one example or simple question of Diuision & Multiplication in stead of many which is this.

There are foure brasse peeces: The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, & the fourth 2 pounds. They are all appointed against the battery of a hold, & there is allowed by the Master gunner 700 pounds of powder to be spent by these foure peeces in this assault. The question is twofold: The first how many shot each peece shall iustly make about with this 700 pounds of poweer? And lastly how many pounds of powder ought iustly to be allowed to each peece for his true proportion?

Scholar. Why sir, you make me smile, to beare me in hand, that these two demands may be simply resolued by Multiplication and Diuision.

Master. Truly that they may: and that you may by and by worke your selfe with a little labour: First add together their quantities of powder, ỹ is 9 pounds, 5 pounds, 4 pounds, & 2 pounds, all which make 20: Diuide the 700 pounds of powder by that 20. & your quotient giueth 35, as here appeareth, which

which sheweth for most certainty that they shall make iust 35 shottes about.

x
700
200 (35
z

Scholar. Sir, all this haue I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To trie the truth of the same, multiply ỳ first peece that spends 9 pounds by 35, and you shal see his allowance, which is 315 pounds of powder. Multiply also the second peece that spends 5 pounds by 35, and you shall finde 175 pounds his allowance: then 4 by 35, and you shall finde 140 pounds his allowance. Lastly, multiply 2 by 35, and you shal finde 70 pounds his allowance. All which foure particular sums you shall adde together by Addition, as here appeareth, & it maketh iust 700 pounds, and so is the question truely absolued.

315
175
140
70
700

Scholar. Truly sir, these excellent conclusions do wonderfully moue and moue make me in loue with the Art.

Master. It is an Art that the further you trauell, ỳ more you thirst to go on forward. Such a fountaine that the more you draw ỳ more it springs. And to speake absolutely in one word (excepting the study of diuinity which is the saluatiō of our soules) there is no

no ſtudy in the world comparable to this for delight in wonderful and godly exercise: For the ſkill hereof is well knowne immediately to haue flowed from the wiſedome of God into the heart of man, whom he hath created the chiefe image and inſtrument of his praiſe and glory.

Scholar. The deſire of knowledge doth greatly encourage me to be ſtudious herein: and therefore I pray you ceaſe not to inſtruct me further in the vſe hereof.

Maſter. With a good will. And now therefore for the further vſe of theſe two later, that is, Multiplication and Diuiſion; I will briefely ſhew you the feate of Reduction.

Reduction.

REduction is, *by which all ſummes of groſſe denomination may be turned into ſums of more ſubtile denomination.* And contrariwiſe, all ſummes of ſubtile denomination, may be brought to ſummes of groſſer denomination. Reduction what it is.

Scholar. What call you groſſe denomination and ſubtill denomination.

Maſter. That I cal a groſſe denomination which doth containe vnder it many other ſubtiler. Groſſe denomination.

subtiller or smaller: as a pound (in respect to shillings) is a grosse Denomination; for it is greater than shillings, and containeth many of them. And shillings (in comparison to pounds) are a subtile denomination, for because they are lesser then pounds, and many of them are contained in one of the other: and so likewise of other things, whatsoever thing is compared to other, if it be a greater, & containeth many of them, it is a grosser denomination: but if it be lesser (so that many of them are in the other) (then are they called the subtile denominations: whereby you may perceiue that one denomination may be called a grosse denomination, and also a subtile (that is to say a great & a small) in diuers comparisons. For shillings compared to pounds, are a subtile or smal denominatiō: but compared to pēce, they are a grosse or great denomination.

Subtile denomination.

Scholar. Now I vnderstand the name, I pray you teach me the vse.

Master. The vse is easily learned, if you remember what you haue learned before. For if you will reduce any sum of a grosse denomination into a sum of a smaller or subtiler denomination, you must consider how many of that subtiler denomination do make one of the grosser denomination, & by that number or Numerator do you multiply the other sum: as if you would reduce 20 pound into

To reduce grosse denomination to subtile.

into ſhillings, you muſt conſider that in a pound are included 20 s. therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may know that in 20 l. are contained 400 ſhillings. Likewiſe, if you would reduce 30 ſhillings into pence, conſidering that in a ſhilling are 12 pence, you muſt multiply 30 by 12. and it will be 360. whereby you find, that in 30 ſhillings are contained 360 pence. And thus may you reduce any groſſe denomination into a more ſubtiller, by Multiplication, if you know how many of the leſſer doe make the greater: of which thing I will anone giue you a briefe table for the moſt accuſtomed kinds of money, weights, meaſures, & time, & ſuch like: whereby you may know how often each ſubtile denomination is contained in the groſſer, when you ſhall need it for the foreſaid kind of Reduction. And alſo ỹ ſame ſhal ſerue you, if you would reduce any ſum of a ſubtiller denomination into a ſum of a groſſer denomination. For in ſuch Reduction you muſt conſider (as in the other form) how many of the ſmaler doe make ỹ greater. & by that number you muſt diuide the other ſum, and the quotient will declare how many of the greater denomination are comprehended in that ſum. as for example. If you would know how many ſhillings are contained in 3240 d, conſider that 12 pence

To reduce ſubtile denomination to groſſe.

do make 1 ſ: you muſt diuide that 3240 by 12, & your quotient will be 270, whereby you know that ſo many ſ. are in 3240 d. But, and if you would know further how many pounds are in theſe 270 ſhillings, ſeing that euery pound containeth 20 ſ. diuide that 270 by 20, & it will be 13, and 10 remaining, whereby you may know that in 3240 d. (or 270 ſhillings) are 13 pounds & 10 ſhillings. For euermore the remainer muſt be named by the name or denomination of that ſum that was diuided, which in this place were ſ. And thus may you do with any other kinds of Denominations.

☞

¶ Wherefore to the intent you may haue certaine light or knowledge in moſt common coynes, weights, & meaſures (which is ỳ moſt chief & principalleſt thing in traffick to be knowne) I haue in each reduction as they come in order, ſet downe certaine inſtructions incident thereunto. And firſt I haue hereunto added this Table, wherin is comprehended not onely our currant & cõmon coynes, but alſo the moſt part of ỳ vſuall coynes of Chriſtendome, with their iuſt weyghts & value currant in this Realme of England, intending at the latter end of my Addition to this Booke, to write of the ordinary mony vſed in diuers places, & their common values currant for traffick, with the manner of their exchanges from place to place, &c.

A Table of the names and now valuation of the moſt vſuall Gold-coynes throughout Chriſtendome, with their ſeuerall weight of Pense and Graines: and what they are worth of currant Engliſh money this preſent yeere 1617.

The names and titles of the Gold.	The weight, Pence,	Grains.	The value Shil.	Pence.
Great Soueraigne.	10	0	33	0
Double Souer. K. H.	8	1	22	0
Double Souer. of Q. E.	7	7	22	0
Royall.	4	23	16	6
Halfe Royall.	2	11.d	8	3
Olde Noble.	4	6	14	8
Halfe Noble.	2	3	7	4
Angell.	3	8	11	0
Halfe Angell.	1	16	5	6
Snlute.	2	5	6	11 ob.
2 parts of Salute.	1	11	4	7
George Noble.	3	0	9	9 ob.
Halfe George Noble.	1	12	4	11 q.
Firſt crowne K. H.	2	9	6	11 ob.
Baſe crowne K. H.	2	0	5	6
Souer. K. H. beſt.	3	14	11	8 ob.q
Soueraigne K. H.	4	0	11	0
Edward Souer.	3	15.d	11	0
Elizabeth Souer.	3	15.d	11	0
Elizabeth Crowne.	1	19	5	6
Halfe Crowne.	0	19.d	2	9
Vnite.	6	12	22	0

Double Crowne.	3	6	11	0
Brittaine Crowne	1	15	5	6
Thistle Crowne.	1	7	4	4 ob.
Halfe Crowne.	0	19. d	2	9
Crosse dagger called the 6 li. peece.	3	6. d	11	0
Halfe crosse dagger.	1	15	5	6
Rose Royall.	8	21	33	0
Spurre Royall.	4	10. d	16	6
The Angell.	2	23. d	11	0
Halfe Angell.	1	11. d	5	6

¶ *All the seuerall peeces of Gold heretofore mentioned are set downe according to their valuation by the Kings Maiestie Proclamation for Gold, Dated the 23. of November, 1611.*

A Table of forraine gold coyne, according to their ancient valuation and seuerall weight in pence and graines.

The names and titles of the Gold.	The weight. Pence, Graines.		The value. Shil. Pence.	
Vnicorne of Scot.	2	10	6	0
Scottish Crowne.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French Crownes.	2	5	6	0
Old French crowne.	2	5	6	0
Flanders Rider.	2	6	6	6
Gelders Riders.	1	2	3	6
Philips Royall.	3	10	10	0
Philips Crowne.	1	5	5	0
Collen Gilder.	1	2	4	8
New Andr. Gild.	1	3	5	0

The names and titles of the Gold.	The weight, Pence,	Grains.	The value, Shil.	Pence.
Flaunders Noble.	4	10	12	0
Halfe Flaun Noble.	2	6	5	0
Flem. Angell beſt.	3	6	9	0
Flaunders Royall orkey.	3	10	10	0
Carolus Gilden.	1	21	3	4
Flaunders Royall.	2	6	5	0
Saxon Gilden.	1	2	4	8
Flaunders Crowne	2	5	6	0
Philips Gilden.	2	3	4	2
Halfe Phil. Gilden.	1	1	2	1
Golden Lyan.	2	16	7	8
½ parts of gold Lyon.	0	21	2	5
¼ parts of gold Lyon.	1	19	4	11
Dauids Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old Andre Gilden.	1	8	4	10
Cruſa long croſſe.	2	6	6	0
Cruſa. ſhoot croſſe.	2	6	6	2
Milreyes.	4	10	9	4
Halfe Milrayes.	2	10	4	8
Portigue 1 ounce.	2	16	68	0
Golden Caſtilo.	2	21	8	10
Ducket of Aragon.	2	6	6	6
Hungarie Ducket.	1	7	3	4
Double Piſtolet.	4	9	11	8
Single Piſtolet.	2	4	5	10
Ducket of Florens.	2	4	4	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Double Ducket of Rome.	4	13	13	8

It is to be vnderstood (gentle Reader) that whereas in these tables, the weight is called by the name of a pennie, it is not meant a pennie of siluer money, but a pennie of goldsmiths weight, which containeth 24 barley-cornes. Concerning which, see Troy weight in folio 163.

So if a man haue not the weight wherewith to weigh any peece of gold, he may doe it with barley-cornes being drie and as it is said, fol. 162.

The prices of Gold which the bringers in of forraine Gold shall receiue at the Mint according to the Kings Maiesties Proclamation, Dated the 14 of May in 1612.

For an ounce of French crownes being 22 Carrots fine.	3 ℔, 6 s.
For euery ounce of Spanish Pistolets being 21 Carrots, 3 grains and a halfe fine.	3 ℔, 6 s.
For Duckets of Spaine, being 23 Carrots, 6 graine fine at least the ounce.	3 ℔, 8 s, 8 d
For Milreas Crusado long crosse. Crusado short crosse, the ounce.	3 ℔, 6 s, 2 d
For *Hungarie* Duckets being 23. Carrots, 1 gr. fine at least the ounce.	3 ℔, 9 s.
For the Checkeen of *Venice* being 23 Carrots, 1 gr. fine at least the ounce.	3 ℔, 10, s.
For *Barbarie* gold being 23 Carrots di g. fine, at the least the ounce.	3 ℔, 9 s.

¶ And if the said Barbarie gold be of lesse fineneſſe, a abatement to be made according to the rate.

For

For Sultains being 23 Carrots 1. gr. fine, at leaſt the ounce. } 3 l. 8 s. 8 d.

For all other gold being 22 Carrots fine the ounce. } 3 l. 6 s.

¶ *And being finer, a greater price according to that rate, and being courſer a leſſe, ſo that the bringer in ſupply the leſſe fine, with the more fine, in ſuch ſort, that in the totall it make good the ſaid rate of 22 Carrots fine.*

The prices of Siluer which the bringers in of forraine Siluer ſhall receiue at the Mint, according to the Kings Maieſties aforeſaid Proclamation.

For the Ounce of Spaniſh ſiluer money of *Ciuill*. } 5 s.

For the Ounce of *Mexio* money } 4 s. 10 d.

For Ingots of Siluer, being 11 Ounces, 2 d. weight fine, according to the Standard of *England*, the ounce } 5 s.

¶ *And for other Siluer of more fineneſſe, a better price according to that rate, and for courſer a leſſe: ſo that the bringer in ſupply the leſſe fine with the more fine, in ſuch ſort that in the totall it make good the ſaide rate of 11 Ounces, 2 pence weight fine, according to the Standard of England.*

Of siluer coynes currant in this Realme.

The *Edward* Crowne of 5 s.
The *Edward* halfe Crowne of 2 s. 6. d.
The *Edward* Shilling, halfe Shilling and the three pence.
Philip and *Maries* Shilling, and halfe Shil.
The *Mary* Groat, and *Mary* two pence.
Queene *Elizabeths* Shilling 9 d, 6 d, 4 d, 3 d, 2, d, 1 d, three farthings, and halfe penie.

HEere would I now expresse the values of sundry other coines of diuers countries, but for three causes I now refraine. The first and chiefest is, because they are not currant by the statutes of this Realme. Another cause is, by reason they are so vncertaine, that they be neuer long at one rate. And againe, they are so different in so many places, that it were matter enough for a great booke, to speake sufficiently of them all. Howbeit because you shall not bee altogether ignorant of them, I will shew you the values of some that are most in vse, and first of France.

French Coynes. The most common money are Deniers, Soulx, and Frankes: 12 Deniers make 1 s. 20 Soulx make 1 Franke, so that as you see these 3 kindes are like in the rate to pence, shillings, & pounds with vs, but that this is the difference, that their Denyer is but the

the ninth part of our penyes to their Soulx, (commonly called Sowses) go 9 to our shillings, and 9 of their Frankes to an Englishe pound of mony. So that 8 of their Frankes make a Noble. And by those three may you practise how to reduce French money into Englishe money, accordyng as I haue set forth here followyng.

2160 } 3240 } 8352 } Deniers make { 240 d. or 20 s. / 360 d. or 30 s. / 960 d. or 80 s. 17 s. 4 d.

2160 Soulx make 240 shillings. And so of other in like rate. As for the rest of their coynes I omit them till hereafter that you haue some vnderstãding in broke nũbers.

But now as for the coines of Flanders, they be so changeable that you must know them from time to time else you cannot reduce them into our mony certainly: but yet that you haue an example of their money to exercise you withall, you shall take those that be most common: as Stiuers both single and double, Groats Flemish, Carolus, and Gyldens. A Flemish Groat is a little aboue 3 Farthings Englishe. A single Stiuer is 1 d. ob. q. halfe farthing. The double Stiuer is 5 d. farthing. The double Stiuer Carolus is 48 d. ob. halfe farthing. Then there is also the Carolus Gylden, which is worth 20 stiuers. And the Flemish Noble is worth 3 Carolus Gyldens and 12 stiuers. Flanders Coynes.

So

So, that if you would conuert Flemish money, or any other kind of mony whatsoeuer it be, iustly into sterling, you must reduce it first into the smallest part of English mony that is in that coyne, as for example. If I would reduce 368 double stiuers into English money (considering that a double stiuer containeth 3 ō farthing), you shall first take how many farthings be in the double stiuer, and you shall find them 13: therefore multiply the sum of the Stiuers by 13, and then haue you their value in farthings, which is 4784. Now, if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in 1 shilling) & then will the Quotient declare the sum of the shillings.

Likewise if you would reduce any sum of single Stiuers into English mony, you must multiply the sum first by 13, and then haue you reduced them into a certaine sum, that is to wit: halfe farthings, which sum if you diuide by 8, then will amount the sum of pence: or if you diuide it by 96, the sum of shillings will appeare.

But marke this in all Diuision: when ye doe reduce to bring one Denomination into another, if there be any Remainer after the Diuision, that must be named by the Denominatiō of the grosse sum that was diuided. As for example. I would bring 254 farthings

things into pence, therefore I doe diuide that 254 by 4 (for so many farthings make a penny) & the quotient is 63, which is the sum of the pence, & then remaineth yet 2, which are farthings still, as one may proue by diuiding. And this must be marked in all Diuision, namely, when it is done for Reduction. Note well.

¶ Touching Danske money, they haue their Soulx, whereof 20 is a Liuer: which is 2 s̄ sterling. They haue also their Grash, whereof 80 make a Gilden, which is 4 s̄ sterling. They haue also Dollors, and their common or old Dollor is 35 Grash. New Dollors they haue, which be diuers, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Authour, touching Danske money. Danske money.

Concerning Spanish money, whereof the most common are Cornados, Marueides, Ryals, and Duckets: fire Carnados make a Marueide, 4 Marueides make 1 Ryall, and 11 Rials make one Ducket, so the Ducket containeth 374 Marueides, which is about 5 s̄, 10 d̄, sterling. Therefore if you would conuert 124 l̄, 5 s̄, sterling into Duckets, consider that pence is the least value or denomination named in this question: therefore reduce 124 l̄, 5 s̄, into pence, and it maketh 29820 pence: which if you diuide by the pence that a Ducket is worth, (which is Spanish money.

70,)

70,) you shall haue for your quotient 426 Duckets, your desire.

Thus much haue I said of money. Now will I shew you in like sore, the distinction of weights.

Weights.

Troy weight. A peny weight. An ounce. *After a Statute made, Anno 11, H 7. there ought to be but one sort of weights. As 24 Barly cornes drie, and taken out of the middest of the eare, doe make a peny waight, 20 of those peny waights make an ounce, & 12 ounces a pound of troy waight: by which is weighed Bread, Gold, Siluer, Pearle, Silke & such like.*

A pound Troy. Haberdepoise. But cõmonly there is vsed another weight called Haberdepoise, in which 16 ounces make a pound. Therefore when you would reduce ounces into pounds, you must consider whether your weight be Troy weight or Haberdepoise: and if it be Troy weight, you must diuide your ounces by 12, to bring them to pounds, but if it be Haberdepoise, you must diuide them by 16. Now againe, there be greater weights, which are called an hundred, halfe a hundred, a quarterne, and also a halfe quarterne, &c.

A hundred weight.

Scholar. Why so there may be reckoned 20 pound, 40 l. 200 l. & such innumerable.

Master. All these are numbers of weight, but they haue not common weights made to their rate, as the other haue. And againe these that I did name are not iust in number as they sounde by their name, for an hundred

dred is not iust 100, but is 112 pound. And so the halfe hundred is 56: the quarter 28, & the halfe quarter 14. And these be the common weights vsed in most things that are sold by weight.

Howbeit there are in some things other names, as in wooll, 28 pound is not called a quarterne but a Todde: and 14 pound is not named halfe a quarterne but a Stone, & the 7 pound halfe a stone. These names because they differ in many places, and agree in few, I let them passe. Wooll. Weights. Todde. Stone.

But a sacke of Woll by the Statutes, is limitted to be 26 stone: Sacke of Wooll.

¶ Now in Cheese, though it be sold by the hundreth, and by the stone in some places, yet the very weights of it are Cloues and weyes. So that a cloue containeth 8 pound and a wey 32 Cloues, which is 256 pound, that is 12 stone & 16 l. & so much weigheth a wey of Suffolke Cheese, & the like weight is or should be the Barrel of Suffolke butter. Cheese weights.

The Wey of Essex Cheese containeth fiue stone and fifteene pound: and so much is also the Barrell of Essex Butter.

Moreouer this weight is vsed by the Apothecaries in their Phisical compositions & mixture of medicine wherin the least is a graine.

G

The Apothecaries waits { 20 Graines, 3 Scruples, 8 Dragmes, 16 Ounces. } make { A Scruple, A Drachme or Dragme, An Ounce, A Pound. } thus characterized, { ℈ ʒ ℥ lb } The Apothecaries weights.

Measures for liquor.

Now of weights are made other measures both for graine and liquor. For a pound in Troy weight, maketh a pint in measure, so that 8 pound or 8 pints do make a gallon: halfe a gallon is named a pottle, a halfe pottle is called a quart, which containeth two pints. Now aboue a Gallon the nert measure is a Firken: then the Tercian, a Kilderkin, or halfe Barrell, and a Barrell. And by those measures are sold commonly Ale, Beere, VVine, and Oyle, Butter and Soape, Salmon, Herrings and Eeles.

A Pint. Gallon. Pottle. Quart.

Ferken. Tertian. Kilderkin. Barrell.

Soape

But as these be vnlike things, so the measures of their Ale are as followeth.

Ale measures.

Of Ale { *the Firkin.* *the kilderk.* *the Barrel.* } *conteineth.* { 8 16 32 } *gallons.*

Beer measure.

Of Beer { *the Firkin.* *the kilderk.* *the Barrel.* } *conteineth.* { 9 18 36 } *gallons*

Sope measures.

Sope measures, both Frking, Kilderkin, and Barrell should be equall to Ale measure

Moreouer the Statutes doe limite the weight of euery of those three vessels being empty.

Barrel

{ *A Barrell* *Halfe a Barrell* *A Firken* } *to weigh empty* { 26 13 6½ } *pounds.*

Herrings.

Herrings also be sold by the same measures that Ale and Soape be sold by.

Her-

Herrings also are sold by the tale, 120 to the hundred, ten thousand to the Last.

Salmon and Eeles have a greater measure. Salmon & Eeles.

Salmon & Eeles	the Butte	holdeth	84	gallons.
	the Barrell		42	
	halfe Bar.		21	
	the Firkin		10½	

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels.

Now as for wine vessels they are seldome smaller then Hogsheds; which are of 63 Gallons: Euery Hogshed, is two Barrels: yet there are many other wine vessels, but of them all see this Table; and marke the measures one by another. Wine measures.

Of wine and oyle	the Ronlet	holdeth	18½	Gallons.
	the Barrel		32½	
	the Hogsh.		63	
	the tercian		84	
	the Pipe.		126	
	the Tunne.		252	

soape

18½
32½
63
84
126
252

But you shall marke that there bee other kinds of Tertians: for there be Tertians, (that is to say) Thirds of Pypes, of Hogsheds, and of Barrels, as wel of other things as of wine. Tertians.

Also

But. Also Malmeseyes, and Sacke, &c. the halfe Tun is not called a Pipe, but rather a Butte.

And thus much haue I thought meet to tell you at this time.

Scholar. And is that alwaies true?

Master. I haue told you how it should be, but how it is, I may not say: how they do differ daily from their iust measure, that Gaugiers can tell you better then I. But I will let this passe now, and speake briefly of the other measure.

Drie measures. And as of weights there did spring the liquid measures, (whereof I spake last) so of the same springeth drie measures, as Pecks, Bushels, Quarters, and such like, whereby are measured corne and like graines, also Salt, Lime, Coales, and other like. And this is the ozter and quantitie of them.

A Pecke. A Pecke in the measure of two Gallons.

A Bushell, A Quarter, A Wey: } containeth { foure Pecks, eight Bushels, sixe quarters.

A Bushell Quarter: Wey:

These are the common names of measures, but in diuers places there be diuers sorts.

The Bushell in many places is two Bushels: but then is that Bushell there called a Strike, and in some places halfe a quarter is called a Cornocke. But these diuersities are so many to tell you briefly them al, and againe, sith they are against the law and statutes, I count them vnmete to be vsed.

Strikes: Cornock.

But

But now remaineth yet another kind of measures, whereby men meate length and breadth, and thickeneſſe, and thoſe are, an Inch, a Foote, and ſuch other: whoſe names and quantities this Table ſheweth. Meaſure to meate length, breadth, & thickeneſſe.

3 *Graines of Barly in length make an inch.* An Inch.

12 *Inches* } *make* { *a Foote.*

3 *Foote* } *make* { *a Yard.* Foote. Yard.

3 *Foote and 9 Inches* } *make* { *an Elle.*

5 *Yards and a halfe* } *make* { *a Pearch.* Elle. Perch.

1 *Perch in breadth and 40 in length doe make a rodde of Land, which ſome call a rood, ſome a yard land, and ſome a Farthendele.*

2 *Farthendels* } *make* { *half an acre of grou̇d.*

4 *Farthendels* } *make* { *an Acre.* Acre.

¶ *More, 40 rods in length doe make a furlong, 8 furlongs make an Engliſh mile, which containeth 320 Perches.*

So that an Engliſh mile, grounded vpon the Statute, is in length 1760 yards, 5280 foote, and 63360 Inches.

Somewhat greater then the Italian mile of 1000 paces, and 5 foote to a pace.

Here might I tell you many things elſe touching meaſure, and alſo how to reduce ſtrange meaſures to our meaſures, but becauſe it cannot bee well done without the knowledge of Fractions, which as yet you haue not learned, I will let them paſſe til another time that I haue taught you the knowledge of broken numbers.

The parts of time.
A day.
An houre.
Weeke.
Moneth.
Yeere.

Scholar. But yet sir of the parts of time, I pray you tell me somewhat.

Master. You know that a naturall day hath 24 houres, and euery houre hath 60 minutes. It needeth not to tell you, that 7 daies make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeere, lacking one day, and certaine houres, and minutes: but of that I shal instruct you hereafter.

Here will I make an end of Reduction for this time, which though it be counted no kinde seuerall of Arithmeticke, you see it is no lesse needfull to be knowne, or easier to be done, then any of the other.

Scholar. Marry sir, it seemeth vnto me much harder thē any other sort, for it requireth the knowledge of so many things: but now sir when you see time, I am ready to learne forth: for as much of Reduction as you haue taught me, I remember; but if I doe at any time forget, I shall haue recourse to the tables which you set forth for me.

Master. So doe you, for it will not be remembred without exercise. But in as much as you vnderstand so much as we haue intreated of, I will now instruct you in Progression.

Progres-

Progreſsion,

Although vntill this day the moſt part of Writers haue defined Progreſſion as a compendious kind of Addition, yet truly it is not ſo: for Progreſſion (as the very nature of the word doth informe any man) is a going forward and proceeding in numbers, and that regularly and orderly, whoſe place is aptly choſen to be very neere, or rather next after the expoſition of the foure principall parts of Arithmeticke, for in it after a moſt eaſie manner, are all the foure former parts exerciſed and practiſed: and not onely Addition, as cuſtomably is done. Which cuſtome hath beene the cauſe, why it hath ſo ſpecially beene named a kind of Addition, and deſined to be a quicke and briefe Addition of diuers ſummes, proceeding by ſome certaine and reaſonable order.

You ſhall alſo vnderſtand that there are infinite kinds of Progreſſions, but for you (as yet) two are ſufficient to be exerciſed in, of which the one I call Arithmetical, and the other Geometricall.

Arithmetical Progreſſion.

Arithmeticall Progreſſion is a rehearſing or placing downe of many numbers, number after number, in ſuch ſort that betweene euery

two next numbers rehearsed or placed downe, the difference, diuersity, or excesse, be equall and alike.

Scholar. Sir, I thanke you for that you haue both opened vnto me what Progression is truly, and also why it is here placed. But I pray you with an example make plaine your definition.

Master. Examples cannot want, seeing all reasonable creatures naturally vse the order of one kind of Arichmetical Progression (which therefore is also named naturel whensoeuer they distinctly doe count or number any multitude by one, saying, 1, 2, 3, 4, 5, 6, whereby the proceeding from the number to number, and euerie one surmounting and exceeding his fellow next before by a like quantitie (which here is 1) declareth the same to be Arithmetical Progression. And for the more plainenesse, I set it downe in this manner.

The common excesse. 1 1 1 1 1

1 2 3 4 5 6

The Progression.

Scholar. This is most euident. And I thinke that I am able to tell you now of any Progression Arithmeticall proposed, what is that common excesse or difference whereby it proceedeth, if this order be kept in it.

Master.

Maſter. What ſay you of 3, 6, 9, 12, 15?

Scholar: They exceede eache other by 3. And that may I ſet downe in ſuch euident order as you did your Example of naturall Progreſſion in this wiſe.

The common exceſſe. 3 3 3 3

The Progreſſion. 3 6 9 12 15

Maſter. And doe you not alſo now perceiue, that the whole Table of Multiplication may be made by the order of Progreſſion Arithmeticall? either if you will begin at the firſt number of any of them on the left hand, and ſo proceed right ouerthwart: or at any of the firſt numbers of the vpper row, and goe directly downeward.

Scholar. I pray you let me conſider the thing a little, and I will anſwere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now very wel, for the commõ exceſſe or difference betwẽ any two next, is continually as much as the firſt number of euery row, either frõ the left hand ouerthwart taken, or from any of the vppermoſt ouerthwart rowes downeward

To know the total ſumme of an Arithmeticall Progreſſion.

Maſter. Now then of any ſuch Progreſſion, you would ſpẽdily know the total ſum much quicklier then by common rules of Additiõ: firſt tel how many numbers there are (which numbers here wee call places or parcels) and if they be odde, write their ſum downe by it ſelfe, as in this example, 1, 4, 6, 8, 10, 12, 14, where the numbers are 7 as you may ſee, therefore ſet down 7 in a place alone, then adde together the firſt number and the laſt, as in this example: adde 1 to 14 and that maketh 16, take halfe of it, and multiply by the 7 which you noted for the number of the ſame places, and the ſumme that amounteth, is the ſumme of all thoſe figures added together, as in this example, 8 multiplied by 7 maketh 56, and that is the ſumme of all thoſe figures.

Scholar. What will I worke by another example. I would know how much this ſumme is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places and there are 9, that I note. Then I put the firſt number 5, and the laſt 29, together, and they make 34, I take the halfe of it, that is, 17, and multiply by 6, and

it

it maketh 153. That you say is the summe of all the numbers.

Master. So shall you find it if you trie it.

Scholar. How shall I trie it?

Master. By your common Addition, for if you adde all the parcels together, you shal se the same sum amount, if you did worke well. And that manner of Addition trieth all kinds of summing any Progression.

Scholar. Then can I summe a Progression, if the numbers of the parcels be odde. But what if they be euen: as in this example, 1,2,3,4,5,6,7,8?

Master. When the number of the parcels is euen, then note that also as you did before, and likewise adde the first sum to the last, and by the halfe of the number of the places doe you multiply it: as in our example, the parcels are 8, that I note: then adding the first sum to the last, there amounteth 9, that doe I multiply by halfe of the parcels, that is, by 4. and maketh 36, which is the sum of the parcels.

But if you will take one rule for these both, doe thus; multiply the halfe of the one, by the other whole, and the sum will amount al one. For sometime it chanceth that the number of the parcels be odde, so that their halfe cannot be taken: and sometime it chanceth the Addition of the first number & the last, doe bring forth an odde number, so that the

A generall rule.

halfe of it cannot be taken: but they will neuer be both odde.

Scholar. Then I perceiue this, if there beno more belonging to it.

Master. As accustomably it hath béene taught, this hath béene the chiefe & only exercise in Progression vsed. But that you may perceiue how diuers waies, & to how great profit so simple a thing (as this Arithmeticall Progression is) may be considered and vsed, I will here propound you six propositions: of which, foure of them were inuented by a friend of mine, & neuer before this published: and the two first, were neuer to my knowledge written of, but by 3 men.

Scholar. This doth greatly encourage me to be attentiue vnto your words, seeing I shall not onely be instructed at your hands in the common knowne rules of this excellent art, but besides that, so abundantly in other new rules enformed, as my very enterance shal seeme to passe a great many mens further study, and longer continuance. Therefore sir I beseech you, let me know your sixe propositions.

Master. These they are.

1 *To know the last number without proceeding by continuall Addition, till you come vnto it, so that the common excesse, the first number, & the number of the places be known.*

2 *The first number of the Progression and the last*

laſt being knowne, with the common exceſſe to finde the number of the places.

The exceſſe being giuen, and the firſt or laſt, to know the quantitie of any middle number, whoſe place is giuen from the firſt or laſt. 3

The totall ſum being giuen, and the firſt and laſt, to find out the number of the places. 4

The totall ſumme of any Arithmaticall progreſſion being giuen, and the firſt and laſt, to finde out the common exceſſe. 5

The totall ſum being giuen, and the mutuall exceſſe, with the number of the places, to giue the firſt or laſt number of the ſame progreſſiō. 6

Many moe conſiderations could I propound you in theſe *Arithmeticall progreſſions*, but theſe are ſufficient, to giue you occaſiō to think, that rulers of knowledge & *Arts* are infinitely capable of enlargement.

Sch. Happy were I, if I did but well vnderſtand that which is already inuented and written, And yet in my ſimple fantaſie theſe things offer themſelues (in manner) to be ſtudied for about progreſſion, therefore I pray you to proceede to the rules anſwering to theſe propoſitions.

Maſter. I will orderly for euery of theſe ſixe propoſitions giue you rules, and with euery one an example, vnleſſe the plainneſſe and eaſineſſe néede no farther exemplifying.

For the Solution of the firſt, Multiply the exceſſe by a number leſſe by 1 than the number 1 Propoſition.

ber of the places, and the off- come adde to the first number, so you shall haue the last number, which is sought for.

As for example: If there were seuen places in a progression Arithmeticall, whose continuall increase or mutuall excesse were 4, and the first number were 5, and I would know what the last and seuenth number is: I multiply 6, which is one lesse than 9, (the number of the places) by 4, thereof commeth 24 which I adde to 5, that maketh 29: and that is the last number which I desire to know. And this you may straight way proue, by continuall proceeding from 5 tyll the seuenth place, encreasing euery one by 4, as thus.

5 9 13 17 21 25 29.

Lo heere, the last, being also the seuenth, is 29.

Scholar. I perceiue already one good propertie in this rule, which in all workes is to be desired: that is, it wil ease one from great labor, if a progression were propounded of a hundred or two hundred places, or mo. And also it is very easie to worke, and most necessarie for the totall summe finding, in a very long progression.

Proposition. Master. *The second rule is this. From the last subtract the first, the remainder diuide by the common exces, to the* quotient *adde* 1, *and you haue the number of the places, which*

you

you would know: As in this progreſſion,

6 11 16 21 26 31

If I know onely 6 and 31, and that they encreaſe by 5, then according to the rule, from 31 I ſubtract 6, there remaineth 25: which 25 I diuide by 5 (the common exces) the quotient commeth forth 5, to which I adde 1, that maketh 6: and ſo many are the places, as you ſée.

Scholar. This rule is ſo eaſie, that I were much to blame, if I could not remember it.

Maſter. *The third propoſition may alwaies thus be ſolued. Multiply the* exces *by a number leſſe by one, than the diſtance of the* place *is from the* firſt, *or the* laſt uumber giuen: *the off come adde to the* firſt, *if the* diſtance *be reckned from the* firſt, *and the* firſt *alſo known, or ſubtract from the* laſt *if the diſtance be from the* laſt counted *& the* laſt giuen *alſo, and that which commeth forth, either in that* Addition to the firſt, *or* ſubtraction from the laſt, *is the nnmber* ſought. As for example, *I propound you this* Progreſſion. 3 Propoſition.

8 15 22 29 36 43 50 57

And for the apt conſidering the manner of this queſtion, I will note ouer euery place his diſtance from the firſt, and vnder euery place his diſtance inclusiuely frō ỹ laſt, thus.

1 2 3 4 5 6 7 8

8 15 22 29 36 43 50 57

8 7 6 5 4 3 2 1

Now

Now if the exceſſe whereby this progreſſion ſtandeth, be knowne to be 7, & firſt number giuen, being 8, I would know what number ſtandeth vnder 4, that is to ſay in the fourth place. I multiply 7 by 3 (which is leſſe by 1 than the number of the place propounded) that yeideth 21 to which I add 8 ỳ firſt number ſo commeth 29: which I ſay to belong to the fourth place, as ye ſée in the example it alſo doth: or if in the third place from the laſt, you would know what number in this example ſhould ſtand the laſt number being known to be 57, and the common exceſſe 7, then by 2 which is leſſe by 1 than the place propounded, I multiply 7, that giueth 14: which I ſubtract from 57, ſo remaineth 43: which appertaineth to the third place incluſiuely reckned from the laſt, and ſo my example giueth you.

Scholar. I perceiue right good vſe of this rule now: for if I had forgotten what the firſt number were, and remember ſtill but the laſt, the common exceſſe, and the number of the places, then might I come by the knowledge of my firſt number againe.

And me thinketh ỳ it differeth not much frõ the firſt propoſition, ſauing that which you make héere a middle number, there was made the laſt: and alſo in this point it differeth, that in it the laſt was only ſought, and no conſideration had in numbring the places

ces from the last, as heere I marke in your numbers noted vnder your progression.

Master. And thinke you not, the middle numbers of Progression standing of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continual encreasing from the first by the common excesse, or abating from the last continually the common excesse as the very finall numbers in a shorter progression would doe?

Sch. Yes sir, that I thinke right wel and therfore I am glad of this new framed proposition, & the manner of the working of it.

Master. *The rule of the fourth is this. Adde the first and the last together, and by the ofcome diuide the total summe. Double the quotient, and that will be the number of the places.* 4 Propesition.

Scholar. Then if in a Progression, whose summe were 207, and the first number 12, and the last 57, if I adde 57 & 12 together, that maketh 69, & by it I diuide 207, the Quotient will be 3, which I double, & so I haue 6, & so many must bee the number of the places that this Progression standeth on.

Master. Whether it be so or no how wil you trie?

Scholar. Halfe 6, which is 3, being multiplied by 69, must make 207, the toral sum; if 6; bee the number of the places. For so

ſo the whole worke of your rule in ſumming any Arithmeticall Progreſſion did enforme me. I will then multiply 69 69, by 3, thus, 3

It commeth forth iuſtly. 207

Maſter. I muſt much herein commend your promptneſſe, both in memorie and in well applying your rule: although in manifeſt words it did containe no ſuch matter.

Scholar. Sir I pray you heare me frame one example more.

Maſter. I am well pleaſed, ſo that ye be ſhort, for yon make me more longer heere than willingly I would haue beene: but I cannot perceiue how I could haue omitted any thing as yet, without your great lacke thereof.

Scholar. If I had receiued 85 pounds of certaine men, but of how many I haue forgotten, I yet I remember that the firſt gaue me 7 l. and the laſt 27 l. & euery payment after other did riſe by a like ſum. And the man for whom I receiued this money, conditioned with me, that of euery payment I ſhould haue twelue pence for my labour: now vnleſſe I can by Art finde the truth of this caſe, I am like to loſe the moſt part of my reward.

Maſter. I perceiue you can handſomely frame an example, which ſhould concerne your owne gain: I pray you let me ſee how

you

you would do iuſtice in this point.

Scholar. I adde the firſt and the laſt together, that maketh 34: by which I diuide 85, thus.

1		
27		17
85	(2	
34		34

Why how now Sir, here is a remnant of 17, in which 34 cannot be had: ſo that now I am in the byars for doubling of my quotient, & farewell then both my Iuſtice, and a good lump of my gains.

Maſter. Yée are neuer the farther from the matter, though it fall into a Fraction. For you ſhall vnderſtand, that the Fraction which of any ſuch worke proceedeth, is euer halfe of one ſuch, as the vnits of the quotient before are. And that you may trie, if you double that which ſo remaineth, for then it will be equall to your Diuiſor, as if ye double 17 (the remnant) it maketh 34, and your Diuiſor alſo was 34, this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard example. For with it I haue a general rule for the Fraction that may hap in this worke. So that the quotient being two and a halfe, I double that, and it maketh 5, therefore ſhould my gaine be 5 ſhillings. And to bee ſure, (by your leaue) I will trie it, for I will multiply halfe of 34. (which is the firſt and laſt number ioined together) by 5, thus.

17	
5	
85	

It

It is most true (I sæ) that I should læse nothing by the former working.

5 Proposition.

Master. *The fifth proposition hath this rule appertaining vnto it: By the fourth rule find the number of the places that being done, from the last subtract the first, and the residue diuide by a number lesse by 1, then the number of the places, and the quotient will shew the excesse which is sought for.*

An example hereof shall be this: If ye had disbursed 685 pounds to a certaine number of men, you neither can tell how many they were, or how much the ones money exceeded his next before but you are sure that the excesse was equall betweene euery two next: and also you remember that the first had 19, & the last 118 pounds, how would you find both the number of the men and the excesse, continually obserued in the succession of their payments?

Scholar. Your rule doth plainly bid, first to find the number of the places, which I will doe according to the fourth rule. I adde 19, and 118 together thus.

118
19
137

By this 137, I diuide 685, thus.

137 | 685 (5
137

Seeing there is no fraction but a whole number, being 5, I double that, and then must the number of the pla-

ces bee 10. Now from the last I subtract the first, as 19 from 118, thus: And so remaineth 99

118	
19	
99	

This 99 I divide by a nnmber lesse by 1 then the number of the places, and seeing the places were 10, I divide 99. by 9, thus

99	
99	(11

the quotient is 11, and so was the excesse, if I have followed your rule right.

Master. You have wrought every part of this question both well in order, and truely in the practise of your rules.

Scholar. I will then set it downe also formably, so that the number of the places, the excesse and the totall sum may streight appeare, as your first example stood.

11	11	11	11	11	11	11	11	11	11	ii
19	30	41	52	63	74	85	96	107	118	

The common excesse.

That the places be 10, and that from the first to the last the common excesse is 11. I perceive most evidently, but whether the totall sum be 685, I have not yet proved, which I will now doe: I adde 19 and 118 together, that maketh 137: I multiply that by halfe the number of the places, thus.

The Progressions

137	
5	
685	

All things agree most exactly, so that I am perfect enough in these rules, if I forget them not again.

Master

Master. Wise maketh all things perfect.

6 Proposition. *Your sixth rule is this. By the number of the places divide the totall summe, double the quotient, and that will be the first and the last ioined in one summe. Then by a number lesse by 1 then the number of the places, multiply the excesse, that of come subtract from the first doubled quotient, and the halfe of the residue is the first number. The last number you may diuersly find out, as by the first of our sixe rules, or by subtracting this first number from the sum which here contained both the first and the last iointly, (or thirdly) by continuall adding the excesse.*

Scholar. I pray you make this somewhat more plainer with an example.

Master. If euery moneth in the yere (counting them now as 13) you gained clerely 40 shillings more then you did the moneth next going before, and at the yeeres end you finde the whole gaine 5720 shillings, but ye remember not how much either the gaine of the first moneth or the last was, by this rule it may be tried out.

Scholar. So that here ye seme to apply the 13 moneths to thirteene places, the 40 shillings euery one more then the other next before it, to be the common excesse, and 5720 s, to be the totall summe.

Master. It is true: by 13 then I diuide

5720

5720. in this manner.

I double this Quotient, so haue I 880 for the first and the last summe ioyned together, by 12, which is lesse by one then the number of the places; I multiply 40 (the common excesse) so commeth 480.

This 480 I subtract from 880, so remaineth 400: halfe whereof is the first number which we desired to know: that is 200.

And as for the last number I can giue you it three waies: As by the first of my fixe rules. I multiply the excesse by a number lesse by 1, then the number of the places, as 40 by 12, that giueth 480, which I adde to the first, being 200, so shal the last be 680.

The same summe commeth forth, if yee subtract 200 from 880:

And thirdly, if I beginne at 200, and so proceed, encreasing by 40, I shall at the thirteenth place haue 680, as thus:

200, 240, 280, 320, 360, 400, 440, 480, 520, 560, 600, 640, 680

Scholar. I thanke you most heartily for these fixe rules. Now if it be your pleasure I would heare e learne somewhat of Progression Geometricall.

Master. There are yet very many rules and propositions, which fall into this Arithmeticall Progression: but these shall suffice for this time.

And in Geometricall Progression I will be more briefe, both because I haue beene so long in this part of Arithmeticall Progression, and also for that it would require the knowledge of Rootes, & numbers surd, (whereof ye haue yet learned nothing) if I should frame the like propositions in them as I haue done in these. Therefore I will onely teach you two practises, about it, & so end the considerations and workes of these progressions.

Progression Geometricall. Progression Geometricall is when the numbers increase by a like proportion, that is, if the second number containe the first, 2, 3, or 4 times, and so forth: then the third containeth the second so many times also: and so the fourth the third, and the 3, 6, 12, 24, 48, fift the fourth, wherefore 1, 3, 9, 27, 81, I set these 3 examples. 2, 10, 50, 250,

Here in the first example you see, that euery number containeth the other (that goeth next before him) two times: and in the second example three times, & in the third example fiue times. Now if you will know how to find easily the sum of any such numbers doe thus: Consider by what number they be multiplied, whether by 2, 3, 4, 5, or

any other, and by the ſame number doe you tiply the laſt ſumme in the Progreſſion.

Scholar. I pray you worke it by this example, 2, 8, 32, 128, 512. 2048, which I haue framed by proceeding from 2, and continuall multiply by 4.

Maſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, & it will be 8192. Now muſt I abate from this ſum the firſt number of the Progreſſion, which here is 2, then reſteth 8190. which ſum I muſt diuide by 1 leſſe then was the number that I multiplied by. Seeing then I multiplied by 4, I muſt diuide by 3, ſo dividing 8190, by 3, the Quotient will be 2730, which is the ſumme of all the Progreſſion. And now to proue whether you can doe the ſame, I giue you theſe numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot wel tell by what number this Progreſſion doth increaſe.

Maſter. In any ſuch doubt doe thus: Diuide the ſecond number by the firſt, and the Quotient will ſhew you the number that engendreth the Progeſſion.

Scholar. Then is that number in this example 5, for ſo many times is 3 in 15.

Maſter. So is it. Now worke as I taught.

Scholar. The laſt number is 46875, which I multiply by 5, & it proueth 234375, from which I abate the firſt number of the progreſſion, that is 3, & there reſteth 234372,

which I diuide by 4, for that is one lesse then 5, and the quotient is 58592, which is the whole summe of the Progression.

Master. If you remember well this, you haue learned the Art of Progression both Arithmetical, and also Geometricall, which you may proue either by subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding together of all the parcels, for so will the same summe amount.

¶ And now for the vse and better vnderstanding of both these rules, I will propone vnto you certaine pleasant and necessary questions Arithmeticall and Geometrical, and to the performance of their workings, such necessary rules and documents, as are requisite for the vnderstanding of them, or any such like.

A question of Velvet. A certaine Mercer sold 20 yards of Velvet to be paid in 12 weekes, by Arithmeticall proportion: that is to wit to receiue the first weeke 6 shillings, the second week 12 shillings, the third weeke 18 shillings, and so forth, encreasing the number of weekes by 6 shillings, til the twelfth and last weeke were expired. The question is how many pounds he had for 20 yards of velvet.

To the performance of this question and such other the like, I set forth the 12 payments in such sort, as for example, here appeareth in the margent. Then

Then touching the adding together of theſe ſummes, without the aid of Addition, accoꝛding to the rules I taught you in Pro- greſſion Arithmeticall, I note the number of the places, which are 12. then adding the leſt number of the Progreſſion, which is 72. and the firſt number together make 78, and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhil- lings, and in pounds maketh 23 l' 8 s. And ſo much hath the Mercer for his 20 yards of veluet, which is nye about 23 s, 5 d, a yard:

Scholar. I underſtand this woꝛke very well: but is there any pꝛoſe foꝛ the iuſtify- ing hereof as you haue of other woꝛkes?

Maſter, The woꝛke of it ſelfe (being ſo perfectly woꝛought) that in your pꝛoceeding and going foꝛward from number to num- ber, each number exceeding his fellow by an equall oꝛ like quantity, is all that is de- manded foꝛ iuſtifying of the ſame: yet not- withſtanding becauſe your requeſt is rea- ſonable, I will pꝛopone an example foꝛ the pꝛoſe hereof.

A certaine man is bound to pay for 20 yards of veluet, the ſum of 23 l' 8 s: and it is to be paid weekly in 12 weekes oꝛ termes by Arithmetical Progreſſion. The queſtion is theꝛefoꝛe to know with what number the ſame Progreſſion is to be begun & continu- ed in ſuch equall pꝛopoꝛtion Arithmeticall,

The proof of the laſt queſtion.

that in 12 weekes the same may iustly be accomplished.

A general rule.

For the resolution whereof, and of all such other like, reduce 23 l' 8 s̄, all into shillings, which maketh 468 s̄.

Then adde 1 vnto 12, the number of the termes, it maketh 13, which 13 you shall multiply by halfe the number of the termes, which is 6, it maketh 78, then diuide 468 by 78, and you shall find 6 in the quotient, which is the true number that shall begin & continue the said Progression. That is to say, the first weeke 6 s̄, the second 12 s̄ the third weeke 6 s̄ more, which is 18 s̄ and so euery weeke as they rise, 6 s̄ more then the weeke before, as is manifest in the question aforesaid.

A question of a Farme.

A Farme is to be solde to be paid by the weeks in a yeere, the first weeke to pay 4 s̄. the second weeke 8 s̄, the third weeke 12 s̄, & so forth, increasing each number by 4. till the number of 52 (which are the number of weeks in a yeer) be expired. The question is what the price of the Farme commeth to.

Scholar. I doubt not, but by that you haue already taught me, to end this question very wel, wherefore I set forth the Progression with his excesse 52 times.

Master. May stay a while. And here for your further ease, (to abridge you of great labor that appeareth to fall out in this question,

ſtion, & ſo may doe in any other the like, if a queſtion were proponed of 100 or 200 places or more, and that this queſtion nor any other ſ like can be ended, vnleſſe you know absolutely what the laſt number of the Progreſſion at the 52 place is or ought to be) I will giue you a generall rule how to know the laſt number of any Progreſſion Arithmeticall, aſwell as if you had ordinarily proceeded by continuall Addition, till you had come to the laſt worke, which is this.

Multiply the exceſſe by a number leſſe by one than the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall haue your deſire. A generall rule.

Sch. This rule is wel worth the noting: for if I vnderſtand you aright I conſider that my exceſſe is 4, which I multiply by 51, which is one leſſe than ſ number of the places, & it maketh 204: whereunto I adde the firſt number of the Progreſſion, which is 4, and then it is 208: which you ſay is, or ſhould be the laſt number of the Progreſſiō.

Maſter. This is a moſt approued truth if there were neuer ſo many places.

Scholar. This rule is ſo eaſie, that I were much to blame, if I do not remember it. For by the benefit hereof I haue ſuch an eaſe & light into this excellent Art, that my firſt entrance doth ſeeme to paſſe a great many mens further ſtudie & longer continuance.

Maſter.

Master. Many moe considerations could I proponed you in these Arithmetical Progressions: but these are sufficient for a taste, to giue you occasion to thinke that rules of knowledge and Artes are infinite capable of enlargement.

Scholar. Happie were I, if I did but well vnderstand that which is already inuented and written. But these things, in my simple fantastie, offer themselues to be greatly beneficiall vnto the ayde of Progression. Therefore now I will goe forward with your question.

Now considering that the 52 and last place is 208, I add thereunto the first number of the Progression, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 shillings. And so much is the totall summe or addition of this Progression: which maketh 275 l. 12 s. as appeareth here by my tables.

Master. I like well your labour and commend you for your diligence, I will here propone one example more, & therewithall for this time wil end progression Arithmatical.

A question of Holland.

A certaine man bought 20 els of Holland to be paid in 17 weekes or termes by progression Arithmeticall. And the first weeke to pay 1 s. 8 d, the second weeke 3 s. 4. d, the third week 5 s. the fourth week 6 s. 8 d.

&

& so forth each weeke succeeding 20 pence more than the weeke before. The question is, what the sum of his 20 els commeth to.

Scholar. Because here is mention made both of shillings and pence, I feare there is some harder matter contained herein, than in the other before: therefore I pray you worke it your selfe, and I will diligently marke your labour.

Master. There is no more to be done in this then in the other before, but because your request is so reasonable, be attentiue vnto me.

First by the generall rules I seeke to find out the last number of the 17 place, what this Progression ought to be. Therefore here in my tables multiplying the excesse 20 by 16, which is one lesse then the number of the termes or places, and it commeth to be 320, and thereunto adding the first number of ꝑ progression which is 20 pence, all is 340 pence, or 28 s, 4 d: for so much ought the last number of the paiments to be. Then finally to know what the whole 17 places amount vnto, I adde the first number of the Progression and the last together, which make 360. Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360, which is 180: and multiplying 180 by 17, commeth to 3060 pence, which maketh as you see by Diuision

12 l. 15 s. And so much is the buyer to pay for his 20 elles of Holland. Which 3060 pence if you diuide by 20, the number of elles that was bought, you shall find 12 s. 9 d. and so much paid he for an elle one with another.

The Proofe.

Aquestion of debt

A certaine man doth owe 12 li. 15 s. to be paid in 17 weekes or termes by Arithmeticall Progression. The question is, to know with what number he shal begin and continue the Progression in such equal proportion, as the same may be truely paid and satisfied in 17 weekes.

The Answere.

First I reduce 12 l. 15 s. all into pence, which as you see here in my Tables make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I should multiply by halfe the number of the weekes or termes, which is $8\frac{1}{2}$, which $8\frac{1}{2}$ multiplied by 18 cannot well be done vnlesse you were acquainted with Fractions or broken numbers, therefore you shall let that passe & multiply 17 by the halfe of 18: which

which is 9, (for that is all one with the multiplication of $8\frac{1}{2}$: and the multiplication of 9 into 17, maketh as you see 153, with which number you shall divide the 3060 pence before said: and the quotient bringeth forth 20 pence, which is the first number or payment to begin the progression with all: and so each weeke succeeding to rise 20 s more than the weeke before, and thereby in 17 weekes shall 12 l 15 s, be paid: as before was sufficiently declared. Thus much for Progression Arithmeticall.

Scholar. Certainly sir, I know not how to render you condigne thanks for these benefits shewed me, which me thinketh are so easie, delightfull and pleasant, that I count my selfe happy to be in your company.

Master. I am glad you delight so well herein, which is an art of wonderfull dexterity to al sorts of men of what degree or profession soever they be. And now will I propone a question or two of progression Geometricall.

A Mercer hath 12 yards of Satten which he valueth at 16 s. the yard, and selleth the same 12 yards to another man to be paid as followeth. That is to wit, for the first yard to haue one shilling, for the second yard 2 s: for the third yard 4 s, for the fourth yard 8 s, &c. doubling each number following, til the twelfth & last yard. The question is, who

A question of Satten.

who hath made the better bargaine of the buyer or the seller.

First you may set downe 1 s the number of the yards, as you see here in this example. And against each number the number of shillings due to be paid, as the order of Duplation, or Multiplication by two teacheth.

s	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12

When resorting to the adding vp or summing of this progressiõ, where I consider that the increase of this summe proceeded by the Multiplication of 2, and therefore after I haue drawne a line vnder the 12, I worke and multiplie the last summe by two also, and it yeeldeth 4096: from whence I abate the first number of the Progression which is one, and than resteth 4095: which I should diuide by one lesse than I did multiply by, but seeing it is one, I need not to diuide it: for 1 (as I haue said before) doth neither multiply nor diuide, therefore I take that summe 4095 for the whole summe of the shillings, which by reduction amounteth to 204 l, 15 s, and so much hath the Mercer for his twelue yards

yards of satten: which is 17 li, 1 s, 3 d, a yard. But I thinke you will buy none so deare.

Scholar. No sir by the grace of God this yeare.

Master. Then what say you to this question? If I sold vnto you a horse hauing 4 shoes, and in euery shoe 6 nailes, with this condition, that you shall pay for the first nayle one ob: for the second nayle two ob: for the third nayle foure ob: and so forth, doubling vntill the end of all the nayles. Now I aske you how much would the price of the horse come vnto. A question of a Horse.

Scholar. First to know the number of the nayles, I must multiply 6 by 4, and it maketh 24. Then will I doe thus: I will write the number of the nayles euery one in order from 1 to 24, and against each number of the nayles the summe of halfe pence duely, as the order of Duplation or Multiplication by two teacheth, and as in this next figure following appeareth.

Then doe I resort to the rule of summing vp the Progression, where I consider that the increase of this summe proceedeth by the Multiplication of two, as the last example did. And therefore multiplying the last summe by two also, and it yeeldeth 16777216, from which I abate the first number which is 1, and then resteth 16777215,

1	1	16777215, which I ſhould
2	2	diuide by one leſſe than I
4	3	did multiply: but ſeeing ỹ
8	4	it is 1, I need not to diuide
16	5	it, for 1 (as you haue be-
32	6	fore ſaid) doth neither mul-
64	7	tiply nor diuide, therefore I
128	8	do take ỹ nũber 16777215
256	9	for the whole ſumme of the
512	10	halfe pence, which by Re-
1024	11	ductiõ I find to be 699050
2048	12	ſhillings and 7 d, halfe pe-
4096	13	nie: that is 34952 pound
8192	14	10 s. 7 d, ob.
16384	15	Maſter. That is wel done,
32768	16	but I thinke you will buy
65536	17	no horſe of the price.
131072	18	Scholar. No ſir, if I be
262144	19	wiſe.
524288	20	Maſter. Well then an-
1048576	21	ſwere me to this queſtion.
2097152	22	A Lord deliuered to a
4194304	23	brick-layer a certain nnm-
8388608	24	ber of loades of bricke,

A queſtion of bricks.

whereof he willed him to make twelue walles, of ſuch ſort, that the firſt wall ſhould receiue two thirdels of the whole number: and the ſecond two thirdels of that which was left: and ſo euery other two thirdels of that that remained: and ſo did the Brick-layer: & when the 12 walls

were

were made, there remaineth 1 load of brick.

Now I aske you, how many load went to the wall, and how many load was in the whole?

Scholar. Why Sir, it is impossible for me to tell.

Master. Nay it is very easie if you marke it well. Marke well that I said, that every wall should receiue two thirdes of the sum that was left. Now take away two thirdes from any sum, and you must néeds grant that that which remaineth is 1 thirdie of the summe last before: Example of 9, from which if you take two thirdes, there will remaine three, which is one thirdie of 9. Likewise from three bate two thirdes, and there will remaine 1.

Scholar. This is true, and now I perceiue that the least wall had but two load of bricke.

Master. And by the same reason may you know how many loade every wall had according as this figure following doth shew, and likewise what the whole sum of brickes was: for if you make 12 summes, multiplying by 3, still from the last remainer, as you may see here on the left side of the Table, there will appeare all the remainers of the whole wall: and if you multiply the last of those 12 summes by 2 also, then will that bee the summe of the loads which

was deliuered to the Brickelaier.

The remayner	1	1	2	*Loads due to*
after euery wal.	3	1	6	*each wall.*
	9	1	0	18
	27		9	54
	81		8	162
	243		7	486
	729		6	1458
	2187		5	4374
	6561		4	13122
	19683		3	39366
	59049		2	118098
	177147		1	354394
Summe of the	531441			*loads deliuered.*

Againe, if you doe double euery remayner, as you may see at the right side of this Table, those numbers will shew the summe of loads that went to each wall, whereby you may perceiue that each wall was three times so great as the next lesser.

Scholar. Lo now it appeareth ease enough. Now surely I see that Arithmetick is a right excellent Art.

Master. You will say so when you know more of the vse of it: For this is nothing in comparison to other points that may be wrought by it.

Scholar. Then I beseech you cease not to instruct me further in this wonderfull cunning.

The

The Golden Rule.

Master.

Y order of the Sciance (as Men haue taught it,) there should follow next the extraction of Rootes of vnmber which because it is somewhat hard for you yet, I will let it passe for a while, and will teach you the feate of the Rule of Proportions, which for his excellency, is called the Golden Rule. Whose vse is, by three numbers knowne to find out any other vnknowne, which you desire to know, as thus. *If you pay for your boord for three moneths sixteene shillings, how much shall you pay for eight moneths?*

The Rule of proportion.

The Golden Rule.

To know this and all such like questions, you shall consider which two of your numbers be of one denomination, & set those two the one ouer ỹ other, so that the vndermost be it that the question is asked of: as in my question 3, & 8 be both of one denominatiō; for they both be moneths: & because 8 is the number that the question is asked of, I set the one ouer the other, and 8 vndermost, thus, with such a crooked draught of lines. Then doe I set the

Question of boarding.

3
8

Z

other number which is 16, against 3 at the right side of the line, thus.

And now to know my question, this must I doe: I must multiply the lowermost on the left side, by that on the right side, & the sum that amounteth, I must diuide by the highest on the left side: or in plainer words, thus, I shall multiply the number of which the question is asked (which is called the third number) by the number of another denomination (which is called the second,) and the sum that amounteth, must I diuide by the sum of like Denomination, (which is called the first.) Then for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, which I diuide by 3. and it yeldeth 42 s, & 2 s, remaineth, which I turne into pence, and they be 24 pence, of which the third part is 8 pence, so the third part of 128 s, is 42 s, 8 d, which sum I write at the right hand of the Figure against 8 thus.

Note.

The third number.

The second number.

The first number.

Hereby I know that if three moneths boording, doe come to 16 s, that 8 moneths boording will come to 42 s, 8 d: and likewise of any other like question.

But here must you marke, that the first number, and the third be of one denomination, and also the second & the fourth, for which

which you seeke: or else be of such denominations, that you in working may bring them into one. As if a man should aske me this question.

Twelue weekes iourneying cost me 14 French crownes at 6 s. the peece, how many pounds is that in one yeere? Here you see no two numbers of one denomination, but yet in working you may turne them into like denomination: as thus, turne the one yeere into 52 weekes, and the fourth summe will be French Crownes, by the order of the working. Then to know this question, multiply the third sum 52, by the second 14, & the summe will be 728: that diuide by your first number 12, and the quotient will be 60 Crownes, & 8 Crownes remaining: which if you turne into shillings, they will be 48 s; which if you diuide by your first number 12, the quotient will be 4, which signifieth 4 s: put those 60 French Crownes, which make 18 pounds with the 4 shillings, for the sum that answereth 12 14 to the question: & it the iust expences of a yeere: And the summe will be thus. 52 60-4 s

Question of expences.

And take this euermore for a generall rule touching this whole Art, that the doubt or vnknowne number that you would be resolued of, shall alwaies be set in the third place. Note also the first number and the third must euer

A generall Rule.

be of one nature and denomination, or else must in working bee brought to like denomination, and then of necessity must the other number be in the second place.

Remember also that the place of the first number is the highest on the left side: and the place of the second right against it on the right side; the place of the third number is vnder the first, as by those examples you haue seene.

Scholar. This I trust I can doe.

Master. But and if the question be asked thus: In 8 weekes I spend 40 s. how long will 105 shillings serue me? Here you see that 8 wæekes answeres himselfe, & saith 40 shillings. But how long time 105 shillings will serue you know not. Therefore you shall set 105 in the third place, according as I told you euen now. And the first place must alwaies be of the same nature or denomination that the third is of, which here is 40. The 8 must 8 needs be that other. Now multiply 105 by 8, & it will be 840, which if you diuide by 40, it will yeild 21, which is the fourth number, and sheweth how many wæekes 105 shillings will serue, if you spend 40 s. in 8 wæekes.

The figure of this question is this: as if you should say: if 40 s. serue for 8 weeks; 105 will serue for 21 weeks,

Shillings.	*Weeks.*
40	8
105	21

Other

Other diuersities there be of worcking by this rule, but I had rather that you would learne this one well, then at the beginning to trouble your mind with many formes of worcking, sith this way can doe as much as all the other, and hereafter you shall learne the other more conuenientlye.

¶ And for your further aid and instruction, to make you better acquainted with this Golden Rule, I haue here proponed six questions and their answeres, which I think most conuenient and meete to preferre the desirous to perfect vnderstanding. The first foure are all branches of one question sprung out of the best tree (for a yong learner to taste of) that groweth in this *Ground* of *Arts*: for that no manner of question in the Rule of 3 whatsoeuer it be, can be proponed, but it must be comprehendred vnder the reason or stile of one of these foure. Note.

The Questions.

If 15 *elles of cloath cost* 7 ℔, 10 ß: *what comes* 27 *elles to at that price?* Answere 13 ℔ 10 ß.

If 27 *elles cost* 13 ℔, 10 ß: *what are* 15 *elles worth?* Answere: 7 ℔, 10 ß.

If 27 *elles cost* 13 ℔, 10 ß: *how many elles shall I haue for* 7 ℔, 10 ß? Answere: 15 *Elles.*

If I sell 15 *elles for* 7 ℔, 10 ß: *how many*

elles are to be deliuered for 13 *l*, 10 *s*, Answere: 27 *Elles*. *If* 8 *pound of any thing cost* 16 *s*,6 *d*: *what money is to be receiued for* 49 *pound?* Answer,5 *l*,1 *s*,0 *d* $\frac{3}{4}$.

If 4 *l*, *of any thing cost* 17 *d*: *what money will* 876 *pound of that commodity cost*? Answer,155 *l* 4 *s*,3 *d*,q;

Of all which questions, I omit the worke of purpose, that you shall whet your wit thereby at conuenient leasure, to clime each branch, and gather the fruit of them, and do mind now, before we make an end of this rule, to giue you some instructions of the backer rule of 3, whose order is quite contrary to this that you haue learned.

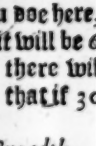
For in this rule hitherto, euermore loke how much the third number is greater then the first, so much the fourth number is greater then the second. And contrariwise: loke how much the first summe is greater then the third, (if it doe chance so) so much is the second summe greater then the fourth. But in this rule, there is a contrary order, as this: That the greater the third summe is aboue the first, the lesser the fourth summe is beneath the second: and this rule therefore you may cal the Backer rule, as in example.

If I haue bought 30 yard of cloath of two yards breadth, and would haue Canuas of three yards broad to line it withal, how many yards should I need?

Scholar.

Scholar. Why, there is none so broad.

Master. I doe not care for that, I do put this example onely for your ease vnderstanding: For if I should put the example in other measures, it would be harder to vnderstand. But now to the matter: If you would know this question, set your numbers as you did before: but you shall multiply now the first number by the second, and that ariseth thereof, you shall diuide by the third: which thing if you doe here, I meane if you multiply 30 by 2, it will be 60: which sum if you diuide by 3, there will appeare 20: whereby I know; that if 30 yards of cloath of two yards broad, should be lined with canuas of three yards broad, 20 yards of canuas would suffice, as this figure sheweth. Note.



And now because ye found fault with my Example, how say you, perceiue you this?

Scholar. Yes Sir, I suppose.

Master. *Then answere me to this question: how many elles of canuas of elle breadth, will serue to line 20 yards of Say, of three quarters broad?*

Scholar. In good faith Sir I cannot tell, for I know not how to bring the sums to like denominations. Ma-

Master. Then will I tell you, sith there is mention here of quarters, and againe euery one of the measures both elles and yards may be parted into quarters, doe you part them so both in the breadth and length, and then put forth the question by quarters.

Scholar. Then I shal say thus. How many quarters of canuas of 5 quarters broad, will line 80 quarters of 3 quarters broad?

Master. Now answere to the question.

Scholar. First I will set them downe in their forme thus, for 5 is ioyned with the question, and is therfore the third number: then is 3 the number of the same denomination, I meane because they be both referred to breadth. Now I multiply 80 by 3, and it is 240, which I diuide by 5, and it yeeldeth 48. Then say I that 48 quarters of 5 quarters broad, will suffice to line 80 quarters of three quarters broad.

Breadth. Length.

3 — 80

5 — 48

Master. Turne the quarters againe into elles and yards.

Scholar. Then I say, that 9 elles 3 three quarters of a yard of elle broad, will serue to line 20 yards of three quarters broad, as this figure sheweth.

Breadth. Length.

3 — 80

5 — 48

¶ Master. Now what say you to this question

queſtion: I lent my friend 400 ℔ for 7 moneths, how much money ought hee to lend me againe for 12 moneths, to recompence my curteſie ſhewed him? Can you anſwere to this?

Scholar. Yes Sir, I ſuppoſe, for I will ſet downe my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I diuide by 12, and it yeldeth 233 ℔, and there is 4 ℔ remaining of my Diuiſion, what doe I therewith?

Moneths.	*Pounds*
7	400
12	

Maſter. Turne the ſame 4 pound into s̄, and then diuide it by 12 as you did before.

Scholar. Well Sir, it ſhall be done, ſo haue I 6 s̄ for my quotient, and yet remaineth 8 s̄ vpon my Diuiſion.

Maſter. You muſt alſo reduce that 8 s̄, into pence, which maketh 96, & diuide that alſo by your Diuiſor.

Scholar. So haue I done, and I find 8 d̄ for my quotient, and nothing is left.

Maſter. This muſt you alwaies doe when any thing remaineth vpon your Diuiſion: whether it be money, weight, meaſure, or any kind of thing whatſoeuer. This Rule is ſo profitable for all eſtates of men, that for this Rule onely (if there were no more but it) all men were bound highly to eſteeme Arithmeticke.

By

By this rule may a Captaine in warre worke many things, as Master Digges in his Stratiaticos doth declare: Onely now in this my simple addition, for a taste and incouragement, I will inlarge the Author with a question or two more, wishing you and euery my Countreymen or Gentlemen whatsoeuer, that by nature be any thing giuen to Military affaires, to be familiar and acquainted with this Excellent Art, the which he shall find not onely at the Sea, but also in the Campe and Field-seruice, and abundantly to aid him, either in fortification, paying of Souldiers wages, charges of Ordinance, Powder, Shot, Munitions, and Instruments whatsoeuer, as for example.

If it should chance a Captaine which hath 40000 *Souldiers to be incloſed with his Enemy, that he could haue no freſh purueyance of victuals, and that the victuals which he had would ſerue that Army but onely three moneths, how many men ſhould hee diſmiſſe to make the victuall to ſuffice the reſidue eight moneths.*

Scholar. As you taught mee, I set the numbers thus, saying: If three moneths, suffice 40000, to how many will eight moneths suffice.

Moneths.	*Men*
3	40000
8	

To know this; I multiply the first number

ber 3 into the ſecond 40000, and it yeelds 120000, which ſumme I diuide by 8, and there will be in the quotient 15000, which if I doe ſubtract from *Moneths. Men.*
40000, the remainer will declare that hee must diſmiſſe 25000 as this figure sheweth.

M. Maſter. Now anſwere me to this queſtion: If 136 Maſons in a moneth be able to build a Fort to preſerue the Souldiers from the Enemy, and ſuch expedition requireth that I would haue the ſame finiſhed in eight daies: How many workemen ſay you is there to bee appointed.

Scholar. As you taught me; I ſet the numbers thus ſaying: If 28 dayes require 136 Maſons, what number of men by the like proportion will 8 dayes require.

To know this, I multiply the firſt number 28 into 106; and it yeelds me 3808: which I diuide by 8. And my quotient is 476, which is the iuſt number of Maſons that ſhall ſupply this worke. And now me thinke theſe queſtions are very eaſie.

Maſter. Truely if you take delectation herein, you ſhall find this Art not onely eaſie, but wonderfull pleaſant & profitable. Now therefore one queſtion more I will propoſe,

pose, and so leaue off this Rule in whole numbers, vntill we come to the vse of it in broken numbers: for had you the vnderstanding of broken numbers perfectly, not onely in this Rule, but in all other, the question that in sight or apparance seemeth to be 100 times more harder to absolue, may thereby be wrought as soone or sooner then this.

Scholar. Your words doe greatly encourage mee to bee studious to attaine whole numbers: but might I once attaine to bee a practitioner in broken numbers, I should thinke my selfe happy.

Master. *What say you then to this question? If 48 Ioyners in two daies make 200 light horsemens staues, (esteeming they worke but 12 houres a day) and such need requireth that 384 Ioyners are set to the finishing of those 200 staues; in what time say you, will they make them vp?*

Scholar. I see here that I must turne my 2 dayes into houres. And so doing I set my numbers thus.

48 24
384

Saying if 48 men are 24 houres, 384 men will make an end quickely. For it is grounded vpon an old Prouerbe, *Many hands make quicke speed.*

I multiply 48 into 24, and it amounteth to

to 1152, which I diuide by 384, & my quotient is three houres, which is my desire.

I take this for a note worthy the marking, either in the rule of three, forward or backeward, when the two numbers are multiplied together, the Produst is of the same nature and denomination, that the second number is of. Note.

Master. Wel, sith you perceiue now the vse of this rule, I will shew other which ensue of the same, and first the double rule which is so called, because there is in it double working, by which thing onely it differeth from this. The double Rule.

Scholar. Then by an example I shall vnderstand it well enough.

Master. *So shall you, and let this be the example: If the carriage of* 100 *weight,* (*that is* 112 *pound*) 30 *miles doe cost* 12 *ů, how much will the carriage of fiue hundred weight cost, being carried* 100 *miles?* Of carriage.

Scholar. I pray you shew me the working of it.

Master. You must make two workings of it: the first thus. If 1 *C weight* cost 12 ů, how much will fiue hundred *weight* cost? Set your figures thus.

C Weight.	*Pence.*
1	12
5	

And multiply 5 by 12, & thereof amounteth 60, which if you diuide by one, the quotient will be still 60, that is the price of

of fiue hundred weight for 30 miles.

CC miles

Then begin the second worke, saying: If 30 miles cost 60 ō, how much will 100 miles cost? Set you figures thus.

Miles.	Pence.
30	60
100	

Then multiply 100 by 60, whereof amounteth 6000, which being diuided by 30, will yeeld 200 ō. Then you may say, that so many pence shall cost the carriage of 500 pound weight 100 miles, after the rate of 12 pence for the 100, carried 30 miles.

Scholar: Now I perceiue it also.

¶ Scholar. These and such other like questions of the double Rule of three, are to be answered much sooner, at one only working by the Rule of three composed of fiue numbers, which anone I will shew you, and then when you haue the vse thereof, you may vse which way you thinke good.

Scholar. Sir, I thanke you much for your courtesie, and I long now till this rule be ended, that I may see how I shall behaue my selfe with that new rule of fiue numbers: for that I haue euer since you taught me hitherto, in the Golden Rule both forward & backeward wrought but with three numbers onely.

Master. But yet a while we will goe on forward with this Rule of three, therefore answere to this question: thirty bushels of wheate

wheate sowed, yeelded in one yeere 360, how many wil 80 bushels yeeld in 7 yeeres? I meane sowing euery yeere of those seuen, still 80 bushels. Question of sowing.

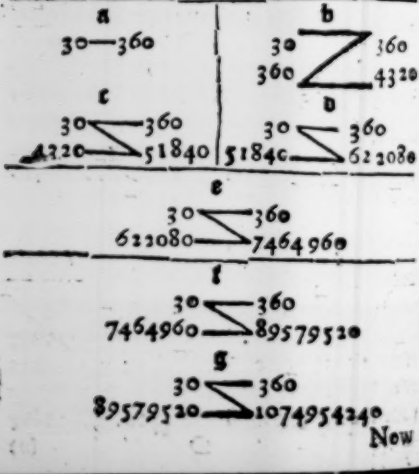
Scholar. First I say, that if 30 bushels will yéeld 360 in one yeere, then 80 bushels will yéeld 960 in one yeere. Then for the second worke I say: If one yeere yéeld 960, then 7 yeeres will yéeld 6720: as these two figures doe shew.

Seede.	*Encrease.*	*Yeere.*	*Encrease.*
30	360	1	960
80	960	7	6720

But now Sir, if I set forth 30 bushels of Corne to another man for 7 yeeres, agreeing so that he shall sow euery yeere the whole increase of the Corne, and I at the end of those 7 yeeres to haue the halfe of the whole increase: I would know how many bushels will there amount to my part, supposing the increase to be after the rate of the last question, for 30 bushels in one yeere to yeeld 360. Question of Corne.

Master. In such a question you must haue so many seuerall workings as there be yeeres, as for example: in the first yeere 30 bushels yéeld 360: the to know the yéelding of the second yeere, I must say: If 30 yéeld 360, how many yéeldeth 360? Worke by your Rule, & you shall find 4320. Then say

for ỹ 3 yere. If 30 yeld 360, how many wil 4320 yeld? you shal haue 51840, & so euery yer multiplying the whole increase by 360 & diuiding it by 30, the increase of the next yer wil amount, as these 7 figures following do orderly declare: where I haue set 7 letters for the 7 yeres, of which the first is set without art, because that is the increase which you do presuppose: and the last number of each other doth shew the increase of ỹ yere that it standeth for, which the letters do declare, so that the increase of the seuenth yere, is 1074954240 bushels: how many quarters that is, and also how many waies you may by Reduction some finde.



Now

Now with one queſtion more I will proue you. *If ſixe Mowers doe mow 45 acres in fiue daies, how many mowers will mow three hundred Acres in ſix daies.* Queſtion of mowing

Scholar. If 45 acres require 6 Mowers, then 300 Acres require 44. For againe: if fiue daies require 40 Mowers, then 6 dayes require but 33 Mowers.

Maſter. Why doe you not make mention of the 2 that remaineth in the laſt Diuiſion? for the laſt part of the queſtion is wrought by the Backer Rule, where the firſt number 5 is multiplied into the ſecond, that is, 40, whereof amounteth 200, which if you diuide by the third number 6, the quotient will be 33, as you ſaid: but then will there remaine two, which cannot well be diuided into 6 parts: howbeit you may vnderſtand by the 6 part of 2, the third part of one Mans worke, which you muſt put to the 33: or elſe you muſt ſay, that 33 workemen will end all the 300 acres in 6 daies, ſaue 2 mens worke for one day, or 2 daies worke for one man. But ſuch broken numbers called Fractions, you ſhall hereafter more better perceiue, when I ſhall wholly inſtruct you of them.

¶ Maſter. *Yet one queſtion more of field matters I will propone, and ſo I will make an end of this double Rule of three.*

Scholar. With all my heart Sir I thanke

you, and I wil dispatch it as soone as I can, because I would faine see the order of the next Rule of 5 numbers.

Question of entrenching.

Master. *If a Captaine ouer a band of men did set 300 Pioners a worke, which in eight houres did cast a trench of 200 Rods: I demand how many labourers will bee able with a like trench in three houres to intrench a Campe of 3400 Rods.*

Scholar. I thinke I am now in the Backhouse ditch: for I know not wel which way to goe about it. And besides that truely I thinke I shall neuer come to preferment that way, my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and service into the fauour of your Prince, for the auaile of your Countrey.

Example for Nauigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, is not the tallest man, and yet hath made as great a venture for the honor of his Prince and Country, as euer Englishman did

Scholar. Sir, I thanke you for your good encouragement. My mind, though I be little, is as desirous of knowledge, as any other: I haue pondred now a little of it, and thus I set forth the worke.

Rod.	Men.
200	300
3400	

Saying,

Saying, if 200 Rod require 300 men, what shall 3400 rods require? I multiply 3400 by 300, and it yeldeth 1020000, which I diuide by 200, and my quotient is 5100 men.

Then must I say for my second worke, if in 8 houres 5100 men, be able to discharge it, how many shal performe the same in three houres? Now if I would worke by the Golden rule of proportion forward, I should find a lesse number of men: because three houres is lesse then 8 houres: but because reason teacheth me that the lesser the time is, wherein the trench must be made, the more Labourers I ought to haue, therevpon I vse now the Backer Rule, as in example. And I haue in my quotient 5000. So many Pioners must I haue to entrench the Campe in three houres.

Master, You haue answered the question very artificially: And truely I commend you for your diligence and apt vnderstanding: and now according to my promise, I will (in whole numbers) giue you a little taste of the Rule of 3, compounded of fiue numbers.

The Rule of 3. compound of 5 Numbers.

THis Rule of three composed, is distinct for most needfull questions, into two seuerall parts or workings: And there belongeth vnto it alwaies fiue numbers, whereof in this rule being the first part, the second number and the fift are alwaies of one nature and like denomination, which Rule is to be wrought thus: you must multiply the first number by the second, and that shall be your Diuisor: Then againe, multiply the other 3 numbers, the one by the other, and their product shall be your dividend.

And now according to my promise, we will first worke the question of weight & carriage, which I deliuered you in the double rule of three, to be absolued by this rule, which was this.

If the carriage of 1 C weight 30 miles cost 12 d, what will the carriage of 5 C weight stand me in being carried 100 miles?

C. weights. Miles. Pence. C. weight. Miles.

1 ——— 30 ——— 12 ——— 5 ——— 100

Now marke well how these fiue numbers stand: Then multiply the first number by the second

second, as 30 by 1, which maketh but 30, & that number kéepe for your Diuisor. Then multiply the other three numbers, the one into the other: that is to wit, 12 by 5, which maketh 60: Lastly 60 by 100, which as you sée here in our Tables, ariseth to 6000, which 6000 you shall diuide by the product of the two first numbers, which here is 30. And as you sée there is found 200 ℔, which is the duety that you ought to pay for the carriage of 500 waight 100 miles, after the rate of 12 ℔ a hundred, and agréeth with the conclusion of the double Rule of three.

Scholar. Sir I thanke you, it is euen so.

Master. *Yet note this for a generality in this rule, looke what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwaies your quotient.* Note this.

Scholar. Well now & if it please you, by your patience, I will sée how I can end the question next following of 30 bushels of wheate sowed, which in one yeere yelded 360: how many then will 80 bushels yéeld in 7 yeer, sowing euery yeer of those 7 still, 80 bushels, & according to your reasons I set my numbers thus.

Bushels.	*Yeer.*	*Bush.*	*Yeer*
20	1	360	80
			7
			80
			28800
			7
			201600

Where I multiply 30 by 1, & it maketh 30 my Diuisor: then multiplying the other 3 numbers the one into the other, as here appeareth in my tables, they make 201600 which I diuide by 30: and my quotient is 6720 bushels my desire, for so much also it came to at 2 workings by the Rule of three.

Master: Yet one question more I will propound vnto you, and so leaue this rule till it please God hereafter, that I may make you worke it in broken numbers.

What comes the interest of 258 *l. for five moneths to, after the rate of* 8 *l. taken in the* 100 *l. for* 12 *moneths?*

Scholar. Sir, this is yet within the compasse of some reasonable skance. Therefore to minister equity in this case, I will see how I can worke the same, which I *l. moneths.* l. *l. moneths* set odwne thus, 100-10--8-248-5 praying you if I haue not done well, to shew me mine error.

Master. Proceed, you haue done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yeldeth 1200, and the 3 others numbers multiplied together produceth 10320, w̄ I diuide by 1200: and my quotient is eight pounds. Thē according as you haue taught me heretofore, I turne the 720 *l.* that I left, into

into shillings: and dividing it by the first number, my quotient is 12 s. So I answere that the loane of 25 l for 5 months, after the rate of 8 l in the 100 l for a yeere, comes to 8 l, 12 s.

Master. You say true, I commend your diligence: now behold the manner of the second part of this rule.

Master. *In the second part of this rule of three composed, the third number is like vnto the first. And the rule is to be wrought as thus: you shall now, contrary to the last rule, multiply the third number and the fourth together, and that Product shall be your Diuisor. Then multiply the fift by the second, and the Product thereof by the first: and that is the number that shall be diuided. For example, I propound this question, for a proofe of my last question of interest. A Merchant hath receiued 8 l, 12 s, for interest of certaine money for 5 months terme, which he receiued after the rate of 8 l in the 100, for a yeere. The question is now, how much money was deliuered to raise this interest.* Behold therefore the manner, how the question is

l	moneths.	l	moneths.	l	s
100	12	8	5	8	12

forth.

The 2 part of the rule of 3 compound.

The proof of the last question.

Scholar. Sir I perceiue it very well: and according to the doctrine which you prescribed for the working thereof: it please you now

now it is set downe, I thinke I can solue the worke.

Master. May stay a while, and before you worke, marke well how I deliuer a reason for the perfect vnderstanding of this rule, which is thus: If 8 £ in 12 moneths doe yeeld me 100 £, to take 8 £, 12 s, for 5 moneths, must needs yeeld a great deale more.

Note

So vpon y knowledge that I haue in this Art, the first part of this rule is answerable to the rule of three forward: And this latter part accordeth to the rule of three backward.

Scholar. Sir, I yéeld you most hearties thanks for these your last instructions, they haue giuen me great light into these two rules, whereby I may the better by deliberation conceiue how to vse them hereafter, when occasion shall require.

Master. You say well, goe to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings, for your more ease in worke.

Note.

Sch. If it please you to behold me a little, I will quickly end it: for I haue but my first, my second, and my last number to be multiplied together for my diuidend: And my third into my fourth for my Diuisor.

100 £

Moneths. *l*, *Moneths.* *l*, *s.*

100—12—8——5——8—12

20	20	20
2000	160	172
12	5	
4000	800	
2000		
24000		
172		
48000		
168000	4128000 (	
24000	8 00	
4128000		

Which 4128000 I diuide by 800, and my quotient is 5160 s̄, which in pounds réeldeth 258, my desire.

Master. I will héere for this time in whole numbers end this rule, and I will instruct you in the rules of fellowship. You may at your conuenient leasure for your exercise work the same by the Rule of three at twice. And for your aide & incouragement therin, I set downo héere a proffer how to apply it.

A			B	
Moneths.	l'	s.	*Pound.*	l'
	8	12	8	100
12	413 4/5 s̄		412 4/5	258 l'

The

The rule of Fellowship.

But now will I shew you of the Rule of Fellowship or Companie, which hath sundrie operations, according to the diders number of the Companie. This Rule is sometime without difference of time, and sometimes there is in it difference of time. First I will speake of that without differnnce of time, of which let this be an example.

A question of companie.

Foure Merchants of one companie made a banke of money diuerſly: for the firſt laid in 30 l', the ſecond 50 l', the third 60 l', and the fourth 100 l': which ſtock they occupie ſo long, till it was increaſed to 3000 l'. Now I demaund of you what ſhould each receiue at the parting of this money?

Scholar. I perceiue that this rule is like the other, but yet there is a difference, which I perceiue not.

Maſter. Then will I ſhewit to you. Firſt by Addition you ſhall bring all the particular ſums of the Merchants into one ſum, which ſhall be the firſt ſumme in your working by the Golden Rule, and the whole ſumme of the gaines by that ſtocke ſhall be the ſecond ſum. Now for the third ſumme you ſhall ſet the proportion of each man one after another, and then worke by the Golden Rule, and the fourth ſumme will ſhew you each mans gaines: as in example.

30
50
60
100
240

The

The parcels of those foure Merchants make in one sum 240 pounds: set that in the first place, the gaines in the second, and the first mans portion of stocke in the third place thus.

Now multiply the second by the third, & it will be 90000, which you shall diuide by 240, and there will appeare 375 l. thus.

And that is the gains for the first man.

Now for the second man, set the 50 l. that he brought, in the third place, & worke as before: and his part will be 625 l: as this figure sheweth.

Likewise for the third man, set his money w̄ was 60 l, & his part of gains wil be 750 l, as here appeareth.

And so for the fourth man, if you set his sum which is 100 l, his gaines will be 1250 l, as ẏ worke will declare.

Scholar. This I perceiue: but is there any way to examine whether I haue well done or no?

Master.

Maſter. For the triall hereof, adde together all their foure portions, and if their addition make the whole ſum of their gaines, then is the worke well done.

Proofe. Scholar. That will I trie by and by, the foure parcels are theſe,

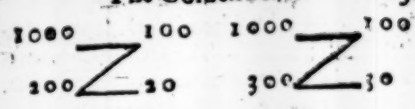
which added together,	375
make 3000, which is	625
the iuſt ſum of money	750
that they gained, where-	1250
by I know the worke is	3000
well done.	

Maſter. Well, now another example will I put to you, not of gaines, but of loſſe: for one reaſon ſerueth for both.

A queſtion of loſſe. If three Merchants in one ſhip, and of one fellowſhip, had bought Merchandiſe, ſo that the firſt had laid out 200 l', the ſecond 300 pound, and the third 500 l', and it chanced by tempeſt that they did caſt ouer board into the Sea merchandiſe of the value of 100 l', how much ſhould each man beare in this loſſe?

Scholar. If I ſhall do in this, as you did in the other queſtiõ, then muſt I ioine their three portions together, 200, 300, and 500, which maketh 1000. Then ſay I, if 1000 loſe 100, then ſhall 200 loſe 20, and 300 ſhall loſe 30, and 500 ſhall loſe 50, as by theſe three figures it doth appeare plaine.

1000



Master. Well, sith now you haue done these, I will propound a question of more importance, which shall make you not only the abler to vnderstand this rule, but also it wil greatly aide you in the next Rule of Fellowship with time, if such neede be that your money be of diuers denominations.

For this may not be forgotten in al such questions: if the number be of diuers kinds, you must by Reduction bring it into one kind, that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of diuers kindes, as some yeeres, some moneths, weekes, and daies, you shall make all moneths, weekes or daies, according as the last name of time in the question is, as for example. Note.

First in diuersitie of money. Three companions bought 2000 sheepe. & paid for them 241 l'—13—4 d. of which summe one paid 101 l'—1—8 s. The second 82 l', 17 s, 10 d. And the third paid 52 l', 5 s, 6 d. How many sheepe A question of sheepe.

sheep must each of them haue? *Answere*: The first shall haue 840. The second 686. And the third 474. And that must you work thus.

Solution. First, considering that your money is of diuers denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pence; and: so will the totall sum be 58000 pence.

Now, if you turne each mans money into pence also, the first mans sum will be 24360 pence: The second mans sum 19894 pence: And ỹ third mans money will be 13746 pence.

Now to know how many sheepe euery man shall haue, let the whole summe of money, that is, 58000 pence be set in the first place, and in the second place set the number of sheepe, and then orderly in the third place set each mans money, and then multiplying the third and the second summes together, and diuiding that that amounteth by the first, there will appeare the number of sheepe that each man ought to haue: as these three figures doe shew.

a

58000 — 1000
24360 — 840

b

58000 — 1000
19894 — 686

c

58000 — 1000
13746 — 474

Scholar

Scholar. Why doe you set the money in the first place, seeing in the question you say 2000 sheepe cost 58000 pence, and not thus, 58000 cost 2000 sheepe.

Master. You remember I taught you at the beginning of this Golden Rule, that the first and third numbers must bee of one name, and of like things: and euermore the number that the question is asked of, must be set in the third place. Now is the question plainly this: If foure men bought 2000 sheepe for 58000 pence, how many sheepe shall each man haue.

But seeing in this question, there ought more respect to be had to the sum of money then to the summe of the persons, (for in the sums of money is their proportion toward the sheepe, & not in the number of persons) therefore must we turne the question thus. If 58000 pence bought 2000 sheepe, how many did 24360 buy? Againe, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceiue it reasonably, and so shall I doe in all like questions.

Master. Euen so. But for easinesse of the worke, marke this: Whensoeuer the first & second numbers haue cyphers in the first places, you may both in ỹ Multiplication & in the Diuision leaue out those cyphers, so that you leaue out like many out of both

Note.

summes, as in this question, the first number 58000 hath three cyphers, and so hath the second, that is 2000: therefore cast away their cyphers, and so will the first number be 58, and the second 2: set them in their places, and worke according to the Rule, & you shall perceiue that it will be all one, sauing that this is the shorter and easier way, as these three figures doe shew.

a

b

c

And this you see is both easier, and also the more certain way to know the answer to this question.

Scholar. Truth it is as you say: but sir me seemeth I might aske a further question here, not onely how many sheepe each man should haue, but also what euery sheep cost.

Master. That question doth not onely belong to this Rule, but may also be discussed by Diuision, especially if the questions number be one onely, as thus: Diuide the totall sum 58000 pence by 2000 (or 58 by 2, omitting the cyphers) and the quotient will be

29

29 pence, that is 2 s̄, 5 d̄; howbeit, by this rule you may doe it, and best when the number of the question doth excéede 1; as if I should ask this question,

2000 shéep cost 58000 d̄, how much doe 20 cost? 2000 — 58000 / 20

Then shall I set my figure thus.

And doing after the rule there will amount 580 pence, that is, 2 l̄, –8 s̄, –4 d̄, the price of one score: but if you will vse that easie way that I did teach you now may change the first and second number thus. 2 — 58 / 20

Thus doe you perceiue the vse of the rule without time. And that you may as well perceiue the same with diuersity of time, I propose this example. The Rule of fellowship with time.

Foure Merchants made a common stocke, which at the yeeres end was increased to 55145 l̄. Now to know what shall be each mans portion of gaines, you must know each mans stockn and time of continuance. Question of a banke.

The first man of these foure laid in 669 l̄, whith he did take from the stocke againe, at the end of 10 moneths. The second man laid in 810 l̄, for eight moneths. The third laid in 900 l̄, for seuen moneths. And the fourth laid in 1040 l̄, for 12 moneths.

This question shall you examine as you did the other before, sauing that whereas in the third place of the figure you did set each Note. A generall Rule.

mans summe alone, here you shall set the same being multiplied by the number of their time, and likewise in the first place of the figure you shall set the number which amounteth of their whole summes so multiplied by their time, and added into one whole summe, as thus.

236
895

The first mans summe is 669 l. which I multiply by 10 (that was the number of his time) & it maketh 6690. The second mans summe 810 l. multiplied by 8, (which was his time) maketh 6480. The third mans summe 900 l. multiplied by 7 (for that was his time) yeldeth 6300. The fourth mans summe was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

These foure sums thus multiplied by their time, must be set orderly in the third place of the figure, and in the first place must be set the whole summe of all foure, which is 31950, and the gaine must be in the second place, which is 35145. Now to end the question, I say first, If 31950 did get 35145, what did 6690.

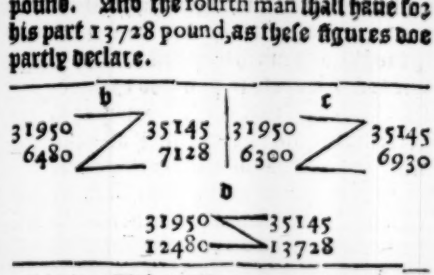
get? Answere. a

7359 l. as by this figure appeareth.

31950 | 35145
6690 | 7359

Likewise the second man had to his part 7128 pound, the third must haue 6930 pound.

pound. And the fourth man shall haue for his part 13728 pound, as these figures doe partly declare.



Scholar. This I like very well: but what proofe is there of this worke?

Master. The same that I taught you for the other, to adde all the portions together, and it may agree to the whole summe, then is your worke well done. Another proofe.

Scholar. Then will I proue in this example: The foure parcels are these, which if I adde together, there will amount 35145, and that was the whole summe, whereby I perceiue the worke is well done.

	7359
	7128
	6930
	13728
	35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then doe I vnderstand this worke also very well: but what haue I now to learne?

Mstaer. There are many other excellent parts behind, of which I will not as now

make mention, because that without the knowledge of fractions they cannot be duely taught, and much lesse vnderstood. Therefore will I propose to you two or three questions more, (that thereby you may better perceiue the vse of this rule & all other the like) and so make an end for this time.

A question of losse.

¶ Three partners, by some ill aduenture sustained the losse of 160 l, whereof the first laid into the common stocke 200 l, for ten moneths. The second laid in 350 l, and the third 100 l, but for how long the two latter is vnknowne. But breaking off their partnership, the first found himselfe a loser 80 l, the second 56 l, & the third 24 l. The question is, for how long time was the money of the two latter in company?

For the solution hereof, and of such other like, you must also multiply the first mans 200 l, that he put into the stock by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme if his money that lost 80 l, multiplied by his time, make 2000: what shall his money make that lost 56 l, and his that lost 24 l, which 2 numbers I commit to the triall of the rule of three, at two workings thus: If 80 giue 2000, what giueth 56? And againe, if 80 giue 2000, what giueth 24.

80 — 2000
56 — 1400

80 — 2000
24 — 600

To

To conclude, if you now diuide 1400, the second mans portion by 350, which was his stocke that he laid into Company, you shall find in your quotient 4 moneths, and for so long time did the second man put his money into the common stocke.

Lastly, if you diuide the third mans now laying in, which was 600 by 100, which was his stocke that he put into Company: the quotient declareth his time of continuance, which was 6 moneths. And thus is the quotient resolued.

Scholar. Sir, I haue attentiuely beheld your working, and the more we trauel herein, the more me thinke I am in loue with this excellent Art.

Master. Then what say you to this question?

There is in a Cathedrall Church 20 Cannons & 30 Vicars, those may spend by yeer 2600 l, but euery Canon must haue to his part fiue times so much as euery vicar hath: how much is euery mans portion, say you? A question of Cannons

Scholar. I pray you make the answere your selfe also, so shall I perceiue best the meanes to answere to such other like.

Master. In this question you must doe as in those before said, that haue diuersity of time, for here is diuersity of portions. Therefore shall you multiply the number of the persons by their difference of portion: (as

you did in the other by time:) Then must you multiply the 20 (which is the number of Canons) by 5 (for that is the number of their portion) so will it be 100: Then 30 (that is the number of vicars) by 1, (that is the number of their portion) and it will be 30: put these two summes together, and they make 130: then say thus: If 130 spend 2600 l, what may 100 spend? The rule sheweth 1000 pound.

Vicars. Againe for Vicars: If 130 spend 2600 l, what may 30 spend? Answer, 600 l, as these figures shew.

130	2600			130	2600
100	2000			30	600

But if every Canon should have so oftentimes 4 l, as the Vicar should have 3 l, then should I multiply 20 by 4 (that were 80) & 30 by 3 (that were 90) and then both were 170. Then should the figures be set thus.

l	s	d		l	s	d
170	2600			170	26000	
80	1223	-10.7		90	1376	-9--5

But this sort is too hard for you, by reason of the fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend, which summe if you divide by 20, you shall see each Canons portion: and so of the Vicars, if you divide their summe by 30, the quotient will declare every Vicars portion.

The

The second Dialogue.

The counting by Counters.

Master.

NOW that you haue learned the common kindes of Arithmeticke with the pen, you shall see the same Art in Counters: which feat doth not only serue for them that cannot write and reade, but also for them that can doe both, but haue not at sometime their pen or tables readie with them.

This sort is in two formes commonly. The one by lines, and the other without lines. In that that hath lines, the lines doe stand for the order of places: and in that that hath no lines, there must be set in their stead so many counters as shal need, for each line one; and they shall supply the stead of lines.

Scholar. By examples I should better perceiue your meaning.

Master. For example of the lines, loe here you see sixe lines, which stand for sixe places, so that the neather

—100000—
—10000—
*—1000—
—100—
—10—
—1—

Numeration by Counters.

nethermost standeth for the first place, and the next above it for the second, and so vpward till you come to the highest, which is the sixt line, and standeth for the sixt place.

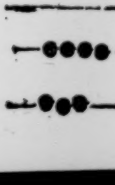
Now what is the value of euery place or line you may perceiue by the figure, which I haue set on them, which is according as learned before in Numeration of figures by the pen: for the first place is the place of vnites or ones, and euery counter set in that line, betokereth but one: and the second line is the place of 10, for euery coũter there standeth for 10. The third line the place of hundreds, ỹ fourth of thousands, & so forth.

Master. Sir, I doe perceiue that the same order is here of lines, as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

Master. If you doe vnderstand it,—— then how will you set 1543? *1 ——

Scholar. Thus as I suppose. 5 ——

Master You haue set the places truly, but your figures be not mete for this vse: for the meetest Figures in this behalfe, is the Figure of a Counter round, as you see here, where I haue expressed that same summe. 4—— 3——



Scholar.

Scholar. So that you haue not one Figure for 2 nor 3, nor 4, & so forth, but as many Digits as you haue, so many Counters you set in the lowest line, & for euery 10 you set one in the second line, & so of other. But I know not by what reason you set that one Counter for 500 betweene two lines.

Master. You shall remember this, that whensoeuer you need to set downe 5, 50, or 500, or 5000, or so forth any number whose Numerator is 5, you shall set one counter for it in the nert place aboue the line that it hath his denomination of: as in this example of that 500, because the numerator is 5, it must be set in a void space, & because the denominator is a hundred, I know that his place is the voide space nert aboue hundreds, that is to say, aboue the thirde line.

And further you shall marke, that in all working by this sort, if you shall set downe any summe betweene 4 and 0, for the first part of that number you shall set downe 5, and then so many Counters more, as there rest numbers aboue 5. And this is true both of Digits & Articles. And for eram ple I wil set downe this sam summe 297965, which sum if you marke well, you

nert

need none other examples for to learne the numeration of this forme.

But this shal you marke, that as you did in other kindes of Arithmetick, set a pricke in the places of thousands, in this worke you shall set a Starre, as you see before.

Scholar. Then I perceiue numeration: but I pray you how shall I doe in this Art to adde two summes or more together.

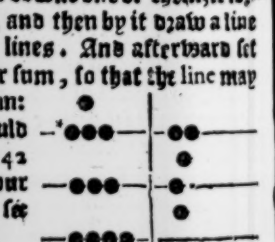
Addition.

Master.

He easiest way in this Art is to adde but two summes at once together: Howbeit you may adde more, as I will tel you anone. Therefore when you will adde two sums you shall first set downe one of them, it forceth not which, and then by it draw a line crosse the other lines. And afterward set downe the other sum, so that the line may be betweene them: as if you would adde 2659 to 8342 you must set your sums as you see heere.

And then if you list, you may then adde the one to the other in the same place:

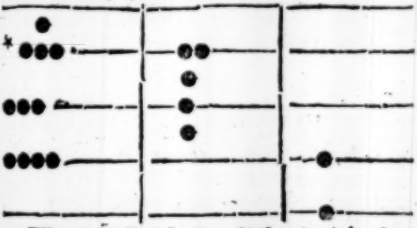
4 Addition of two summes.



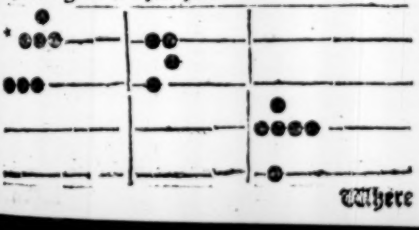
or

or else you may adde them both together in a new place: which way because it is most plaine, I will shew you first.

Therefore will I beginne at the vnites, which in the first summe is but 2, and in the second summe 9, that maketh 11. Those doe I take vp, and for them I set 11 in the new roome, thus.



Then doe I take vp all the Articles vnder a hundred, which in the first summe are 40, and in the second summe 50, that maketh 90: or you may say better, that in the first summe there are foure Articles of 10, and in the second summe 5, which maketh 9, but then take heede that you set them in their right lines, see here.



Where

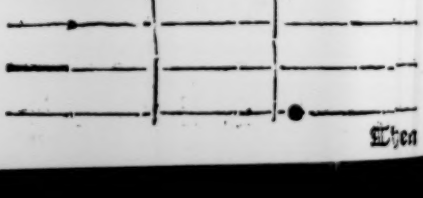
Where I haue taken away 40 from the first summe, and 50 from the second, and in their stead I haue set 90 in the third rome, which I haue set plainly, that you might well perceiue it: howbeit, seeing that 90 with the 10 that was in the third rome already both make 100, I might better for these 6 Counters set 1 in the third line thus.

For it is all in one summe.

☞ as you may see, but it is best neuer to set fiue Counters in any line, for that may be done with one Counter in a higher place.

Scholar. I iudge that good reason, for many are vnnedfull where one will serue.

Master. Well, then will I adde forth of hundreds: I find 2 in the first summe, and 6 in the second, which maketh 900, then doe I take vp, and set in the third rome, where is 100 already, to which I put 900, and it will be 1000: therefore I set one Counter in the fourth line for them all, as you see here.



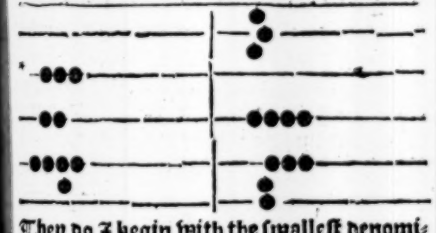
Then

Then ad I the thousands together, which in the first summe are 8000, & in the second 2000, that maketh 10000: them doe I take vp for those two places, and for them I set one Counter in the fift line, and then it appeareth as you se to be 11001, for so many doth amount of the Addition of 8342 to 2659.

To adde summes together.

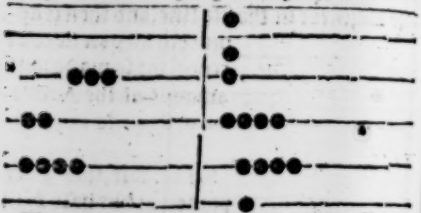
Schol. Sir, this I doe perceiue: but how shall I set one summe to another, not changing them to a third place?

Master. Marke well how I doe it: I will adde together 65436 and 3245, which first I set downe thus.



Then do I begin with the smallest denomination, which is 1 in the second sum, & set it in his place: then do I find 5 in ỳ first sum, & 5 in the second sum, which I put together saying ỳ 2 counters cannot be set in a void place of 5, but for them both I must set 1 in the

the second line, which is the place of 10, therefore I take vp the fiue of the first sum, and the 5 of the second, and for them I set 1 in the second line, as you see here.

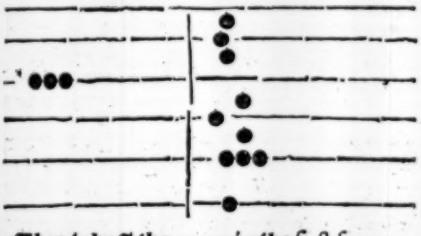


Then doe I likewise take vp the 4 Counters of the first summe & second line (which maketh 40) and adde them to the 4 Counters of the same line in the second sum, & it maketh 80: but as I said, I may not conueniently set aboue 4 Counters in one line, therefore to those 4 that I take vp in the first sum, I take one also of the second sum, and then haue I taken vp 50: for which 5 Counters I set downe one in the space ouer the second line, as here doth appeare.

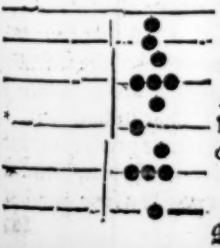
And

And then is there 80, as well with those 4 Counters, as if you had set downe the other 4 also.

Now doe I take the 200 in the first sum, & adde them to the 400 in the second sum, and it maketh 600, therefore I take vp the two Counters in the first sum, and three of them in the second summe, and for them 5, I set 1 in the space aboue, thus.



Then take I the 3000 in the first summe, vnto which there are none in the second sum agreeing, therefore I doe onely remoue those three Counters from the first sum into the second, as here doth appeare.

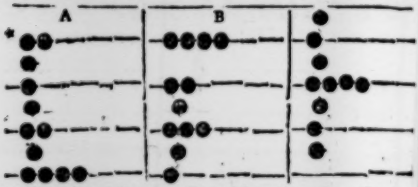


And so you see the whole summe that amounteth of that Addition of 65436 with 3245, to bee 68681.

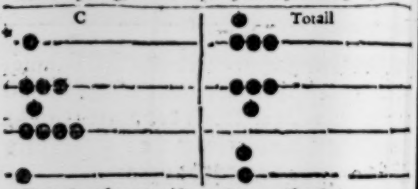
And if you haue

marked these two examples well, you need no further instruction in Addition of 2 only summes: but if you haue more then two summes to adde, you may adde them thus.

First adde two of them, and then adde the third and fourth, or more if their be so many: as if I would adee 2679, with 4286, and 1391. First I adde the two first sums, thus.



And then I adde the third thereto, thus.



And so of more if you haue them.

Scho. Now I think it best that you passe forth to subtractiō, except there be any way to examine this manner of addition then I thinke that were good to be kowne vnto.

Master. There is the same prose here that is in the other addition by the pen, I meane subtraction; for that onely is a sure way, but

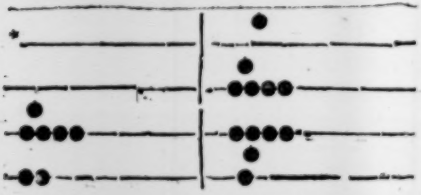
considering that Subtraction must be first knowne, I will first teach you the Art of Subtraction, and that by this example.

Subtraction.

I Would Suburact 2892 out of 8746. These summes must I set downe as I did in Addition: but here it is best to set the lesser number first, thus,

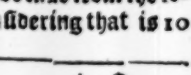
Then shal I begin to subtract the greatest numbers first (contrary to the vse of the pen) that is, the thousand in this example: therefore I find amongst the thousands 2, for which I withdraw so many from the second summe (where are 8) and so remaineth there 6 as this example sheweth.

Then doe I likewise with the hundreds, of which in the first summe I find 8, and in the second summe but 7, out of which I cannot take 8, therefore this must I doe: I must loke how much my summe differeth from 10, which I find here to be 2, then must I abate for my summe of 800, one thousand and set downe the excesse of hundreds, that is to say, 2, for so much as 1000 is more then I should take vp: therefore from the first summe I take that 800, and from the second summe (where are 6000) I take vp one thousand, and leaue 5000, but then I set downe the 200 vnto the 700 that are there already, and make them 900, thus.

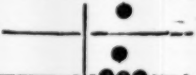


Then come I to the Articles of tennes, where in the first summe I find 90, and in the second sum but onely 40. Now considering that 90 cannot be abated from 40, I loke how much that 90 doth differ from the next sum aboue it, that is, 100 (or else which is all to one effect) I loke how much 9 doth differ from 10, and I find it to be 1: then in the

the stead of that 90, I doe take from the second sum 100: but considering that is 10 to much, I set downe 1 in the next line beneath for it, as you see here.

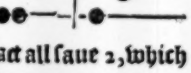


Saving that here I haue set 1 Counter in the space in stead of 5 in the nert line.



And thus haue I subtract all saue 2, which I must abate from 6 in the second summe, and there will remaine 4, thus.

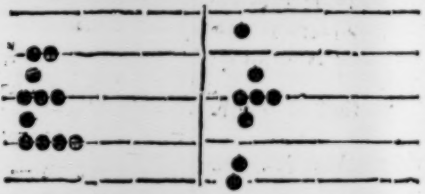
So that if I subtract 2892 from 8746, the remainer will bee 5854.



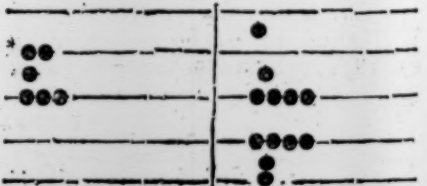
And that this is truely wrought, you may proue by Addition: for if you adde to this remainer the same summe that you did subtract, then will the former summe 8746 amount againe.

Scholar. That will I proue, and first I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus. A proose of Subtraction.

Then doe I adde the first 2 to 4, which maketh 6: so take I vp 5 of those Counters, and in their steade I set 1 in the space, and one in the lowest line, as here appeareth.

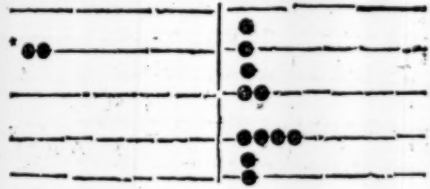


Then doe I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vp those 6 counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus.

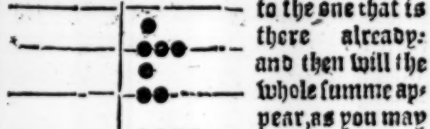


Then

Then doe I come to the hundreds, of which I find 8 in the first sum, and 8 in the second, that maketh 1600, therfore I take vp those 8 counters, and in their stead I set 1 in the fourth line, and 1 in the space next beneath, and 2 in the third line, as you may see here.



Then is there left in the first summe but onely 2000, and in the second 5000, which is 7000, which I shall take vp from thence, and set in the same line in the second sum



to the one that is there already: and then will the whole summe appear, as you may well see, to be 8746, which was the first grosse summe, and therefore I doe perceiue that I had well subtracted before.

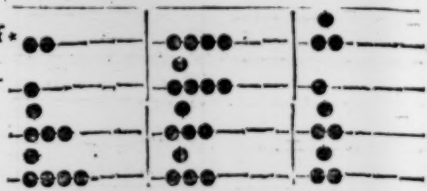
And thus may you see, how Subtraction may by tried by Addition.

Scholar. I perceiue the same order here with Counters, ỹ I learned before in figures.

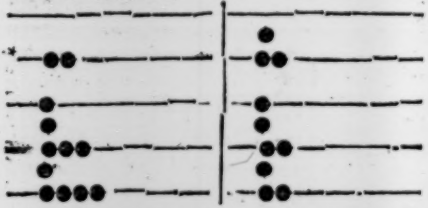
Maſter, Then let me ſee how you can try Addition by Subtraction.

Scholar. Firſt I will ſet forth this example of Addition, where I haue added 2189, to 4988. And the whole ſumme appeareth to be 7177.

Proofe of Addition by Subtraction.

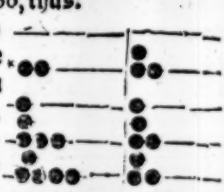


Now to try whether that ſum be well added or no, I will ſubtract one of the firſt two ſummes from the third. And if I haue well done, the remainer will be like that other ſumme: as for example, I will ſubtract the firſt ſum from the third, which ſet thus in their order.

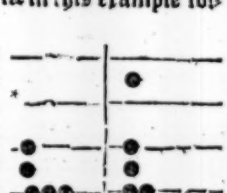


Then doe I ſubtract 2000 of the firſt ſumme from the ſecond ſumme, and then remaineth

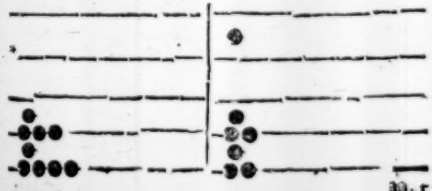
remaineth there 5000, thus.



Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100 alſo: and then in the third line reſteth nothing, as you may ſee in this example following.

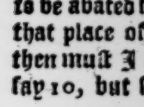


Then in the ſecond line with his ſpace over him I find 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but onely 70 I muſt take it out of ſome higher ſumme, which is here onely 5000: therefore I take up 5000: and ſeeing that is too much by 4920, I ſet down ſo many in the ſecond rome, which with the 70 being there already, doe make 4990, and then the ſummes doe ſtand thus.



Put

Yet remaineth therein the first sum 9, to be abated from the second sum, wherein that place of vnites doth appeare onely 7: then must I abate a higher sum, that is to say 10, but seeing that 10 is more then 9 (which I should abate) by 1, therefore shall I take vp one Counter from the second, and set downe the same in the first line or lowermost line as you see here.



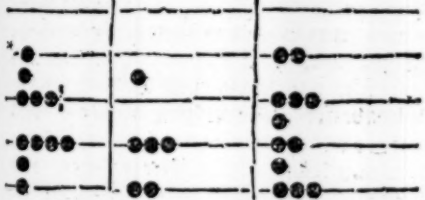
And so haue I ended this worke and the summe appeareth to be the same which was the second summe of mine Addition, and therefore I perceiue I haue well done.

Another way of Addition.

Master. To stand longer about this, it is but folly: except that this you may also vnderstand that many doe begin to subtract with counters, not at the highest sum, as I haue taught you, but at the nethermost, as they doe vse to adde, when the sum to be abated in any line appeareth greater then the other, then doe they borrow one of the next higher roome, as for example. If I should abate 1846 from 2378 they set the summes thus:

First

First they take 6, which is the lower line, and his ſpace from 8 in the ſame rooms in the ſecond ſumme, and yet there remaineth 2 counters in the loweſt line. Then in the ſecond line muſt 4 be ſubtracted from 7 and ſo remaineth there 3. Then 800 in the third line and his ſpace, from 3000 of the ſecond ſumme cannot bee, therefore doe they abate it from a higher roome, that is, from 1000. and becauſe that 1000 is too much by 200, therefore muſt I ſet downe 200 in the third line, after, I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſumme, which if I withdrawe from the ſecond ſumme, then doe all Figures ſtand in order, thus: 532.



So that (as you ſée) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as ſome men like that one way beſt, ſo ſom like the other: therfore you now knowing both, may vſe which you liſt.

Multipli-

Multiplication

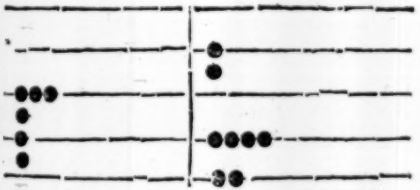
But now touching multiplication: you shall set your numbers into two roomes (as you did in those other kindes) but so that the multiplier be set in the first rowme, then shall you begin with the highest numbers of the second roome, and multiply them first after this sort.

☞ Take the ouermost line in your first worcking as it were the lowest line, setting on it some moueable marke (as you list) and lok how many coũters be in him, take them vp, and for them set downe the whole multiplier so many times as you toke vp counters: reckoning (I say) that line for the vnites. And when you haue done with the highest number, then come to the nert line beneath, and doe so euen with it, and so with the nert, till you haue done all. And if there be any number in a space, then for it shall you take the Multiplier 5 times: and then must you reckon that line for the Vnites, which is nert beneath that space. Or else after a shorter way ye shall take onely ☞ halfe the multiplier, but then shal you take the line next aboue the space for the line of vnites. But in such worcking, if by chance your multiplier be an odde number, so that you

you cannot take the halfe of it iuſtly, then muſt you take the greater halfe, and ſet downe that, as if that it were the iuſt halfe: and further, you ſhall ſet one Counter in the ſpace beneath that line, which you reckon for the line of Vnits, or elſe only remoue forward the ſame that is to be multiplied.

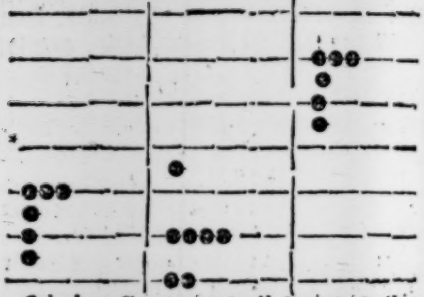
Scholar. If you ſet forth an example hereof, I thinke I ſhall perceiue you.

Maſter. Take this example, I would multiply 1542 by 365, therefore I ſet my numbers thus.



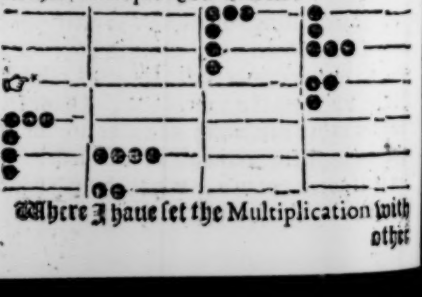
Then firſt I begin at the 1000 in the higheſt roome, as if it were the firſt place, and I take it vp ſetting downe for it ſo often (that is once) the Multiplier, which is 365, thus as you ſee here: where, for the one Counter taken vp from the fourth line, I haue ſet downe other fiue which make the ſumme of the multiplier, reckoning the fourth line, as if it were the firſt, which thing I haue marked by the ſtarre ſet at the beginning.

Scholar.



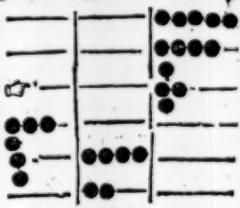
Scholar. I perceiue well, for indeede this ſumme that you haue ſet downe is 365000: for ſo much doth amount of 1000: multiplied by 365.

Maſter. Well then goe forth, in the next ſpace I find one Counter, which I remoue forward, but take it not vp, but (as in ſuch a caſe I muſt) ſet downe the greater halfe of my multiplier (ſeeing it is an odde number) which is 182, and here I doe ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſee.



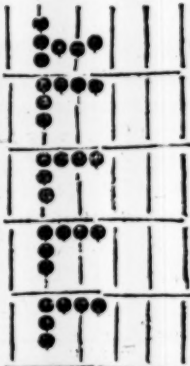
Where I haue ſet the Multiplication with other

other; but for the ease of your vnderstanding, ☞ I haue set a little line betweene them. Now shuld they both in one summe stande thus:

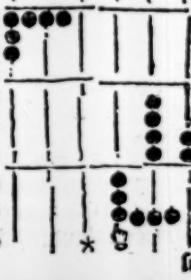


Another forme of Multiplication.

Howbeit another forme to multiply such counters in space is this: first to remoue the finger to the next line beneath the space, and then to take vp the counter, and to set downe the multiplier fiue times, as here you se.



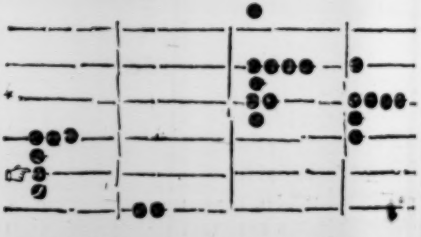
Which sums if you do adde together into one sum, you shall perceiue that it will be the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter,



so this is plainer to reason, for such as haue had small exercise in this Art.

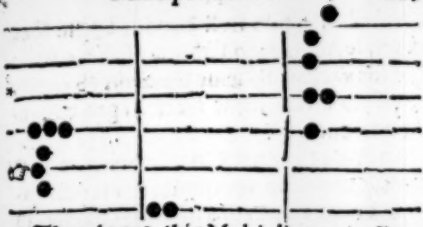
Notwithstanding you may adde them in your mind before you set them downe: as in this example you might haue said, fiue times 300 is 1500, and 5 times 60 is 300, also 5 times 5 is 25, which all put together doe make 1825, which you may at one time set downe if you list.

But now to goe forth, I must remoue the hand to the next Counters which are in the second line, and there must I take vp those foure Counters, setting downe for them my multiplier foure times seuerally, or else I may gather the whole sum in my mind first and then set it downe: as to say, 4 times 300 is 1200: 4 times 60 are 240: and foure time 5 make 20, that is in all 1460: that shall I set downe also, as here you see.

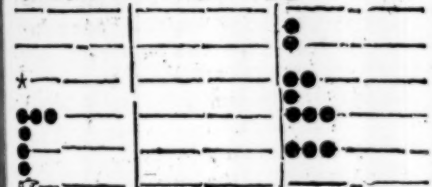


Which if I ioyne in one summe with the former numbers, it will appeare thus.

Then



Then to end this Multiplication, I remoue the finger to the lowest line, where are only 2, them doe I take vp and in their stead doe I set downe twice 365, that is 730, for which I set one in the space aboue the third line for 500 and 2 more in the third line with that one that is there already, and the rest in their order, and so haue I well ended the whole summe thus:



Whereby you see, that 1542 (which is the number of yeeres since Christ his incarnation) being multiplied by 365 (which is the number of the daies in one yeere) doth amount to 562830, which declareth the number of daies since Christs incarnation vnto the end of 1542 yeeres, beside 385 daies and twelue howres for leape yeeres.

Scholar. Now will I proue by another example, as this: 40 Laborers (after 6 ß the day for each man) haue wrought 28 dayes, I would know what their wages doth amount vnto.

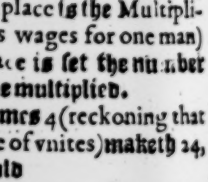
A question of wages.

In this case must I worke doubly: first I must multiply the number of the Laborers, by the wages of a man for one day, so will the charge of euery day amount.

Then secondly shal I multiply the charge of one day by the whole number of dayes, and so will the whole sum appeare: First therefore I shall set the summes thus.

Where in the first place is the Multiplier (that is one dayes wages for one man) and in the second space is set the number of the workemen to be multiplied.

Then say: If 6 times 4 (reckoning that second line as the line of vnites) maketh 24, for which sum I should set two Counters in the third line, and foure in the second, therefore doe I set two in the third line, and let the foure stand still in the second line thus.

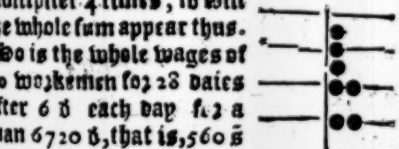


So appeareth the whole dayes wages to be

be 2408, that is 208.

Then doe I multiply againe the same sum by the number of daies, and first I set the numbers thus: then because there are Counters in diuers lines: I shall beginne with the highest, and take them vp, setting to them the Multiplier so many times as I toke vp Counters, that is twice, then will the summe stand thus.

Then come I to the second line, and take vp those 4 Counters, setting to them the multiplier 4 times, so will the whole sum appear thus.



So is the whole wages of 40 workemen for 28 daies after 6 d. each day for a man 6720 d. that is, 560 s. for 28 pound.

Master. Now if you would proue Multiplication, the surest way is by Diuision: therfore will I ouerpasse it til I haue taught you y art of Diuision, which you shal worke thus.

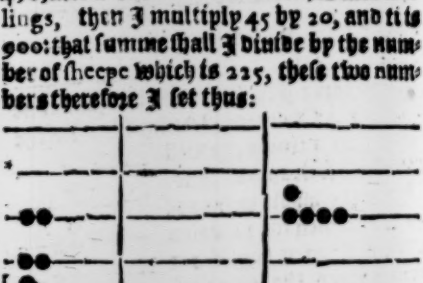
Diuision

Diuision in Fractions.

First set downe the Diuisor, for feare of forgetting, and then set that number that shall be diuided at the right side so far from the Diuisor that the quotient may be set betweene them as for example.

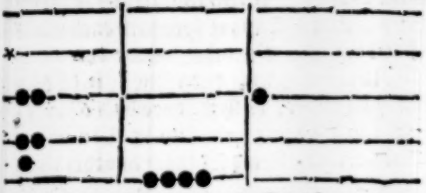
Example of sheepe.

If 220 sheepe cost 45£, what did euery sheepe cost? To know this I would diuide the whole sum, that is 45£, by 225, but that cannot be, therefore must I first reduce that 45£, into a lesser deuomination, as into shillings, then I multiply 45 by 20, and it is 900: that summe shall I diuide by the number of sheepe which is 225, these two numbers therefore I set thus:



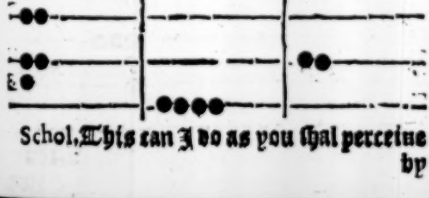
Then begin I at the highest line of the diuidend therein, and that I may doe foure times: then say I foure times 2 are 8, which if I take from 9, there resteth but 1, thus:

And



And becauſe I found the Diuiſor 4 times in the diuidend, I haue ſet, as you ſée, 4 in the middle roome, which is the place of the quotient: but now muſt I take the reſt of the Diuiſor as often out of the remainer therefore come I to the ſecond line of the Diuiſor, ſaying: two times 4 make 8, take 8 from 10, and there remaineth 2 thus;

Then come I to the loweſt numher which is 5, and multiply it 4 times, ſo is it 20, that take I from 20, and there remaineth nothing, ſo that I ſée my quotient to be 4, which are in valne ſhillings, for ſo was the diuidend: and thereby I know that if 225 ſheepe coſt 45 l, euery ſheepe coſt 4 ſ.



Schol. This can I do as you ſhal perceiue by

by the example. If 160 ſouldiers doe ſpend euery month 68£, what ſpendeth each man?

ample wages

Firſt becauſe I cannot diuide the 68 by 160, therfore I will turne the £, into ß by multiplication, ſo ſhal there be 16320 ß, now muſt I diuide the ſum by the number of ſoldiers, therfore I ſet them in order thus

Then begin I at the higheſt place of the diuidend, ſeeking my Diuiſor there, which I find once, therfore I ſet 1 in the nether line.

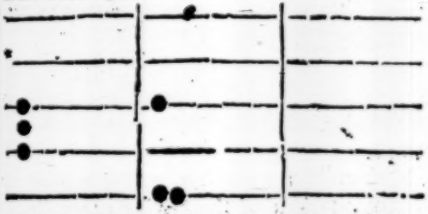
Maſter. Not in the neather line of the whole ſumme, but in the neather line of that worke, which is the third line.

Scholar. So ſtandeth it with reaſon.

Maſter. Then thus doe they ſtand.

Then

Then séeke I againe the rest, bow often I may find my diuisor: and I sée that in 300 I might find 100 thrée times: but then the 60 will not be so often found in 20, therfore I take 2 from my quotient: thē take I 100 twice from 300, and there resteth 100, out of which with the 20 that maketh 120, I may take 60 also twice, and then stand the numbers thus:



Where I haue set the quotient 2 in the lowest line: So is euery souldiers portion 102 d, that is 8s, 6d.

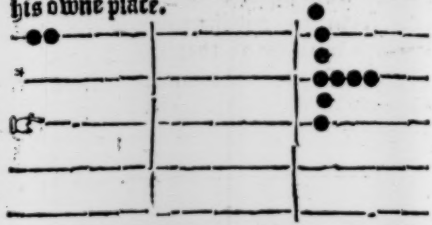
Master. But yet because you may iustly perceiue the reason of Diuision, it shall be good that you set your Diuisor still against those numbers from which you doe take it, as by this example I will declare:

If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost? Example of purchase.

First will I turne the pounds into pence, so will there be 69600 pence. Then in setting downe these numbers I shall doe thus.

First set the diuidend on the right hand as ought

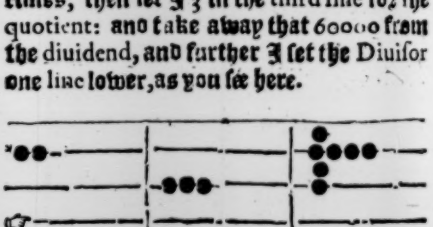
ti ought, e then the Diuiſor on the left hand againſt thoſe numbers from which I intend to take him firſt, as here you ſee where I haue ſet the diuiſor two lines higher then his owne place.



Scholar. This is like the order of Diuiſion by the pen.

Maſter. Truth you ſay, and now muſt I ſet the quotient of this worke in the third line, for that is the line of vnites in reſpect of the Diuiſor in this worke.

Then I ſee how often the Diuiſor may be found in the diuidend, and that I find 3 times, then ſet I 3 in the third line for the quotient: and take away that 60000 from the diuidend, and further I ſet the Diuiſor one line lower, as you ſee here.



And

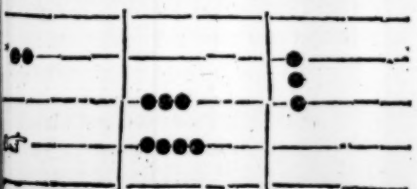
And then ſeke I how often the Diuiſor will be taken from the number againſt it, which will be foure times, and 1 remaining.

Scholar. But what if it chance that when the Diuiſor is ſo removed it cannot be once taken out of the diuided againſt it?

Maſter. Then muſt the Diuiſor be ſet in another line lower.

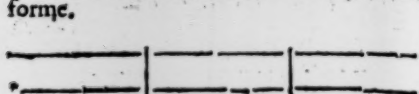
Scholar. So was it in Diuiſion by the pen, & therfore was there a Cipher ſet in the Quotient: but how ſhal that be noted here?

Maſter. Here needeth no taken, for the lines doe repreſent the places, only loke that you ſet your Quotient in that place which ſtandeth for vnites in reſpect of the Diuiſor. But now to returne to the example: I find the Diuiſor foure times in the diuidend, & 1 remaining: for 4 times 2 make 8, which I take from 9, & there reſteth 1 as this figure following ſheweth: and in the middle ſpace for the Quotient, I ſet 4 in the ſecond line, which is in this worke the place of vnites.



Then remoue I the Diuiſor to the next lower

lower line, and seeke how often I may haue it in the Diuidend, which I may doe here 8 times iust, and nothing remaine, as in this forme.



Where you may see that the whole quotient is 3480, that is 29s, whereby I know that so much cost the purchase of one Acre.

Scholar. Now resteth the proues of Multiplication, and also Diuision,

Master. Their best proues are each one by the other, for Multiplication is proued by Diuision, and Diuision by Multiplication, as in the worke by the pen you learned.

Scholar. If that be all, you shall not need to repeate againe that which was sufficiently taught already: and except you will teach me any other feate, here may you make an end of this Art, I suppose.

Master. So will I doe as touching whole number, and as for broken number, I will not trouble your wit with it, till you haue practised this so wel that you be full perfect,

so

o that you néed not to doubt in any point ẏ I haue taught you, and then may I boldly instruct you in the Art of Fractions or broken numbers: wherein I will also shew you the reasons of all that you haue now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings & pounds, proceéding by no grounded reason, but only by a receiued forme and that diuersty, of diuers men: for the Merchants vse one forme, and Auditors another.

The reason of all the former rules.

Merchants vse.

Merchãts Accompt.

BVt first for Merchants forme marke this example here, in which I haue expressed this summe 198l, 19s 11d. *So that you may see that the lowest line serueth for pence, the next above for shillings, the third for pounds, and the fourth for scores of pounds.*

And further you may sée that the space betwéene pence and shillings may receiue but one counter (as al other spaces likewise do) and that one standeth in that place for 6d. Likewise betwéene the shillings and the pounds, one counter stands for 10 shillings.

And

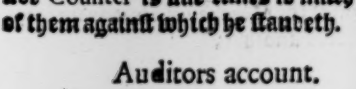
And betweene the pounds and 20 l, one Counter standeth for 10 l.

But beside those, you may see at the left side of shillings that one number standeth alone, and betokeneth 5 s.

So against the pounds, that one Counter standeth for 5 l. And against the 20 pounds, the one Counter standeth for fiue score ponnds, that is, 100 pounds, so that euery fiue Counter is fiue times so much as one of them against which he standeth.

Auditors account.

Auditors Accompt. NOw for the account of Auditors, take this example.



Where I haue expressed the same summe 198 l—19—11 d.

But here you see the pence stand towards the right hand, and the other increasing orderly toward the left hand.

Againe you may see, that Auditors will make two lines (yea and more) for pence, shillings, and all other values, if their sums extend thereto. Also you see that they set one Counter at the right end of each row, which so set there, standeth for fiue of that roome,

and

and on the left corner of the row it standeth for 10 of the same row.

But now, if you would adde, or subtract after any of both those sorts, if you marke the order of the other feate which I taught you, you may easily doe the same here without much teaching: for in Addition, you must first set downe one summe, and to the same set the other orderly, and in like manner if you haue many: but in Subtraction, you must set downe first the greatest sum, and from it must you abate the other, euery Denomination from his due place.

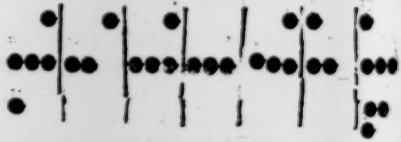
Scholar. I doe not doubt but with a little practise I shall attaine these both: but how shall I multiply & diuide after these formes?

Master. You cannot duly doe any of both by these sorts, therefore in such case you must resort to your other Arts.

Scholar. They that vse such accounts that it excéd 200 in the summe they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand therof, and the thousands they set in farther row yet, and at the left side thereof they set the 5000, and in the space ouer they set the 10000, in a higher row 20000, which all I haue expelled in this example, which is 97869 l 12 s, 9 d, ob, q. Ninety seauen thousand, eight hundred threescore and nine pounds, twelue

twelue ſhillings and 9 pence halfe peny farthing, for I had not told you before where neither how you ſhould ſet downe farthings, which (as you ſee here) muſt be ſet in a void place ſtanding beneath the pence, for a farthing, one Counter, for ob. 2 Counters, for ob. farthing, 3 Counters, and more there cannot be: for 4 farthings make 1 d, which muſt be ſet in his due place.

And if you deſire the ſame ſumme after Auditors manner, loe here it is.



But in this thing you ſhall take this for ſufficient, and the reſt you ſhall obſerue as you may ſee by the working of each ſort, for the diuers wits of men haue inuented diuers & ſundry waies, almoſt innumerable.

The

The second part of Arithmeticke, touching Fractions, briefely set forth.

Scholar.

Lbeit I perceiue your manifold busines doth so occupy or rather oppresse you, that you cannot as yet completely end the Treatise of Fractions Arithmeticall, which you haue prepared, wherein not onl● ſ ndry workes of Geometry, Musicke, and Astronomy be largely set forth, but also diuers conclusions and naturall workes, touching mixtures of Mettals and compositions of medicines, with other strange examples: Yet in the meane season, I cannot stay my most earnest desire, but importunely craue of you some briefe preparation toward the vse of Fractions, whereby at the least I may bee able perfectly to vnderstand the common workes of them, and the vulgar vse of those rules, which without them cannot well bee wrought. Arithmeticall fractions.

Master. If my leasure were as great as my will is good, you should not neede to vse any importunate craning, for the attaining of that thing, whereby I may bee perswaded that I shall any waies profit the cō-

mon

common wealth, or helpe the honest studies of any good members in the same: wherefore while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

What a Fraction is. Therefore first to begin with the explication of this name, Fraction, what take you it to be?

Scholar. Mary sir, I think a Fraction (as I haue heard it often named) to bee a broken number, that is to say, to be no whole number, but a part of a number.

Master. A Fraction indeed is a broken number, and so consequently the part of a nother number: but that must be vnderstood of such another number as cannot be diuided into any other parts then Fractions, for although I may take the third part of 60, or the fourth part of it, and so of other parts diuerslie, yet these parts be not properly, nor ought not to be called Fractions, because they may be expressed by whole numbers. for the third part of it is 20, the fourth part is 15, the twelfth part is 5, and so forth of other parts, which all bee whole numbers.

What a fraction is properly. Wherefore properly a Fraction expresseth the parts or part onely of a vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not be greater then one: and therefore it followeth that

that no one Fraction alone can be so great, that it shall make 1, as by examples I will declare as soone as I haue taught you to know the forme how a Fraction is expressed or represented in writing:

Numeration.

BVt *first to begin with expressing of a* Fraction, *which is the numeration of it: you must vnderstand that a* Fraction *is represented by two numbers, set one ouer the other, and a line drawn betweene them, as thus,* $\frac{1}{3}\ \frac{3}{4}\ \frac{2}{5}\ \frac{10}{17}$, *which 4 Fractions you must pronounce thus:* $\frac{1}{3}$ *one third part,* $\frac{3}{4}$ *three quarters,* $\frac{2}{5}$ *two fift parts,* $\frac{10}{17}$ *ten 17 parts.* Numeration of Fractions.

Scholar. I vnderstand the forme of their expression & pronunciation: but their meaning or valuation seemeth more obscure. Yet I thinke that by the two first fractions I vnderstand the valuation of the two latter fractions, and consequently of other.

Master. Value them then, that I may perceiue your taking of them.

Scholar. $\frac{2}{5}$ Betokeneth two fift parts, that is to say, if one be diuided into 5 parts, that fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be diuided into 17 parts, I must take tenne of

them. And this I gather of the two first examples: for $\frac{1}{3}$ that is one third part, doth easily declare, that if one thing be divided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, doth declare that one being divided into foure quarters, I must take (for this Fraction) three of those quarters.

☞ If there be no more difficultie in their Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kinds of workes. For I understand that the same kindes of workes be in fractions that be in whole numbers.

Master. There are the same kindes of workes in both, albeit the order of them is divers, as I will anone declare: but yet more in Numeration before we leave it. You must understand, that those two numbers which expresse a Fraction, have severall names. The ouermost, which is above the line, is called the Numerator & the other beneath the line, is called ỳ Denominator.

Numerator. Denominator.

Scholar. And what is the reason of their divers names? (For, in mine opinion, both be Numerators, seeing both they do expresse the numeration of the Fraction.

Master. You are deceived: for one onely (which is the ouermost) doth expresse the Numeration: and the Denominator doth declare the number of parts, into which the

unit

vnite is diuided, as in this example, when I say: diuide a pound weight of Gold betweene foure men, so that the first man shall haue $\frac{3}{15}$, the second $\frac{5}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now doe you perceiue that by the Denominator (which is one in all foure fractions) it is intended that the pound weight should be diuided into so many parts, I meane 15, and by the foure seuerall Numerators is limited the diuers portion that each man should haue, that is, that when the whole is parted into 15, the first man shall haue two of those 15 parts: the second man three of them: the third man foure: and the fourth man fiue. And so may you see the seuerall offices (as it were) of those two numbers, I meane of the Nnmerator and the Denominator.

And hereby you perceiue that a man can haue no more parts of any thing then it was diuided into, neither yet aptly so many: so that it were vnaptly said: You shall haue $\frac{15}{15}$ that is 15, 15 parts of any thing, for it were better said you shall haue the whole thing.

Scholar. So doth it appeare reasonably, for the labour is vaine to diuide any thing, and then to apply the Diuision to no vse. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be diuided into 15 parts

onely, it is not possibly to take 16 of them, that is to say, more then altogether.

☞ Master. This is true touching the proper and apt vse of the name of a Fraction: yet improperly, and after a vulgar acceptation (for easinesse in worke) both those formes be called fractions, because they be written like Fractions, although they be none indeede: for $\frac{15}{15}$, and generally all such other, where the Numerator and Denominator be equal, are not Fractions, but the whole thing with all his parts. And so $\frac{14}{13}$ is not to be called a Fraction but a mixt number, of a whole number and a Fraction, for it is as $1\frac{1}{13}$, that is one whole 1 and $\frac{1}{13}$ parts, as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions where the Numerator is eyther equall or greater then the Denominator.

Mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers after that sort like Fractions: but they need not to call them Fractions, but (as they be) whole numbers or mixt numbers, (that is, whole numbnrs with Fractions) expressed like Fractions.

Now must you understand, that as no Fraction properly can be greater then 1, so in smalnesse vnder one the nature of Fracti-

ons

ons doth extend infinitely, as the nature of whole numbers is to increase aboue one infinitely, so that not onely one may be diuided into infinite Fractions or parts, but also euery fraction may be diuided into infinite Fractions or parts, which commonly be called Fraction of Fractions, and they be expressed diuerslie: as for example $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is, three quarters of two third parts of one halfe part. Whereby is signified, that if one be diuided into two halfes, and the one halfe into three parts, and two of those three parts be diuided iointly into foure quarters, this fraction of fractions doth represent three of those quarters. Fractions of Fractions.

Scholar. I pray you let me proue by an example in common money, whether I do rightly vnderstand you or no. One Crowne which I take for an vnite, doth contain 60 s. therfore the halfe of it is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{1}{4}$ is fiftéen pence, so then 15 pence is $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crowne: and so is 3 pence $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceiue this well enough: yet this note I giue you by the way, that the forme of expressing these fractions is voluntarie, and hath no other reason than the will of the Diuisor, which forme many follow: for some expresse them thus $\frac{1}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ without any figure of distinction betwéen them, which forme many follow. Some other do

make lines betweene every fraction, and adde words of distinction, after this sort: $\frac{3}{4}$, of $\frac{1}{2}$, which some is best.

Some other expresse them thus in slope some: to distinct them from seuerall Fractions of one whole number, for if they were set in one right line thus, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, then ought it to be pronounced, three quarters, and two third parts, and a halfe, which maketh almost two whole vnites, lacking but one 12 part. And so is it nothing agreable with the other fraction of fractions: wherefore it is a great ouersight in certaine learned men, which do expresse them so confusedly with such seuerall fractions, that a man cannot know the one from the other.

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

☞

Therefore some men (*as* Stifelius) *doe expresse without a line numbers of proportion, being applied to* Addition *or* Subtraction, *because they must be taken as two, where the line in* fractions *maketh them to be taken for one: For of the* Numerator *and* Denominator *is made one number.*

Three seuerall varieties.

Scholar. When I perceiue there be three seuerall varieties in fractions: first, when one onely fraction is set for one number, as $\frac{4}{5}$, that is foure fift parts. The second is, when there be set two or more seuerall fractions of one number: as $\frac{4}{9}$, $\frac{2}{9}$, that is foure ninth parts, and two fift parts. The third sort

sort is fractions of fractions, as $\frac{4}{9}$ of $\frac{2}{5}$, that is foure ninth parts of two fift parts.

Master. You haue said well, if you vnderstand well your owne words.

Scholar. If it shall please you I will by an example in the parts of an old English Angell expresse my meaning.

Master. Let me here you.

Scholar. The old English Angell did containe 7 shillings 6 d, that is 90 pence Now $\frac{4}{5}$ of it is 72 d. And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$ that is foure nine parts, and 2 fift parts, $\frac{2}{5}$ is 40, and $\frac{4}{9}$ is 36, which both make 76 : but if I take $\frac{4}{9}$ of $\frac{2}{5}$ that is 4 nine parts of two fift parts, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16, for $\frac{1}{9}$ of 36, is but 4, & that taken 4 times maketh 16.

Master. This is plainly expressed, & truly, and hereby (I doubt not) but you doe perceiue, that as great a difference, as is betweene 16 and 76, so much difference is betweene those two Fractions $\frac{4}{9}$ and $\frac{2}{5}$: & $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you vnderstand these varieties, I will proceede to the rest of the workes : first admonishing you, that there is another order to be followed in Fractions, then there was in whole numbers : for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuision, and Reduction : but in fractions (to follow the same aptnesse in proceeding

The order of worke in Fractions.

ceeding from the eaſieſt workes to the harder) we muſt vſe this order of the workes, Numeration, Multiplication, Diuiſion, Reduction, Addition, and Subtraction.

Scholar. That Multiplication, and Diuiſion ſhould go together, and Subtraction to follow Addition, naturall order doth perſwade, but why Multiplication ſhould be firſt in order here next to Numeration, and Reduction in the middle. I deſire to vnderſtand the reaſon.

Maſter. As in the Art of whole numbers Order would reaſonably begin with the eaſieſt, & ſo goe forward by degrées to the hardeſt: euen reaſon teacheth in Fractions the like order. And, conſidering that Addition or Subtraction of Fractions can very ſeldome be wrought without Multiplication and Reduction: and contrariwiſe, Multiplication and Reduction may be wrought without this forme of Addition or Subtraction: therefore was it orderly required, that Multiplication and Reduction ſhould goe before Addition and Subtraction, & the ſame reaſon ſerueth for the placing of Multiplication before Reduction.

Scholar. Then if Multiplication be the eaſieſt, I pray you declare the forme of it firſt by Rule, and then by example.

Maſter. Your requeſt is good.

Multipli-

Multiplication.

Multiplication of fractions.

Heretofore when any two fractions be proponed to bee multiplied together, the Numerator of the one muſt be multiplied by the Numerator of the other: and the ſum that amounteth therof muſt be ſet for a new Numerator: likewiſe the denominator of the one muſt be multiplied by the denominator of the other, and that that amounteth, ſhall bee ſet for the common Denominator: and this new third fraction expreſſeth the product of the multiplication of the two firſt fractions proponed, whereof take this example, $\frac{3}{5}$ *multiplied by* $\frac{5}{12}$, *doth make* $\frac{15}{60}$.

Scholar. I perceiue then that 3, being the Numerator of the firſt fraction, is multiplied by 5, being the numerator of the ſecond fraction, whereof amounteth 15, the numerator of the third fraction. And ſo likewiſe, 5 being denominator of the firſt fraction, is multiplied by 12 the denominator of the ſecond fraction, whereof amounteth 60 the new denominator, ſo that I perceiue how the worke is done, but I doe not perceiue how $\frac{15}{60}$ is greater then $\frac{1}{4}$: for if I ſhall vſe my former manner of examination by the parts of ſome Coyne. I ſée that $\frac{3}{5}$ of a Crowne is 36 d, and $\frac{1}{12}$ of a Crowne is 25 d, whereof the

the one multiplied by the other doth make 900 d, which is 15 Crownes: but by your Multiplication there amounteth $\frac{15}{60}$, which is but 15 pence, and that is much lesse then any other of both the first fractions.

Master. That difference is betweene multiplication in whole numbers, and Multiplication in broken numbers, that in whole numbers, the sum that amounteth is greater then both the other whereof it came: but in fractions it is contrariwise: for the sum that amounteth is lesser then any of the other two fractions whereof it came.

Scholar. I desire much to understand the reason thereof.

Master. Although I purposed to reserue the reasons of workes Arithmeticall for the perfect Booke of Arithmeticke, yet I will shew you this, because of the strangenesse of the worke.

You se in whole numbers, that of two numbers being multiplied together, is made the third number, which third number, doth beare the same proportion to the number multiplied, that the multiplier doth beare to an vnite. And so in fractions the third number which amounteth of Multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an Vnite.

Scholar.

Scholar. Sir, I vnderſtand your words thus: when 40 is multiplied by 12, there doth amount 480, which 480 doth containe 40 ſo many times in it, as 12 doth containe vnites, that is to ſay, twelue times. And ſo it appeareth that 480 doth containe twelue ſo many times alſo as 40 doth containe vnites, that is, 40 times. But now I ſée not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) ſéeing it is leſſer then either of them.

Maſter. No marueil if you cannot ſée that thing which is not poſſible to be ſéene of any man, how the third number in multiplication of fractions ſhould be greater then any of the two former fractions: but yet this may you ſée (which I ſaid) that the third number in fractions ſo multiplied, doth beare the ſame proportion to any of the two former fractions, that the other of thoſe two fractions doth beare to an vnite, as in your example, $\frac{3}{5}$ being multiplied by $\frac{1}{12}$, doth make $\frac{15}{60}$. Now ſay I, that $\frac{15}{60}$ doth beare the ſame proportion to $\frac{3}{5}$ that $\frac{1}{12}$ doth beare to an vnite, as you may in your owne forme of examination by Coyne trie it: for in an old Angell, (which in times paſt was currant for 7 s, 6 d. are 180 halfe pence) which I ſet for the intire vnite, whoſe parts (according to the fractions aforeſaid) are theſe.

for

for $\frac{12}{15}$ set 45 ob, for $\frac{9}{15}$ take 108 ob, and for $\frac{7}{15}$ put 75 ob. Now doth 45 beare the same proportion to 108 that 75 doth beare to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better referred till another time, when the knowledge of proportions in due order shal be taught: yet in the meane season I will shew you howit commeth to passe, that in fractions the third summe must needs be lesser then any of the other two.

Consider this, that when a fraction is proponed, as in the former example $\frac{3}{4}$ if it bee multiplied by more then 1, it will make more then one entire number. As if I multiply $\frac{3}{4}$ by 5: that is to say, if I take it 5 times, it will make three entire unites: example in a Crowne, $\frac{3}{4}$ of it maketh 3 s̄, which if I take fiue times it wil amount to 15 s̄, that is, three entires Crownes: so if I take the same $\frac{3}{4}$ but twice it wil yeeld 6 s̄, that is, one entire Crowne, and $\frac{1}{4}$. Now if I take it but once, it cannot be more then it was before, that is 5 s̄. And if I take it lesse then once, it cannot be so much as it was before. Then seeing that a fraction is lesse then one, if I multiply a fraction by another fraction; it followeth that I doe take the first fraction lesse then once, and therefore the summe

Note:

ſumme that amounteth, muſt needs be leſſe then the firſt fraction.

Scholar. Sir, I thanke you much for this reaſon. And I truſt I do perceiue the thing, as by example of this ſame fraction $\frac{1}{3}$ I wil expreſſe. If I take $\frac{1}{3}$ of a Crowne once, that is to ſay, if I multiply $\frac{1}{3}$ by 1, it will be as it was before, but $\frac{1}{3}$ ſ: ſo if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then will it be but halfe ſo much: likewiſe if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yéeld but 12 pence, that is, the third part of the firſt fraction.

And ſo to make an end: If I take it but the twelfth part of once, that is, If I doe multiply it by $\frac{1}{12}$, it will yéeld but the twelfth part of the firſt fraction, which is but three pence. And it followeth, that if $\frac{1}{12}$ make three pence, then $\frac{1}{15}$ muſt néeds make fiue times ſo much, that is 15 pence, which was the ſumme that hath giuen the occaſion of all this doubt.

Maſter. When I perceiue you haue ſufficient vnderſtanding in this ſort of Multiplication for this time, wherefore I will omit that I might ſay more of Multiplication, till we come to Reduction, and will paſſe to the other workes, and firſt to Diuiſion, whoſe place followeth Multiplication, both by naturall order, and alſo in eaſineſſe of worke.

Diuiſion.

Diuision.

Diuision of fractions.

WHensoeuer two Fractions bee proponed, that one should bee diuided by the other, I must set downe first the fraction that shall be diuided, (which is called the Diuidend) and then after it the other, which is the Diuisor. Then shall I multiply the Numerator of the Diuidend by the Denominator of the Diuisor, & that which amounteth I must put for a new Numerator. Againe, I shall multiply the Denominator of the Diuidend by the Numerator of the Diuisor, and the number that amounteth thereof, I must put for the new Denominator. And this third Fraction is the Qnotient of the said Diuision.

Scholar. This seemeth easie in forme, as by example thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{3}$, first I multiply 5, (being the Numerator, of the Diuidend) by 8, which is the Denominator of the Diuisor, and thereof riseth 30: then I multiply 8 being the Denominator of the Diuidend) by 2, being the Numerator in the Diuisor: and so riseth 16, the which I must make in a third Fraction, thus $\frac{30}{16}$.

Master. Me seemeth you are quicker in vnderstanding now, then you were when I taught you the Art of whole number: but that

that is no marueil: for the more knowledge that any man getteth, the readier shal hee finde his wit, & quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued here for the ease of worke, and lightnesse of vnderstanding, the nature of the Quotient.

Whensoeuer you diuide one fraction by another, eyther they be both equal together, either else the one is greater then the other: if they be equall, their Quotient shall be such, that the Numerator and the Denominator of it shall be equall also. And if the two first fractions be equall, their Quotient shall declare the same by the vnequalitie of the Numerator & Denominator, as in these examples following shall appeare.

First of equal fractions $\frac{4}{9}$ and $\frac{12}{27}$ be equal together, and if the one be diuided by the other, the quotient will be $\frac{108}{108}$, as you may perceiue by that rule aforesaid.

Now in the vnequall fractions, as $\frac{4}{9}$ and $\frac{1}{10}$, the quotient will be $\frac{40}{27}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see the reason why it should be so.

Master. The reason is this: When any Fraction is diuided by another, the quotient declareth what proportion the diuidend beareth to the Diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$ maketh 2, which must be sounded, not 2, but

Note how to know the proportion betweene two numbers.

but twice, declaring that $\frac{1}{2}$ is contained twice in $\frac{1}{5}$.

And note this, that the Numerator in the quotient representeth the Diuidend, and the Denominator representeth the Diuisor. And this is alwaies true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne, till you haue learned the Art of proportions: notwithstanding somewhat of it I will declare in the next Rule of Reduction. But now for the easie remembrance of the quotient in Diuision: as sone as you haue set downe your two fractions the one against the other, then make a straight line for the quotient: and as sone as you haue multiplied the Numerator of the Diuidend, by the Denominator of the Diuisor, set the number that amounteth ouer the said line, and then multiply the other two numbers, and set their totall vnder the same line.

Scholar. I perceiue you would not haue me trust to memory till I were better expert, lest oftentimes I happen by misse remembrance to be abused. This example I take for that declaration.

If I would diuide $\frac{3}{4}$ by $\frac{1}{2}$ I must set the numbers one against the other, (as here both appeare) e then make another line for the Quotient

2 3
3 4

in

in some good distance, where I may set the numbers of the quotient, as some as any of them is multiplied. So then as some as I haue multiplied 2 by 4, which maketh 8, I shal set that 8 ouer that line, thus: 8 And then multiply 3 by 3. which yeldeth 9: & that 9 must be set vnder the same line, and then will the whole quotient appeare thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but how may I perceiue that? Note.

Master. Although you shall better perceiue it by the Rule of Reduction, yet this example may be declared in commō coines, as in a common shilling of 12 d, of which $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 d, and so you may easily see that their proportions doe agree. And if you had taken this example before when you toke the example of $\frac{2}{3}$ and $\frac{3}{4}$, your Quotient would appeare (as this doth) more easie to vnderstand; whereas that Quotient being $\frac{8}{9}$, is not an easie proportion for you to perceiue, being yet little acquainted with proportions: whereof to giue you some taste, I will enter to the rule of Reduction: In which also I will declare other workes, both of Multiplication, and also of Diuision, which now I must for a time omit, as things that doe neede the helpe of Reduction.

Reduction.

Reduction of Fractions.

Fiue kinds of Reductions.

Herefore will I now declare the diuerſities of Reduction of Fractions, which commonly haue fiue formes of Reduction.

1 *Firſt, when there be ſundrie Fractions of intire vnite, they muſt be reduced to one denomination, and alſo into one fraction.*

2 *Secondly, when there be proponed fractions of fractions, they muſt be reduced likewiſe into one fraction: for otherwiſe they cannot be brought into one denomination.*

3 *Thirdly, when an improper fraction is proponed, that is to ſay a fraction in forme, which indeed is greater then an vnite: it muſt be reduced into apt forme, expreſſing the vnite or vnites of it, and the proper fraction diſtinctly. And ſometimes alſo it ſhall be needfull to conuert ſuch a mixt number of vnites with fractions into the forme of a fraction, that is, into an improper fraction: which two formes I eſteeme but as one, becauſe they worke on one kinde of number.*

4 *Fourthly, there happeneth ſometimes fractions to be written in great numbers, which might be written in leſſer nnmbers: therefore is there a meane to reduce ſuch great numbers into their ſmalleſt termes.*

Fifth,

Fiftly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certaine parts, but none of like denomination with that Fraction, then may you reduce the ſaid fractions into another, whoſe denomination ſhall expreſſe the common parts of that whole thing. 5

Scholar. This diſtinction in doctrine delighteth me much, but more with hope then preſent fruit: for as yet I do not underſtand ſcarcely the varieties, & much leſſe the practiſe and vſe of their workes.

Maſter. Reduction is an orderly alteration of numbers out of one forme into another which is neuer done orderly but for ſome needfull vſe, as in euery of the ſaid fiue ſeuerall formes I will diſtinctly declare.

First therefore, when two or more ſeueral fractions of any vnite be propoſed, as for example $\frac{1}{16}$ and $\frac{1}{8}$, becauſe it is hard to tell what proportion of the intire number thoſe two fractions do expreſſe, therfore was Reduction diuiſed, to be a mean whereby theſe ſeuerall fractions might be brought into one denomination and fraction. The firſt forme of Reduction

And in theſe fractions this is the Art for bringing them to one denomination.

Multiply firſt the denominators together, and the totall thereof you ſhall ſet twice downe vnder two ſeuerall lines for two new denominators, or rather for one common Denomi- How to reduce Fractions.

Of diuers denominations into one denomination.

nator. Then multiply the Numerator of the first fraction, by the Denominator of the second, and set the totall thereof for the Numerator ouer the first line. Likewise multiply the Numerator of the second fraction by the Denominator of the first, and set that totall ouer the second line for the Numerator of that fraction, and so are those two first fractions of seuerall denominations, brought to one denomination.

Scholar. If I vnderstand you, as I thinke I doe, my example shall declare the same. The fractions which you proponed were these, $\frac{3}{16}$, and $\frac{5}{6}$, whose denominators (being 16 and 6) I multiply together, and there amounteth 96 which I set vnder two lines thus: $\frac{96}{}$, $\frac{96}{}$.

Then I multiply the Numerator of the first fraction by the Denominator of the second saying, 3 into 6 maketh 18, that I set ouer the first line for a new Numerator, and it will be thus $\frac{18}{96}$:

Likewise I multiply the Numerator of the second fraction by the Denominator of the first, saying, 4 times 16 maketh 64, that I set for the second Numerator, and the fraction will appeare thus, $\frac{64}{96}$.

So that both fractions brought to one denomination, must stand thus $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You haue done well.

Scholar. I beseech, you let me examine it after

after my accustomed forme, by common parts of coyne or other measure.

Master. Goe to.

Scholar. I haue a peece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{12}$, that is, the 16 part, is 6 pence, and $\frac{1}{16}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{1}{2}$ of the same peece of Gold is 16 d. so that $\frac{2}{3}$ part maketh 64 d, that is $\frac{64}{96}$. And so I finde the sums to agree with the other before.

Master. *So haue you now the Art to bring two such fractions into one denomination. And if there be more then two, then must you multiply all the denominators together, and set the totall thereof so many times downe as there be fractions, and then to get for each one a new Numerator, multiply the Numerator of the* first, *by the Denominator of the* second, *and the totall thereof multiply by the Denominator of the* third, *and so forth, if there be more. Likewise multiply the Numerator of the* second, *by the Denominator of the* first, *and the totall thereof by the Denominator of the* third. *And in the same sort multiply the Numerator of the third into the Denominator of the* first, *and the totall thereof into the Denominator of the* second, *and so forth, if there were moe. So these three fractions* $\frac{2}{3}, \frac{3}{4}, \frac{2}{5}$, *doe make by reduction these other 3 fractions of one denomination* $\frac{24}{60}, \frac{45}{60}, \frac{40}{60}$. *All which yet*

Note the reduction of 3. fractions (or more) to one.

may bring into one fraction by adding the numerators together, and putting that totall for the common Numerator, reserving still that same common denominator. And those 3 fractions make one improper fraction thus, $\frac{109}{20}$.

Scholar. All this I perceiue, and also that this last fraction is more then an vnite, and therfore yon did cal it an improper fraction.

Master. There be certaine other formes of working in this Reduction, which I will briefly touch also, to giue you an occasion to exercise your wit therein.

The first varietie of Reduction

The first varietie is this: When you haue made and written downe your common Denominator (as I haue taught before) then to get a Numerator for the first, do thus: Diuide the common denominator by the denominator of the first fraction, & the quotient multiplied by the Numerator of the same yeeldeth a new Numerator for the first new fraction. So likewise do with the second and the third, and with all the residue, if there be more.

Sch. That wil I proue in your last example of these 3 fractions $\frac{2}{5}\ \frac{3}{4}\ \frac{2}{3}$. When the denominators be multiplied, they make 60: for 5 into 4 maketh 20, & 20 by 3 yeldeth 60, that I set downe 3 times, thus: 60, 60, 60: then to haue a numerator for the first I must diuide 60 by 5, (ỹ denominator of the first) & ths quotient is 12, which I must multiply by 2 (the numerator of the first) & that maketh

keth 24, & so haue I for the first fraction $\frac{24}{60}$.
Likewise for the second fraction: I diuide 60 by 4, & there commeth 15: which I multiply by 3, and so haue I 45, and the second fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{40}{60}$.

The second varietie.

Master. *Another way is this: If it happen so, that the lesser denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numerator ouer the lesser denominator, and for the lesser denominator put the greater, as thus in these two fractions, $\frac{3}{12}$ and $\frac{2}{3}$, three being the lesser denominator multiplied by 4, will make 12, which is the greater denominator: therefore by the same 4 I do multiply 2, which is the Numerator euer 3, and that maketh 8: vnder which I do put 12 being the greater denominator, which is also made by multiplication of 4 into 3, and so haue I these two fractions $\frac{3}{12}$, $\frac{8}{12}$, thus shortly reduced, without altering the one fraction.*

Scholar. This I vnderstand.

The third varietie.

Master. *Then marke this third way: if the denominators do not happen so, that one by multiplication may make the other, then looke whether they both may be parts of any other one number, as in $\frac{5}{12}$, and $\frac{7}{18}$, although the lesser taken but twice, be too great to make 18, yet they both may be parts vnto 36: therefore looke how many times twelue is in 36, and that quotient being multiplied by the Nume-*

rator ouer 12, *the totall shall be put in stead of Numerator ouer* 12, *and for* 5 *put* 15, *thus,* $\frac{11}{12}$. *So likewise looke how often is* 18 *in* 36, *and because it is twice, therefore by* 2 *multiply* 7, *which is ouer* 18, *and it will be* 14, *set that for the Numerator, and in stead of* 18 *put* 36, *and then shall your fractions reduced stand thus* $\frac{11}{12}$, $\frac{14}{36}$, *in stead of* $\frac{4}{12}$ *and* $\frac{7}{18}$.

Proofe. And if you will proue whether you haue wrought well or no, that may be proued by Reduction of them againe to their former denominations, which Art shall be taught in the fourth kind of Reduction, where greater termes of Fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction of the same termes or numbers come againe that were before, then is the worke good, els not.

Scholar. Sir, I heare your words, but I doe not vnderstand many of them: which if it may please you declare.

Master. With a good will, when conuenient place serueth, but that must be in the said fourth kinde of Reduction. In the meane season I will declare the second forme of Reduction, which teacheth how to reduce fractions of Fractions into one Fraction, and so to one Denomination.

The second sort of Reduction. *When fractions of Fractions bee proponed, you shall multiply the Numerators of each into other, and set the totall for the new Numerator,*

merator, and then multiply all the denominators likewise, and take their total for the new denominator, and so are they speedily reduced.

Scholar. If that be all, then I vnderstand it already, as by this example I will declare. These be the fractions, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{6}{7}$ of $\frac{7}{9}$, which I would reduce to one denomination. Reduction of fractions of fractions into one denomination

Therefore begin I with the Numerators, and multiply them together, saying: 3 into 2 maketh 6, & 6 by 6 maketh 36, which multiplied by 7, yeldeth 252, that I set ouer a line for the Numerator thus. 252

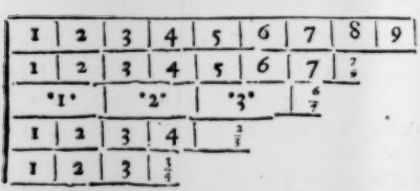
Then I multiply the Denominator, 4 by 3 maketh 12, & that by 7 bringeth 84, which multiplied by 9, yeldeth 756, the new denominator. And so the whole reduced fraction is this, which is to hard a fraction for me to vnderstand yet. $\frac{252}{756}$

Master. You thinke so, and no marueil, but anone you shall learne to iudge it easily: for this fraction is no more indeed then $\frac{1}{3}$, although it be in greater termes, and therefore more straunger, and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kind of fractions, and also the due vnderstanding of the Reduction.

The

The entire measure parted into 9:



Here you see the longest measure, (which standeth for the whole and entire quantity) first parted into 9 divisions, whereof 7 are severed by the second measure: and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being divided into foure parts, the lowest measure doth containe $\frac{1}{4}$, so that the same $\frac{1}{4}$ must be named, not $\frac{1}{4}$ of the whole measure, but indeed is $\frac{1}{4}$ of $\frac{3}{7}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar: This example is so sensible that I cannot chuse but see it. And furthermore I see also that the same fraction is equall to $\frac{3}{7}$ of the entire measure, as the lines which run by and downe doe expresly set forth. Also I see here that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{7}$. And further yet that $\frac{3}{7}$ of $\frac{7}{9}$ is equall to $\frac{7}{9}$ of $\frac{3}{7}$.

Master. I am glad that you see it so well, not doubting but you will gather greater light of knowledge hereby.

But

But now it is time that we come to the third forme of Reduction which teacheth of improper fractions, that is to say, mixt numbers of vnites and Fractions, although they appeare like fractions, as this $\frac{16}{5}$, which doth include 5 vnites wholly, and $\frac{1}{5}$ ouer. Wherefore first you shall know them: by that the Numerator is greater then the Denominator. The third forme of Reduction of improper fractions.

Scholar. **Indeed sir, that appeareth reasonable, that if the** Numerator **doe expresse more parts to be taken of any vnite, then the** Denominator **doth signifie that vnite to be diuided into, it must néeds follow that such a fraction importeth more then the whole, that is to say, the whole with certain parts ouer: but what reductiō is there in it?**

Master. There be two seuerall kinds of reduction, concerning such fractions. Sometimes it shall bee needfull to conuert these fractions into vnites, and the proper fractions that will remaine. And sometimes contrariwise, it shal be meet to reduce mixt numbers, that is, vnites written with fractions, into the forme of one simple fraction, and so be there two wayes. Two seuerall wayes in this Reduction.

Scholar. **What is the meane of the first way to turne improper fractions into vnites with their fractions?**

Master. *That is thus. Your Numerator being greater then the Denominator, must bee diuided by the same Denominator, and the quotient* The first way.

Reduction of improper fractions into vnites, with their proper fractions.

quotient thereof expresseth the vnites: the Remainer shall be put for the Numerator of the fraction that resteth, and the Denominator must be the same that was before.

Scholar. For example, I take $\frac{17}{5}$. And dividing 17 by 5, the quotient will be 3, and there will remaine 2.

Master. That must you write thus, $3\frac{2}{5}$ where (you see) I haue written 3 without any line, as entire numbers ought to bee written, and the 2 that remained I haue set ouer the former Denominator with a line, as a proper fraction. And this number doth signifie now 3 vnites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by vnites here vnderstand crownes, so it were 3 Crownes, and $\frac{2}{5}$ that is 2 s.

The second way.

Master. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this hapeneth sometime that when the reduction is so wrought, there remaineth nothing And then it is not a mixt number, but a simple entire number, represented like a fraction.

Scholar. As $\frac{12}{4}$ will make 3 iust, and $\frac{12}{2}$ will make euen 6. This I will remember. But now, what is the second forme of Reduction, that you speake of, for these sorts of fractions?

Master. *When soeuer you haue any of these two sorts of numbers, that is to say, whole numbers without fractions, or whole numbers with fra-*

fractions, and you would turne them into the forme of a fraction, you must multiply the whole number by that denominator which you will haue to remaine ſtill, and to the totall thereof adde the Numerator, which you haue already, and all that ſhall you ſet for the new Numerator, keeping ſtill the former Denominator: as if you haue 6 ¼ which you would conuert into an improper fraction, you muſt multiply 6 by 4, whereof commeth 24, & therto adde the numerator which is 3 & ſo haue you 27 for the numerator, & 4 ſtill for the denominator.

Reduction of whole numbers, either alone, or ioined with fractions into fractions.

Scholar. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Maſter. Euen iuſt, and ſo backward, (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of theſe Reductions may be the proofe of the others worke.

Note.

Scholar. This I perceiue, but now if you would turne whole numbers without fractions into any fraction, I ſee not how that may be done, becauſe there is no Denominator to make the multiplication by.

Maſter. That was well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication muſt be made: for the proofe whereof I ſet downe 7, which is a whole number. And if you will haue this number conuerted into any certaine fraction, will me to doe it.

Scholar.

Sho. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some fraction.

Scholar. No, I passe not for the sort of the fraction.

Master. Then how can you thinke that you require me to doe any thing certaine, when you leave me to doe as I list? And seing you stand at that stay, whether thinke you that I must first intend in mind what fraction I will make of it, before I can doe it indeede

Scholar. Else you should doe ignorantly.

Master. Then will I limit my selfe (seing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the Denomination of quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the Denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar. Indeed I perceive this to be reasonable, for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turne 8 into 5 parts, it will make $\frac{40}{5}$, which is all one with 8: for 8 crownes turned into 5 parts (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a crowne.

The fourth forme of Reduction.

Master. *Seeing you understand now these three kinds of Reduction, I will declare unto you the fourth kind, that is, when fractions be written*

written in greater termes then they need, how they may be brought to lesser termes.

Scholar. To write any thing in greater termes then needeth, seemeth to be a fault, & so this rule seemeth to amend that fault.

Master. It were a fault to doe any thing without need, which after must be redressed: but in this case it is not so; neither did I say absolutely (as you doe) that it needeth not to expresse those fractions in so great termes, but that the fractions doe not need, I meane for their value to be understood: but yet it may be needfull for the ease of those workes whereto they be applied, as for example: In the first kind of Reduction this was your owne example: $\frac{1}{10}$ and $\frac{4}{8}$, which when you would reduce you were faine to turne them first into one denomination, & so appeared they thus, $\frac{18}{36}$, and $\frac{64}{36}$, where the fractions (for their owne understanding) needed not to be turned out of smaller terms into greater, but yet the easinesse of working needed it.

Scolar. Sir, I understand now, not onely the difference of this need (for the fractiõs might better be understood as fractiõs seuerall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you meane by greater termes and lesser termes, wherof before I was in doubt: for I see you call

Termes of Fractions.

call the Numerator and Denominator, the termes of the fraction.

Maſter. *I am glad you vnderſtand it ſowel. Now then when you would value any fractions (becauſe that they may beſt be done when the termes are ſmalleſt) you ſhall reduce them to the ſmalleſt that you can, which thing you may doe thus: Diuide the greateſt of any ſuch two termes by the leſſer, and if any thing remaine, by that remainer diuide the laſt diuiſor: and if any thing remaine now, by that diuide the firſt diuiſor (which was before the remainer of the laſt diuiſion) and ſo continue ſtill, til nothing doe remaine in the diuiſion: and then marke your laſt diuiſor, for it is the number that will eaſily reduce your fraction, if you diuide both the numerator and the denominator by the ſame number, and put for the numerator the quotient of his diuiſion, and for the denominator alſo his quotient that riſeth by his diuiſion.*

Reduction of fractions into their ſmalleſt termes or leaſt value.

Scholar. I take for example $\frac{96}{18}$, and becauſe 96 is the greateſt number, I diuide it by 18, and the quotient is 5, and there reſteth 6: what ſhall I doe with this Quotient?

Maſter. Nothing in this worke, but now ſeeing there remaineth ſomewhat, by that remainer muſt you diuide the laſt Diuiſor.

Scholar. If I ſhall diuide 18, (which was the laſt diuiſor) by 6, that was the remainer, ſo is the quotient 3, and nothing reſteth.

Maſter.

Master. As for the Quotient I omit him yet: but because there doth remaine nothing, therefore is 6 (which was your last Diuisor) that number by which you may reduce the Fraction proponed.

Scholar. Then as you taught me, I must diuide the Numerator 18 by 6, and the quotient is 3, which I must put for the Numerator ouer a line, thus: $\frac{3}{\ }$ And then by the saide 6 must I diuide also the Denominator 96, and the Quotient will be 16, which I must take for the Denominator, and so is the Fraction $\frac{3}{16}$. And so mee thinketh this Rule doth proue the worke of the first Reduction.

Master. That is true if the first Reduction were made of fractions into their least termes, and else not, without some helpe, as the second number in that place will declare.

Scholar. The second number was $\frac{2}{3}$ which was turned into $\frac{32}{48}$ by that Rule. Now if I shall by this Rule reduce it againe into the least termes, I must diuide 96 by 64, and there remaineth 32, wherefore I must take that 32 for the Diuisor, to reduce the saide fraction. Then doe I diuide 64 by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96 by 32, and the quotient will be 3, and so I haue but $\frac{2}{3}$.

Master. Muse not at the matter, for you

haue done well enough: but you thinke you haue not the fraction that you looked for, that is, $\frac{4}{8}$, yet haue you one equall to it, as by the parts of a shilling you may proue.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$, and $\frac{4}{6}$ and $\frac{2}{3}$, be all three equal. And now I perceiue that because $\frac{4}{8}$ was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now vnderstand I this Rule well. But is there any other way to worke this Reduction?

Another way to worke this reduction.

Master. *Yes, but first note this, that if you find no such Diuisor to reduce the Fraction til you come to* 1, *because one doth make no Diuision: therefore that Fraction is already in his least termes, as by* $\frac{71}{100}$ *you may proue, and so of* $\frac{81}{253}$, *and many other like.*

Note that mediate any number to diuide by two.

But now for your better aide to find the due proportion in least termes, with more ease for a young learner, you shall meditate or take the halfe of the Numerator, and also the Denominator as long as you may vpon a line, alwaies parting them with a right downe dash of your pen as you worke, which may easily be done, if the numbers be euen: as 2, 4, 6, 8, *or* 10, *but if they bee odde (though it be but one of them) then must you abreuiate them by* 3, 5, 7, *or* 9, *&c.*

And because examples doe most instruct, I haue

I haue here set downe the manner of two or three, whose last number at the end of the line sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawne from it thus $\frac{288}{576}$. And because both the Numerator, & the Denominator end in euen numbers, I (for this may be abreuiated by 2, or 4, or 6, &c. Therefore on the other side of the right downe dash toward the right hand, I first take the halfe of the numerator: saying, the halfe of 2 is 1, the halfe of 8 is 4: and again, the halfe of 8 is 4: which 144 is now a new numerator: and therefore I part it with a right downe dash, as before:

Then doe I also take the halfe of 576, in saying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a new Denominator.

Then beginning againe: saying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2, vntill you come to the last worke, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$, which is equal to $\frac{288}{576}$.

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2
	28	14	7	1			
	212	56	28	4			

Abbreuiated first by 5, then by 293.

1465	293	1
4395	879	3

Scholar. Sir I thanke you much, this is very easie and good for a young learner.

Master. So it is, but yet notwithstanding if you can without that diuision by memory espie the greatest number that may diuide exactly both termes of your fraction proponed, then need you not to vse that diuision, as in this fraction $\frac{12}{132}$, I see that 12 is the greatest number that I can diuide them both: and therfore without any worke, by memory onely, I turne that into $\frac{1}{11}$, but this ability in knowledge is got by exercise.

Yet one other way of easie reduction in this kind there is when your fraction hath any ciphers in the first places of both termes, then may you by casting away the ciphers, make a briefe reduction, as thus $\frac{300}{3000}$ here take away the cyphers, and it wil be $\frac{3}{30}$, which is the same in value with $\frac{300}{3000}$.

Sch. And so if I haue $\frac{400}{4700}$ it will be $\frac{4}{47}$.

Master. You are deceiued, for you take away more cyphers from the Numerator then you doe take from the Denominator, which you may not dee.

Scholar. I confesse my fault, which came of

of to much haſt, I was more gladder of the Rule then wiſe in vſing it: but now I vnderſtand it I truſt.

Maſter. Then may I goe in hand with the fift or laſt kind of Reduction, which teacheth how to turne any fraction proponed into any other Denomination that you liſt, or into any parts of common coynes, weights, or meaſures, or ſuch like.

The fift kinde of Reduction

For declaration whereof, firſt you ſhal marke whether your fraction be a ſimple Fraction, either elſe a Fraction of ſundry parts, I meane of more termes then two. And if your Fraction be a Fraction of Fractions, or otherwiſe compound, you muſt reduce it to one ſimple fraction. And then marke well the Denomination of that other Fraction, into which you would turne this: For by that Denominator you muſt multiply the Numerator of your firſt Fraction, and the totall product thereof ſhall you diuide by the Denominator of your firſt Fraction, and that quotient ſhall be the Numerator of the Denominator proponed: as for example, I haue this Fraction $\frac{3}{5}$, *which I would turne into ten parts: therefore I multiply this* 10 *by* 3, *that is the Numerator of my Fraction, and there ariſeth* 30, *which I diuide by* 5, *and the quotient is* 6, *which muſt be the Numerator to* 10 *and ſo* $\frac{3}{5}$ *will be* $\frac{6}{10}$.

Scholar. This is eaſie enough to doe.

Maſter. Then ſhall you ſee another ex-

ample of the same fraction that is not so ca-
ste: as if I would turne $\frac{3}{5}$ into 8 parts, proue
you that worke.

Scholar. I must multiply 8 by 3, & there
amounteth 24, which I diuide by 5, and the
quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull
in this worke?

Scholar. I see that when 24 was diuided
by 5, there remained 4, which I did not
passe of, because we spake nothing of any re-
mainer, but onely of the quotient.

Master. By likelihood you remember what
I said to you in diuision of whole numbers,
that you should not passe of the remainer
there, but onely note it as a sum that could
not be diuided without knowledge of frac-
tions. Wherefore now marke this, that in
all diuisions of whole numbers, when there
is any remainer, you shall set it ouer a line
as a Numerator, and set the Diuisor for the
Denominator, and that fraction doth make
the Diuision compleat, & is part of the quo-
tient: as if I would diuide 48 by 5, the quo-
tient will be $9\frac{4}{5}$: so in your former worke
when 24 was diuided by 5, the quotient
should be $4\frac{4}{5}$, and so the new fraction should
be thus: $\frac{4}{5}$ and $\frac{4}{5}$ of 8, that is, $\frac{4}{5}$ of the entire
number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which
you may proue by example of some coyne.

Scholar. Then I take a crowne, whose $\frac{1}{8}$

is

is 3 s. Now if I would proue whether the 3 s be $\frac{4}{5}$ and $\frac{5}{6}$ of $\frac{3}{4}$, I shall haue a cumbrous worke to doe.

Master. Indeed for whole pence, your example is a little troublesome: yet turning ỹ crown into halfe pence, it is easie enough.

Scholar. That will I trie.

¶ First I see that $\frac{3}{4}$ of a crowne is 3 s, which is 36 d, or 72 halfe pence. Now if I can find that this fraction $\frac{4}{5}$ and $\frac{5}{6}$ of $\frac{3}{4}$ be equall vnto 3 s, then am I fully answered. Because I cannot take $\frac{4}{5}$ of a crowne, I turne tht crowne into halfe pence, as you willed me, which makes 120, wh'ch I diuide by 8, my quotient is 15, which taken foure times, make 60 ob. Now resteth me to haue $\frac{4}{5}$ of the $\frac{5}{6}$ part of a crowne, whereof $\frac{5}{6}$ part is 15 ob the 15 being parted in 5 parts, the quotient is 3, which taken 4 times maketh 12 ob. which with my 60 before, amounteth to 72, wh are then equal to $\frac{3}{4}$ my desire.

Master. I commend you for your diligence, you might haue wrought it thus either $\frac{4}{5}$ being abreuiated as before I taught, is $\frac{1}{2}$. Now, halfe of a crowne is 2 s, 6 d. Then $\frac{4}{5}$ of $\frac{5}{6}$ is a fraction of fractions, which if you doe reduce into one entire fraction, as before you haue learned, in saying, fiue times 8 is 40, for a new Denominator, and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, and abreuiated also make $\frac{1}{10}$. Now the

tenth part of a crowne is 6 d, which put to 2 s, 6 d, make also 3 s, your desire.

But now one example more for this rule, and then we shall end it. If I haue $\frac{7}{15}$ of a Soueraigne (accounting the Soueraigne 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholar. I must multiply 7 by 20, & that maketh 140. which I shall diuide by 15, & ye quotient will be $9\frac{5}{15}$, or else in lesser termes $\frac{1}{3}$

Master. That is 9 s, & one third part of a shilling, that is 4 d, as by the same Rule you may proue. And this for this time shall suffice for Reduction, saue that I must now repeat a little touching Multiplication and Diuision, and so goe forward.

Multiplication.

Multiplication of fractions.

IN Multiplication it happeneth sometime, that there bee whole numbers to be multiplied with Fractions: and may be in two sorts: for either the whole number is seuerall from the Fraction, and is the Multiplier, or else the whole number is ioined with one, or both of the Fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no Reduction, but onely multiply the Numerator of the Fraction by that whole number, and the totall thereof set for the new Numerator.

Reduction of whole numbers into fractions.

Scholar.

Scholar. I understand you thus. If I have $\frac{6}{13}$ to be multiplied by 16, then must I multiply that 16 with 6, which is the Numerator whereof commeth 96, & that must I set for the new Numerator, keeping still 13 for the Denominator, and so the fraction will be $\frac{96}{13}$, that is $4\frac{4}{13}$.

Master. And in this sort of work you may abridge the labour, thus. If it happen the Denominator to be such a number, as may cunnly be divided by the said whole number proposed, then divide it thereby, and set the quotient of that division for the former Denominator: but reserve still the Numerator, and so is the Multiplication ended.

Scholar. Then I faine this example $\frac{7}{20}$ to be multiplied by 5: and because 5 will iustly divide 20, therefore I take the quotient of that division which is 4, and set in stead of 20, and so the fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$ that would have followed of y^e other sort of work.

Scholar. I perceive it very well.

Master. *Now then for the other sort, where the number is mixt take this way: first to reduce the said whole number, and fraction into one fraction improper (as I shewed you in* Reduction*) and then multiply them together as if they were proper fractions.* How to multiply mixt numbers.

Scholar. $13\frac{1}{4}$ being set to be multiplied by 4; first I must reduce the mixt number,

as

as appeareth in the margent, by multiplying 13 by 5, and that maketh 65, whereto I must adde the Numerator 3, and so the fraction will be $\frac{68}{5}$, which two fractions now I shal multiply after the accustomed forme, and it will be $\frac{340}{20}$ or $13\frac{4}{5}$.

Master. You haue done well: and so may you see, that although most part of the formes of Multiplication may be wrought without Reduction, yet some cannot, as namely mixed numbers.

And yet one note more will I tell you of Multiplication, before wee leaue it: That is, whensoeuer you would multiply any fraction by Duplation. *2, which commonly is called* Duplation, *you may do it not onely by doubling the Numerator, but also by parting the Denominator into halfe if it be euen.*

Scholar. Then if I would double $\frac{1}{2}$, I may chuse whether I will make it, $\frac{2}{2}$, or else $\frac{1}{1}$. And indede I see that is all one, but that the diuiding of the Denominator seemeth the better way to make smaller termes of the fraction, and so they shall neede the lesse reduction.

Master. It is so: & now I shall not need to tell you that Multiplication is proued by Diuision, and Diuision likewise by Multiplication: but the like workes that I shewed you in Multiplication, will I shew you in Diuision also.

Diuision,

Diuision.

When any whole number shall be diuided by a fraction, you must multiply the said whole number with the Denominator of the fraction, and set the totall thereof for the new Numerator, *and for the* Denominator *set the* Numerator *of the fraction.*

Diuision of fractions.

To diuide a whole number by a fraction

Scholar. Then 20 diuided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{80}{3}$.

$$\frac{20}{1} \text{ by } \frac{3}{4} \qquad 80 \qquad 3$$

Master. *Euen so: but if you would diuide the fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for the Denominator, without changing the Numerator.*

To diuide the fraction by the whole number.

Scholar. Then to diuide $\frac{20}{3}$ by 4, it will be $\frac{20}{12}$, as here appeareth $\frac{20}{3}$ by $\frac{4}{1}$ in this example. $\frac{20}{12}$.

Master. You say well. And by the same example you giue me occasion to remember another briefe way to do the same: for if you had diuided the said numerator by 4, and set the quotient for the numerator, keeping still the olde denominator. it would haue bene not only as well done, but also in a fraction of lesser termes

Another brief way.

Scholar. I guesse it to be euen so, by a like worke

worke that you taught me in multiplication: And for proofe thereof $\frac{20}{23}$ being the diuidend, and 4 the diuisor, I diuide the Numerator 20 by 4, & the quotient is 5, which I set for 20 ouer 23, thus, $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$ as by diuiding or abbreuiating both these termes by 4, & so reducing them to their least denomination, I may easily proue: as appeareth by this example $\frac{20}{92}|\frac{5}{23}$.

Master. You conceiue it well. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction, and then worke as you haue learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you goe forward to some other thing.

Master. Then take this note yet for Diuision, if the Denominators be like, thē diuide the Numerators as if it were in whole numbers, & the quotient whether it be fraction, whole number, or mixt, is a good quotient for that diuision. And generally if one of the Numerators may iustly diuide the other, by that quotient multiply the Denominator of the lesser Numerator, & set it that both amount in the roome of the same denominator, & then for a numerator to it, set the Denominator of the other fraction.

Scholar. Then if I would diuide $\frac{1}{4}$ by $\frac{12}{17}$

I

I see that 3 will diuide 12, and the Quotient will be 4, by which I must multiply the other 4, that is the denominator vnder 3, & then it is 16, which I set for the denominator 4, & ouer it instead of 3, I must set 17 the other Denominator, & so is it thus, $\frac{17}{12}$.

Master. And so is $\frac{17}{12}$ instead of $\frac{11}{41}$, which would haue risen by the common worke as here appeareth. $\frac{3}{4}$ by $\frac{11}{41}$

And now for mediation (which is to diuide by 2) marke this, if the Numerator be euen, set the halfe of it in his place without the Diuisor, and so haue you done: and if the Numerator be not euen, then double the Denominator. Mediation.

Scholar. That is, if I would mediate $\frac{6}{11}$ I may make the quotient $\frac{3}{11}$, & if I would mediate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Master. Now trust I that you haue sufficient knowledge in Reduction, Multiplication, and Diuision: and therefore will I goe in hand with Addition & Subtractiō, which now will appeare easie enough:

Addition.

Whensoeuer you haue any Fracti-ons to be added, you must consider whether they be of one denomination or not: and if they be of one denomination, then adde the Numerators together, and set that that amounteth for the Numerator, ouer the com- Addition of fractions of one denomination.

mon Denominator, and so haue you done: The reason is because that such differ little in Addition *or* Subtraction *from the worke of vulgar denominations, where the denominaters be of odde numbers: as* 3 *d, and* 5 *d, make* 8 *d, where the* Denomination *is not altred. But if the fractions be not of one Denomination, or any of them be mixt of whole numbers and fractions, then must you first reduce them to one* Denomination, *and after adde them. And if they be many, then adde first two of them, and so the summe that doth amount of the Addition, and the third, & then the fourth &c. if you haue so many.*

To adde fractions of diuers denominations.

Scholar. This seemeth easie enough, now that I haue already learned to multiply & to reduce, without which two I could neuer haue thought this. And therefore now I see good reason, why you did place Multiplication, and Reduction before Addition.

Maister. It is well considered, but yet refuse not to expresse your vnderstanding of it by an example.

Scholar. Then would I adde first, with $\frac{1}{2}$, and because the Denominators are like (and so needeth no reduction) I adde 7 to 5 which maketh 12, and then is my summe $\frac{12}{24}$, that is in smaller numbers, being abbreuiated, $\frac{1}{2}$.

And if I haue many numbers to be added as here $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, first I must reduce them (be-

cause

cause they haue diuers denominators) into one denomination, & then they will be thus $\frac{100}{400}\ \frac{320}{400}\ \frac{360}{400}$, or in lesser termes $\frac{25}{100}\ \frac{32}{100}\ \frac{36}{100}$ which by Addition doe make $\frac{83}{100}$ that is $2\frac{3}{4}$.

Maister. Now may we go to Subtraction.

Subtraction.

Subtraction or fraction.

Subtraction hath the same precepts that Addition had: for if the Denominators be like then must you subtract the one Numerator from the other, and the rest is to be set ouer the common Denominator, and so your Subtraction is ended: but and if you haue many fractions to be subtracted out of many, then must you reduce them to one Denomination, and into two seuerall fractions, that is all that must be subtracted into one fraction, and the residue into another fraction, and then worke as I said before.

Scholar. For the first example I take $\frac{10}{12}$ to be subtracted out of $\frac{12}{12}$ & ỹ rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For another example, I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I subtract 24 out of 28, and there resteth 4, which I set ouer the common Denominator for a Remayner, thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Now

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ & $\frac{9}{10}$: And because their Denominators be diuers, I doe reduce them into one denomination thus $\frac{144}{192}$ $\frac{168}{192}$ $\frac{172}{192}$ $\frac{180}{192}$ $\frac{160}{192}$ $\frac{150}{192}$. Then doe I adde the two first, and they make $\frac{342}{192}$. Also I adde the two last, and they yeld $\frac{340}{192}$. Then doe I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{192}$, that is in smaller termes $\frac{23}{12}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Master. Proue one example more of two fractions of diuers denominations.

Scholar. I take these two fractions, $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced will stand thus $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I cannot.

Master. Then may you perceiue that you misstoke the fractions: for you cannot subtract the greater out of the lesser, although you may adde, multiply, or diuide, the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his termes lesser then $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser fraction: for generally if you multiply the Numerators and the Denominators of two fractions crosse waies, that Fraction is the greatest of whose Numerator commeth the greatest summe, as in this example: 7 multiplied by 24, maketh 168: and 9 being multiplied by 8, yeldeth but 72: there-

The greatest of two fractions.

therefore is the first fraction 7 the greatest of these two, so can you not subtract it out of a lesser fraction.

But if you should subtract a fraction out of a whole number, what would you doe?

Scholar. Marry I would reduce the whole number into a fraction of the same Denomination that my fraction is, and then worke by Subtraction.

Master. So may you doe, but it is easier much, if your fraction be a proper fraction, that is to say, lesse then an vnite, to take an vnite from the whole number, and then turne it into an improper fraction, and so worke your Subtraction. As if I would subtract $3\frac{2}{7}$ from 4. I may take 1 from 4, & turne it into $\frac{7}{7}$, frō which if I abate $3\frac{2}{7}$ there will remaine $\frac{5}{7}$. And if the first Fraction be an improper Fraction, then may I take so many vnites from the whole number, that they may make an improper Fraction greater then that first, and then worke by by Subtraction. As if there be proponed $\frac{10}{5}$ to be subtracted from 6, becaufe $\frac{10}{5}$ is more then 3, and not so much as 4, I must take 4 from 6, and turne them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{5}$, and there resteth $\frac{2}{3}$: so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{5}$ which is $\frac{16}{5}$, from 6, whole: then set 1 vnder 6, as thus $\frac{6}{1}$: And then to reduce those two fractions into one Denomination, as here appeareth $\frac{16}{5}$ from $\frac{6}{1}$

Then $\frac{12}{7}$ from $\frac{15}{7}$ resteth $\frac{3}{7}$ which maketh $2\frac{3}{7}$ your desire. And thus wil I make an end of the worke of cõmon Fractions for this time, not doubting, but you can apply them both vnto the Rule of Progression, and also vnto the Golden Rule, without any other teaching then you haue learned before, which might séeme tedious to repeat, saue that in some speciall diuersities, which be peculiar to fractions, I cannot ouerpasse, but instruct you somewhat by the way.

8

3

The Golden Rule.

The rule of proportion.

THerefore as touching the Golden Rule, for the placing of the 3 numbers proponed in the question whereby to find the third, and for the forme of their worke, with other like notes, I refer you to that which you haue already learned

But this easie forme of working by fractions shall you note, that if your three numbers be fractions, for an apt worke and certaine, multiply the Numerator of the first number in the question, by the Denominator of the second. And all that againe multiply by the Denominator of the third number, and the totall thereof shall you keep for to be the Diuisor. Then multiply the Denominator of the first number by the Numerator of the second, and the whole

thereof by the Numerator of the third, and the totall thereof shall be your Diuidend.

Now diuide this Diuidend by the diuisor which you found out before, and that number shall be the fourth number of the question, which you seek for, as in this example. *If ¾ of a yard of velvet cost ⅔ of a Souereigne, estemed at 20 s, what shall ½ cost?* A question of velvet.

Scholar. If it please you to let me make the answere, I would first place these three numbers, as I learned in whole numbers, thus.

And then according to your new rule, I must multiply 3, being Numerator in the first number, by 3 the Denominator of the second, and thereof commeth 9, which I multiply againe by 6, the Denominator of the third number, and so haue I 54, which I keepe for the Diuisor: Then multiply I 4, the Denominator of the first, by 2 the Numerater of the second, and there ariseth 8, which againe I multiply by 5, the Numerator of the third, and it maketh 40: Then must I diuide 40 by 54, and it will be $\frac{40}{54}$, that is, $\frac{20}{27}$ in lesser termes, and then the figure will stand thus.

But what that is in money I cannot tel, except I shall worke it by Reduction, as you taught me.

Master. It forgetteth not now, you may reduce it when you list, but it were disorderly

done here to mingle diuers workes together, where we do not seeke the value of the thing in common mony, but in an apt number, which ye haue well done. And therefore will I yet shew you another like way of easinesse in worke, how you may change your three fractions into three whole numbers, by which you shall worke as if the question were proponed in whole numbers. The first number you shall find as I taught you: now to find the Diuisor of the second number, take the Numerator for the second fraction: and for the third number take that that ariseth of Multiplication of the Denominator of the first, by the Numerator of the third, and then worke your question.

A question of Siluer.

Scholar. *For example hereof, I put this question. If $\frac{11}{12}$ of 1 l' waight of siluer be worth $\frac{12}{4}$ of a Soueraigne, what is $\frac{1}{2}$ of 1 l' waight worth? For the answere, first I place the fractions in order thus.*

$$\begin{array}{c} \frac{11}{12} \\ \frac{1}{2} \end{array} Z \frac{12}{4}$$

Then to turne these fractions into whole numbers, I multiply 11, which is the Numerator of the first by 4 (the Denominator of the second) and there commeth 44, which I multiply by two the Denominator of the third, and so amounteth 88, which I set for the Diuisor in the first place. Then in the second place I set 12, which is the Numerator of the second fraction, and in the third place I set the summe that amounteth of 12,

being

being the Denominator in the first number, multiplied by one, being Numerator in the third number, add to the figure 88 Z 12 will stand as here you see. 12

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be $\frac{16}{11}$, or in lesser termes, $1\frac{7}{11}$, and then the figure will stand thus: $\frac{12}{12}$ Z $\frac{12}{4}$ $\frac{4}{3}$ Z $1\frac{7}{11}$

Master. These two formes now you understand well enough, and as for any other at this time I will not repeat, onely this shall you mark for the proofe of this rule whether your worke be well wrought or no: Multiply the first number by the fourth, and note what amounteth: Then multiply the second by the third, and marke what amounteth also. Now if those two numbers so amounting be equal, then is your worke well done, else you have erred. And this shall suffice for the former Rule. But in the backer Rule, this shall you note for ease of worke. that you multiply the numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the third, and that amounteth thereof, shall be the Diuidend. Then multiply the Denominator of the first by the Denominator of the second, and that whole by the Numerator of the third, and that that riseth thereof shall bee the Diuisor. Example of this:

The proofe of the golden Rule.

The backer Rule.

A queſtion of Loane.

I did lend my friend $\frac{1}{4}$ of a Porteguiſe ſeauen moneths, vpon promiſe that he ſhould doe as much for me againe and when I ſhould borrow of him, he could lend me but $\frac{5}{13}$ of a Porteguiſe: now I demand how long time muſt I keepe his mony in inſtrecompence of my loane, accounting 13 moneths in the yeere?

Scholar. The firſt number muſt be the firſt mony borrowed, that is $\frac{1}{4}$ of the porteguiſe: the ſecond number the 7 moneths, that is $\frac{7}{13}$ of a yeere: and the third number the money that was lent in recompence, that is, $\frac{5}{13}$ of a Porteguiſe: then I ſet the numbers thus:

$\frac{1}{4}$ $\frac{7}{13}$ $\frac{5}{13}$

Then (as you taught me) I multiply 3 (being Numerator in the firſt number) by 7, the Numerator of the ſecond number, and it maketh 21, which I multiply by 12, the Denominator of the third, and ſo haue I 252 for the diuidend: then I multiply 4 the Denominator of the firſt, by 13 the Denominator of the ſecond, and it yeeldeth 52, which I multiply againe by 5, the Numerator of the third, and it will make 260, that is the Diuiſor. Then muſt I diuide 252 by 260, ſo it will be in the ſmall Fraction $\frac{9}{10}$ of yeere.

Maſter And thus doe you ſee ſome eaſe in working better then to multiply & diuide tediouſly ſo many fractions.

Statute of Aſſiſe of Bread and Ale.

Another queſtion yet will I propoſe, to the intent you may ſee thereby the reaſon of the Statute of Aſſiſe of Bread & Ale, which in all

Statute

Statute Books, in Latine, French, and English is much corrupted for want of knowledge in this Art, for the right vnderstanding whereof I propone this question.

When the price of a quarter of wheat is 2 s̃, the farthing white loafe shall waigh 68 s̃, then I demand, what shall such a loafe waigh when quarter of wheat is sold for 3 s̃? Question of Bread

Scholar. This question must be wrought as it is proponed in whole numbers, and not in Fractions.

Master. You séeme to say reasonably, howbeit in the Statute of Assise, the rate is made by the proportion of parts in a pound waight Troy, else could it not be a Statute of any long continuance, séeing the shillings doe change often, as all other monies doe: but this Statute being well vnderstood, is a continuall rule for euer, as I will anon declare by a new Table of Assise, conuerting the shillings into ounces & parts of ounces.

Wherefore here by a shilling you must vnderstand $\frac{1}{20}$ of a pound waight, and so by a penny $\frac{1}{20}$ of an ounce: wherfore although you might worke this question proponed by whole numbers well enough, for that time when the Statute was made, yet to apply it to our time, & to make it serue for all times generally, it is best to worke it by fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings :and so for three shillings $\frac{3}{20}$, & then will the figure of the question stand thus. Note what a shilling i

$$\begin{array}{c} \frac{2}{20} \\ \frac{3}{20} \end{array} \Big\rangle \frac{68}{20}$$

In

In which question because all the Denominators be like, you shall worke only with the Numerators.

Scholar. Then shall I multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will be 45 1/3: but how shall I make a fraction of that to stand with the other?

Master. Haue you so soone forgotten what was taught you so lately? this is his forme.

Scholar. I remember it now, & then it signifieth 45 twenty parts, and the third deale of one twenty part. 45 1/3 20

Master. So is it that maketh in shillings 45 s. 4 d. whereby you may note one great errour in the Statute Bookes, which haue constantly 48 s in that Assise. And by this Rule if you examine the Statute, you shall find many summes false. Wherefore for the true vnderstanding of that Statute, and such like as I haue made mention of it, & somewhat recognised it, so doe I wish that all Gentlemen & other Students of the Lawes would not neglect this Art of Arithmetick, as vnneedfull to their studies. Wherefore to encourage them thereto, & to gratifie both them & all other in generall, I will exhibite a Table of that part of the Statute in two columes, & in a third colume I will adde the correction of those errors which haue crept into it. Here followeth the Table.

The

The price of a quarter of Wheate.		The waight of a farthing white loafe by the ſtatute bookes.			The correctiõ by inſt Aſſiſe.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 1/5	2	14	4 4/5
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/5
5	0	1	8	2 1/5	1	7	1 2/5
5	6	1	4	8 1/4	1	4	3 6/11
6	0	1	2	6	1	2	8
6	5	0	19	11	1	0	11 1/11
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/5	0	18	1 2/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	3/4	0	14	3 15/19
10	0	0	13	7 1/2	0	13	7/8
10	6	0	12	11 1/4	0	12	11 3/7
11	0	0	12	4 1/4	0	12	4 4/11
11	6	0	11	10	0	11	9 21/23
12	0	0	11	1	0	11	4

In the common bookes there is no further rate of Assise made, then into 12 s. the quarter of wheat, but in an ancient copy of 200 yeeres old (which I haue) there is added the rate of Assise vnto 20 s. the quarter, but yet was that Assise also either wrong cast at the first penning, or else corrupt since that time, for lacke of iust knowledge in the Rule of Proportion, which I wil adde here also to gratifie such as be desirous to vnderstand truth exactly.

The price of a quarter of wheat.		The waight of a farthing white loafe by the Statute bookes.			The correction by iust Assise.		
s.	d.	l.	s.	d.	l.	s.	d.
12	6	0	11		0	10	$10\frac{4}{25}$
13	0	0	11	$0\frac{1}{2}$	0	10	$5\frac{7}{25}$
13	6	0	10	$11\frac{1}{2}$	0	10	$\frac{8}{25}$
14	0	0	9	7	0	9	$8\frac{4}{7}$
14	6	0	9	$2\frac{1}{2}$	0	9	$4\frac{16}{29}$
15	0	0	9	$1\frac{1}{2}$	0	9	$0\frac{4}{7}$
15	6	0	9	$1\frac{1}{4}$	0	8	$9\frac{9}{31}$
16	0	0	9	0	0	8	6
16	5	0	8	6	0	8	$2\frac{10}{16}$
17	0	0	8	3	0	8	0
17	5	0	7	10	0	7	$10\frac{9}{37}$
18	0	0	7	6	0	7	$7\frac{2}{7}$
18	6	0	7	3	0	7	$4\frac{8}{37}$
19	0	0	7	2	0	7	$1\frac{17}{38}$
20	6	0	5	10	0	6	$11\frac{9}{33}$
20	0	0	6	6	0	6	$9\frac{1}{2}$

These two Tables I haue set seuerall, because no man should thinke that I would either adde or take away from any Law those parts which might of right seeme either superfluous, either diminute: but yet I may not bee so curious as to neglect manifest errours, which is not only my part, but euery good subiects duety with sobriety to correct. And for auoiding of offence, I haue rather done it in this priuate Booke, then in any Booke of the Statutes it selfe, trusting that all Men will take it in good part.

Scholar. I would wish so, but I dare not hope so, sith neuer good man that would reforme errour, could escape the venimous tongues of enuious detracters, which because they either cannot, or list not to doe any good themselues, doe delight to barke at the doings of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of iust answers, but I will not seeme so hasty to make the answers before I hear their obiections, but as I trust that men are of a better nature, & more gratefull now then some haue beene in times past, as I haue done in the Statute of Assise for bread in rates of s, so will I set forth the like Table in pounds and ounces and the parts thereof, that it may be easily applied to all times: but I meane not

by

by this to alter any word of the statute (being so good an ordinance, and of so greate continuance) but only to make it as a kinde of exposition and declaration of the said Statute, trusting that thereby the Statute may be better vnderstood, and consequently better put in execution. And here you shall note that I haue accounted the shillings after the rate of 60s, to the pound weight, because I esteeme it the most apt for our time. Wherefore in the first columme you find the price of Wheat directly against it, in the second columme, you may find the weight of a farthing white Loafe, in this our time: and if you double the number (as I haue done in the third columme) then haue you the weight of the halfe penny white Loafe, and so in the fourth columme is set the weight of a penny white Loafe, It needeth not to tell that, that the sight doth testifie, how that euery columme is parted into three smaller pillars, whereof the first columne hath these three titles, pounds, ounces and penny weights, And as in the first columme 12 pence make a shilling, and 20 shillings make a pound, so in the other three colums 20 pence weight maketh an ounce, and 12 ounces doe make a pound.

A pound weight.

Scholar.

GENTLE READER, touching the vnderſtanding of this Table, wherein accoꝛding to our time, Maiſter Record alloweth 60. pence to the ounce, and 3. poun
oꝛ 60 ſhillings to the pound, ⁊ thereupon after the rate of 6c. ſhillings to the pound Troy, doth he frame oꝛ pꝛoduce this his Table, beginning at 3. ſhillings the qua
ter, till he come to 40. ſhillings, 6. pence the Quarter, And this his pꝛopoꝛtion (foꝛ that he hath not ſet downe any one Example to continue the woꝛke) hath bee
hard foꝛ many to conceiue oꝛ compꝛehend, and therefoꝛe the onely chiefe cauſe why I haue written this digreſſion foꝛ the better vnderſtanding of him therein. T
firſt thing therfoꝛe that is ſought foꝛ in this Table, as in the other afoꝛeſaid, is a Maxime, grounded vpon the Statute, which is this. When the Quarter of wheate
ſolde foꝛ two ſhillings, then the Farthing-white-loafe ſhall weigh 68. ſhillings, whereby a ſhilling is meant $\frac{1}{32}$ of a pound, ⁊ by a penny $\frac{1}{32}$ of an ounce. Now ther
foꝛe, foꝛ a generall rule, to finde what weight the Farthing-white-loafe ſhall weigh at 3. ſhillings the Quarter, till you come to 40 ſhillings, 6. pence the quarter,
thus to be wꝛought. Comming to the firſt ground, and woꝛking by the Backer Rule, ſay, if 2. ſhilings the quarter, giue oꝛ allow the Farthing-white-loafe to wei
68. ſhillings, what weight ought the Farthing-white loafe to weigh at 3. ſhillings the quarter? Woꝛke and you ſhall finde 45. ſhillings, 4. pence, as befoꝛe in the coꝛrection of the fi
Table is noted. Then foꝛ the ſecond woꝛke, ſay by the Rule of 3. direct, if 20. pence giue one ounce, what gineth 45. ſhillings, 4. pence? Multiplie and diuide, you ſhall find 544. ounce
which 544. ounces being multiplyed by 3. foꝛ 3. pounds, „ oꝛ 60. ſhillings, yeldeth 1632. ounces, which diuided by 20. pꝛoduceth 81. ounces, ⁊ $\frac{12}{20}$. oꝛ rather $\frac{3}{5}$. of an ounce, equall vn
12. peny weight, which is halfe an ounce, and 2. peny weight, ⁊ ſo maketh in all, 6. pounds, 9½ ounces, and 2. peny weight. Now the nert way to continue this Table, to know the weig
of the Halfe-peny white loafe is thus. Multiply 1632 ounces by 2, and it bringeth foꝛth 3264 ounces, and diuided by 20. it yeldeth 163. ounces, and $\frac{4}{5}$, which is equall to 13 pounds, 7 ou
ces, and 4 peny weight, as M. Record his Table noteth. Thirdly, foꝛ the weight of the Peny white-loafe, multiply 1632 ounces by 4 and diuide by 20. and after by 12. as afoꝛe, and yo
ſhall finde 27. pounds, ½ ounce, and 3. peny weight, &c. This Method, oꝛ elſe by doubling the Farthing-white loafe, foꝛ the weight of the Halfe-peny white loafe, and ſo doubling th
Halfe-peny white-Loafe, foꝛ the weight of the Peny-white loafe is the eaſier to continue the Table to the ende therof.

Thus much foꝛ the vnderſtanding of this Table, here placed by M. Record, a man indued with rare knowledge in Arithmeticall and Geometricall pꝛopoꝛtions, touching the Statut
of Coynage, and the Standard thereof, as appeareth in his Epiſtle of this Booke, dedicated to King Edward the ſixt: Inſinuating vnto his Highnes that the Standard of Coyne is muc
altered from the 14. yeare of King Edward the third, (when this Statute and Aſsiſe was confirmed) to the Standard of this our time. Foꝛ it appeareth that in King Edward the third
time, when this Aſsiſe of Bread and Drinke was eſtabliſh, that a Sterling peny, round without clipping, did then weigh 32 cornes of wheate dr e, ⁊ taken out of the middle of the eare
and 20. of theſe pence made an ounce, ⁊ 12. ounces made a pound Troy. And ſo frō the weight of a peny, to 20. ſhillings ſterling, which then weighed 12. ounces, toke Bread his weigh
and pꝛopoꝛtion: And now finding 60. pence is an ounce. That onely cauſe (I perceiue, foꝛ the zeale of a Common-wealth) moued him to ſet downe this Table in this priuate Boo
meaning not therby to alter any woꝛd of the Statute, being ſo good an oꝛdinance and of ſo long continuance, but as a kinde of expoſition by the way, that thereby the Statute may be bet
ter vnderſtood, and ſo conſequently better put in execution. Which Aſsie of his, is three times greater then the Statute now alloweth. Therefoꝛe alſo, to gratifie ſuch as are deſirou
of knowledge, (accoꝛding to theſe prices of a Quarter of wheate) I haue added to this Author, three other new Tables, and reduced their prices into their iuſt pꝛopoꝛtion of Sterling mo
ney, and alſo reduced the Money into knowne weight Troy, accoꝛding to the Statute. And thereafter accoꝛding to pꝛopoꝛtion in my other three Tables, haue I noted the iuſt weight
that a Farthing, Halfe-peny, and Peny-white-Loafe ought to weigh by the Statute.

Place this Table betweene 340. and 341.

The price of a Quarter of Wheate.

£	s	d
1	0	0
1	1	0
1	2	0
1	3	0
1	4	0
1	5	0
1	6	0
1	7	0
1	8	0
1	9	0
1	10	0
1	11	0
1	12	0
1	13	0
1	14	0
1	15	0
1	16	0
1	17	0
1	18	0
1	19	0
2	0	0
2	1	0
2	2	0
2	3	0
2	4	0
2	5	0
2	6	0

The weight of a Farthing white-Loafe.

Pounds	Ounces	Peny weight.
0	9	12
0	10	8
0	11	4
0	12	0
0	12	16
0	13	8
0	14	4
0	15	0
0	15	16
1	0	8
1	1	4
1	2	0
1	2	16
1	3	8
1	4	4
1	5	0
1	5	16
1	6	8
1	7	4
1	8	0
1	8	16
1	9	8
1	10	4
1	11	0
1	11	16
1	12	8
1	13	4
1	14	0
1	14	16
1	15	8
1	16	4
1	17	0
1	17	16
1	18	8
1	19	4
2	0	0

The weight of a Halfe-peny white Loafe.

Pounds	Ounces	Peny weight.
0	13	4
0	14	8
0	15	12
0	16	16
1	0	0
1	1	4
1	2	8
1	3	12
1	4	16
1	6	0
1	7	4
1	8	8
1	9	12
1	10	16
1	12	0
1	13	4
1	14	8
1	15	12
1	16	16
1	18	0
1	19	4
2	0	8
2	1	12
2	2	16
2	4	0
2	5	4
2	6	8
2	7	12
2	8	16
2	10	0
2	11	4
2	12	8
2	13	12
2	14	16
2	16	0
2	17	4
2	18	8
2	19	12
3	0	16
3	2	0
3	3	4
3	4	8
3	5	12
3	6	16
3	8	0
3	9	4
3	10	8
3	11	12
3	12	16
3	14	0
3	15	4
3	16	8
3	17	12
3	18	16
4	0	0

The weight of a Peny-white Loafe.

Pounds	Ounces	Peny weight.
0	2	8
0	4	0
0	5	8
0	7	0
0	8	8
0	10	0
0	11	8
0	13	0
0	14	8
0	16	0
0	17	8
0	19	0
1	0	8
1	2	0
1	3	8
1	5	0
1	6	8
1	8	0
1	9	8
1	11	0
1	12	8
1	14	0
1	15	8
1	17	0
1	18	8
2	0	0
2	1	8
2	3	0
2	4	8
2	6	0
2	7	8
2	9	0
2	10	8
2	12	0
2	13	8
2	15	0
2	16	8
2	18	0
2	19	8
3	1	0
3	2	8
3	4	0
3	5	8
3	7	0
3	8	8
3	10	0
3	11	8
3	13	0
3	14	8
3	16	0
3	17	8
3	19	0
4	0	8

The price of a Quarter of Wheate.

£	s	d
1	0	0
1	1	0
1	2	0
1	3	0
1	4	0
1	5	0
1	6	0
1	7	0
1	8	0
1	9	0
1	10	0
1	11	0
1	12	0
1	13	0
1	14	0
1	15	0
1	16	0
1	17	0
1	18	0
1	19	0
2	0	0

The weight of a Farthing white-loafe in Sterling money by Assise.

£	s	d
0	0	4
0	0	8
0	0	12
0	0	16
0	1	0
0	1	4
0	1	8
0	1	12
0	1	16
0	2	0
0	2	4
0	2	8
0	2	12
0	2	16
0	3	0
0	3	4
0	3	8
0	3	12
0	3	16
0	4	0
0	4	4
0	4	8
0	4	12
0	4	16
0	5	0
0	5	4
0	5	8
0	5	12
0	5	16
0	6	0
0	6	4
0	6	8
0	6	12
0	6	16
0	7	0
0	7	4
0	7	8
0	7	12
0	7	16
0	8	0
0	8	4
0	8	8
0	8	12
0	8	16
0	9	0
0	9	4
0	9	8
0	9	12
0	9	16
0	10	0
0	10	4
0	10	8
0	10	12
0	10	16
0	11	0
0	11	4
0	11	8
0	11	12
0	11	16
0	12	0

The weight of a Farthing white Loafe in Troy-weight by Assise.

Pounds	Ounces	Peny weight.
0	1	2
0	2	4
0	3	6
0	4	8
0	5	10
0	6	12
0	7	14
0	8	16
0	10	0
0	11	2
0	12	4
0	13	6
0	14	8
0	15	10
0	16	12
0	17	14
0	18	16
0	20	0
0	21	2
0	22	4
0	23	6
1	0	8
1	1	10
1	2	12
1	3	14
1	4	16
1	6	0
1	7	2
1	8	4
1	9	6
1	10	8
1	11	10
1	12	12
1	13	14
1	14	16
1	16	0
1	17	2
1	18	4
1	19	6
2	0	8

The weight of the Halfe-peny white Loafe in Troy-weight by Assise.

Pounds	Ounces	Peny weight.
0	2	8
0	4	0
0	5	8
0	7	0
0	8	8
0	10	0
0	11	8
0	13	0
0	14	8
0	16	0
0	17	8
0	19	0
1	0	8
1	2	0
1	3	8
1	5	0
1	6	8
1	8	0
1	9	8
1	11	0
1	12	8
1	14	0
1	15	8
1	17	0
1	18	8
2	0	0
2	1	8
2	3	0
2	4	8
2	6	0
2	7	8
2	9	0
2	10	8
2	12	0
2	13	8
2	15	0
2	16	8
2	18	0
2	19	8
3	1	0
3	2	8
3	4	0
3	5	8
3	7	0
3	8	8
3	10	0
3	11	8
3	13	0
3	14	8
3	16	0
3	17	8
3	19	0
4	0	8

The weight of a Peny-white Loafe in Troy weight by Assise.

Pound	Ounces	Peny weight
0	1	0
0	2	0
0	3	0
0	4	0
0	5	0
0	6	0
0	7	0
0	8	0
0	9	0
0	10	0
0	11	0
0	12	0
0	13	0
0	14	0
0	15	0
0	16	0
0	17	0
0	18	0
0	19	0
0	20	0
0	21	0
0	22	0
0	23	0
1	0	0
1	1	0
1	2	0
1	3	0
1	4	0
1	5	0
1	6	0
1	7	0
1	8	0
1	9	0
1	10	0
1	11	0
1	12	0
1	13	0
1	14	0
1	15	0
1	16	0
1	17	0
1	18	0
1	19	0
2	0	0

Scholar, Sir, I doe thanke you most hartily for this, not onely in mine owne name, and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in bread, which long time hath beene abused. And if you know any like things more, wherein you would vouchsafe to declare the errors, and set forth the truth, you cannot but obtaine great thanks of all good harted men that loue the Common wealth.

Master. I haue sundry things to declare, but I haue reserued them for a priuate booke by it selfe, yet notwithstanding because the statute of the rate of measuring of ground is so common that it toucheth all men, and yet no more common then needfull, but so much corrupt, that is so farre out of all good rate, not only in the English Bookes of statutes, commonly Printed, but also in the Latine, Bookes, and in the French also (for I haue read of each sort, and conferred them diligently) I will giue you a Table for the restitution of those errors, as may suffice for this present time. And first I will propose one question to you touching the vse of that statute, whereby you may perceiue the order how to examine the whole statute, and curry parcell thereof, and the question is this.

When the Acre of ground doth containe

foure

A queſtion of meaſuring of ground.

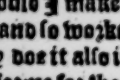
foure Perches in breadth, then muſt it contain 40 Perches in length: Then doe I demand of you, how much ſhall the length of an Acre be, when there is in the breadth of it 13 Perches, but before you ſhall anſwere to this queſtion, I will declare vnto you another Statute, which is the ground of the former Statute. And that Statute is this.

Statute meaſure.

It is ordained that three Barly-cornes dry, and round, ſhall make vp the meaſure of an inch: 12 inches ſhal make a foote, & 3 foote ſhall make a yard (the common Engliſh bookes haue an Elne) fiue yards and a halfe ſhall make a Perch, & 40 Perches in length, & 4 in breadth, ſhal make an Acre. This is that Statute, wherby you may perceiue, that the intent of the Statute is, that one Acre ſhould containe 160 ſquare Perches. Now let me heare you anſwere to the queſtion.

An Acre.

Scholar. As I perceiue by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, ſo could I make thoſe numbers all in fractions, and ſo worke the queſtion, but ſeing I may doe it alſo in whole numbers, I take that forme for the moſt eaſie, therefore thus I ſet the queſtion in forme. Then doe I multiply 40 by 4, and it maketh 160, which I diuide by 13 and the quotient is $12\frac{4}{13}$.



Maſter. Now turne that $\frac{4}{13}$ into the com-

mon

mon parts of a Perch, as they be named in the former Statute: howbeit it shal be best to take one of the least parts in Denomination for avoiding of much labour, as Feete, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. When to returne $\frac{1}{11}$ into Feet, I multiply $16\frac{1}{2}$ by 4, and it maketh 66, which I must diuide by 12, & the quotient is $5\frac{1}{11}$.

Master. So I find that if the Acre hold in breadth 13 Perches, it shall containe in length 12 Perches, 5 foote, and $\frac{1}{11}$ of a Foot, which is not fully an inch, for the inch is $\frac{1}{12}$ of a Foot. But here all the Statute Bookes in Latine and English (that I haue seene) do note it to be 13 Perches, 5 Foot, and one inch, which maketh aboue 12 Perches, too many in the Acre: so that I would haue thought the errour to haue crept into the Printed Books by the great negligence that Printers in our time doe vse, saue that in written copies of great antiquity, I do find the same. Yet haue I one French copy which hath 12 Perches $\frac{1}{2}$ and one foote, and that misseth very little of the truth. Note this errour.

Scholar. Then I see it is true that I haue often heard say, that the truest Copies of the Statutes be the French Copies.

Mast. That is often true but not generally, as I haue by conference tried diuerslye: but in this Statute the French Book is most corrupt: in all other places lightly.

But now to performe my promise, I will set

let forth the Table for measuring of an Acre of ground only by such parts as the Statute doth mention, because at this time I doe of purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table following, I haue not done as in the other Statute before compared by restitution with the Faults crept into the Statute, but onely haue written that true measure, which the equity of the statute doth pretend. For it were vile to indge of so noble Princes and worthy Counsellors, as haue authorised and set forth this Statute, that they would make one Acre in any forme greater then another, but euery one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needfull Arithmeticke is to the Students of Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it none obscurity, that I should need to declare.

The

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of inch
10	16	0	0	0
11	14	9	6	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{2}$
26	6	2	7	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an inch.
28	5	11	9	$\frac{3}{7}$
29	5	8	6	$\frac{12}{29}$
30	5	5	6	0
31	5	2	7	$\frac{29}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$\frac{15}{17}$
35	4	9	5	$\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$\frac{8}{37}$
38	4	3	5	$\frac{13}{19}$
39	4	1	8	$\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$\frac{28}{41}$
42	3	13	4	$\frac{2}{7}$
43	3	11	10	$\frac{32}{43}$
44	3	10	6	0
45	3	9	2	0

Scholar.

Scholar. Indeed Sir. I understand the Table (as I think) by those other which you set forth before. For in the first columne is set the Perches of the breadth of any Acre, and then in the two columnes following appeareth how many Perches, and how many foote that same Acre must haue for his length.

Master. You take it wel: howbeit to speak exactly of breadth and length, and the first colum doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limite his length, and the shorter side doth betoken the breadth; yet it is no great abuse in such Tables, where a man cannot well change the title, to let the name remaine, although the proportions of the numbers do change: for still by the first columne is expressed the measure of the one side, & by the two other Pillars in one columne, is set forth the measure of the other side. And this shall be sufficient now for the vse of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certain other rules, which for their seueral names may seem diuers rules, and distinct from this, but indeed they are but

The Rule of Fellowship.

branches of it: yet because they haue seuerall workings in appearance, but also pleasant in vse, I will giue you a taste of each of them. As for the Rule of Fellowship, both single and double, with time and without time, I shall need to say little more then I haue already said in teaching the workes of the whole numbers, yet an example or two will we haue to refresh the remembrance of the same, and to declare certaine proper vses and applications of it, as this for one.

A question of inequall society.

Foure men get a booty or prise in time of warre, the prize is in value of mony 8190 pound, and because the men bee not of like degree, therefore their shares may not be equall, but the chiefest person will haue of the booty the third part, and the tenth part ouer: the second wil haue a quarter, and the tenth part ouer, the third will haue the sixt part: and so there is left for the fourth man a very smal portion, but such is his lot (whether he be pleased or wroth) he must be content with one 20 part of the pray: Now I demand of you, what shall euery man haue to his share?

Scholar. **You must be faine to answere to your owne question, else it is not like to be answered at this time.**

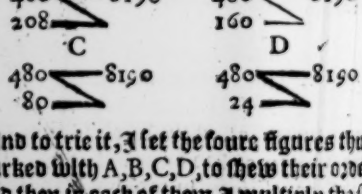
Master. **The forme to vnderstand the solution of this question, and all such like is this: Reduce all the Denominators into one number by Multiplication, except that**

any of them be parts of some other of them for all such parts you may overpasse, and take for them all those nũbers, whose parts they be. As in this example the shares bee these $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{10}$, $\frac{1}{6}$ $\frac{1}{10}$, if I multiply all the denominators together, beginning with 3, & so goe on vnto 20, it will make 144000: but considering that 3 is a part of 6, I shal omit that 3, & likewise 10, which is a part of 20, I may overpasse also, & then is there but 3 denominators to multiply, that is, 4 6, & 20 which make 480: which summe I take for my worke, because all the denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ & $\frac{1}{10}$, for $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one sum for the first mans share, & it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, & $\frac{1}{10}$, which is 48, and that maketh in the whole 168. Now for the third man which must haue $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole sum: so that if the whole pray had borne but 480 £, then were the question answered: but because the sum was of greater value, by this meanes now that I know the partition of it, I must set my numbers by the order of the Golden Rule, putting in the first place the number that I found by multiplying the Denominators, and in the second place the sum of

the booty, And loke what proportion is betwéne the first number and the second, the same proportion shall be betwéen the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third place, one of the parts or shares, and then worke by the former Rule of proportion or Golden Rule. And because I haue foure seuerall parts of the first number, by which I would find out foure like parts of the second number, therfore must I make foure seuerall figures.

The reason of this rule

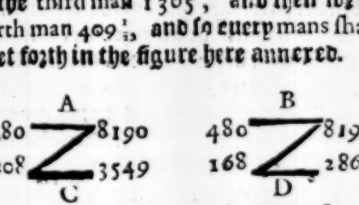
Scholar. Now I trust I can answere to your question, as by your fauour I will proue.



And to trie it, I set the foure figures thus, marked with A, B, C, D, to shew their order. And then in each of them I multiply the second number by the third, and diuide their totall by the first, & so amounteth the fourth summe which I seeke for: For if I doe multiply 8190 by 208, it maketh 170520, which being diuided by 480, maketh in the quotient 3549 for the first mans portion.

And

And so working with the other three figures, I find for the second man 2866 1/3, & for the third man 1365, and then for the fourth man 409 1/3, and so every mans share is set forth in the figure here annexed.



And thus I thinke I haue done well.

The proofe by Addition.

Master. If you misdoubt your working, and list to proue it, adde all the shares together; and if they make the totall, then seemeth it well done.

Scholar. I may set them thus: and then by Addition the iust summe doth amount that is, 8190, and therefore (as you say) it seemeth to be well wrought.

3549	
2866 1/3	
1365	
409 1/3	
8190	

But I beseech you, is there any doubt in this triall, that you vse that word, Seemeth?

Master. You may easily coniecture, that if you did assigne the first mans share to the last, & so change all the rest, and one had anothers share, yet would the Addition appeare

all one, and therefore is not the proofe exact.

But if you will make a iust proofe for the first mans part, take $\frac{1}{3}$ & $\frac{1}{10}$ of the whole sum, & if it agree with the number in the figure, then it is well done. And so doe for the second, third, and fourth summes, and this proofe faileth not. Now will I propound certaine other questions, which haue beene set forth by ceartine learned men, albeit not without some ouersight, which questions I protest heartily, I do not repeat to depraue those good men, whose labours and studies I much praise, and greatly delight in. But only according to my profession, to seeke out truth in all things, and to remoue all occasions of errours as much as in me lieth: and for that cause I will onely name the questions without barting the Authours name. The first question is this.

The iust proofe.

A question of building.

Foure men did build an house, which cost them 3000 *crownes, their shares were such that one man should pay* $\frac{1}{3}$ *of the summe, and sixe crownes ouer: the second should pay* $\frac{1}{4}$ *and* 12 *crownes ouer: the third man must lay out* $\frac{2}{5}$ *abating* 8 *crownes, and the fourth man should pay* $\frac{1}{4}$ *and* 20 *crownes more. Can you answere to this question?*

Scholar. No in good sooth sir, and that you know best of any man, for I know no more then you haue taught me.

Master. Then I dare say you cannot do it,

it, neither yet the best learned man that euer did propose it, for the question is impossible. For declaration whereof, I will bee bold to vse first the representation of the numbers in their aptest forme, (although I haue not yet taught that manner of worke) because it may appeare plainely that the question is not possible. For here I haue set the parts, and added them, & they make the whole sum, and $\frac{1}{4}$ & 30 more. Now, how is it possible to diuide truly either gains, either charges, so that the particulars $1\frac{3}{4}$ shall bee more then the totall?

An impossible question.

$$\left\{\begin{array}{l} \frac{1}{2} - + \ 6 \\ \frac{1}{3} - + \ 12 \\ \frac{2}{3} - - \ 8 \\ \frac{1}{4} - + \ 20 \end{array}\right\}$$

$$1\frac{3}{4} \ + - 30$$

Scholar. It is against the forme of proofe by addition of parts.

Master. You say truth. And (because you shall perceiue it the better) I will trie it after the vulgar forme, as in this figure you see, where the $\frac{1}{2}$ with 6 ouer, is 1506; for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe, do make 5280, where the total summe should be but 3000, which summe of 3000 if you diuide by $\frac{4}{3}$, so shall

1506
1012
1992
770
5280

you

you haue ⅔ of it, that is 2250, & thereto adde 30 more, then will those 3 sums make 5280: whereby you may see how this forme (as wel as the other) doth declare that the particulars in that question would make more then the whole sum by ⅔, and 30 more and therefore can that question not be accepted as a possible thing: but yet doe certaine Learned men propound such questions, and answere to them. Therfore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anone declare what may be said for their defence: but in the meane season, I will propound the question as it may be wrought by good possibilitie. *As if foer men build a house together, and it cost them 3000 crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 crownes, so often the second man shall pay 4, the third man 8 and the fourth man 2. Or else, thus: that the first man shall pay double so much as the fourth, and the second man shall pay ⅓ of the first mans charge: the third man shall pay double so much as the second: (and these two waies are to one end:) but further for their agreement it is appointed also, that the first shall giue 6 crownes ouerplus, and the second 12, and the fourth shall giue 20: but the third man shall giue no ouerplus, but*

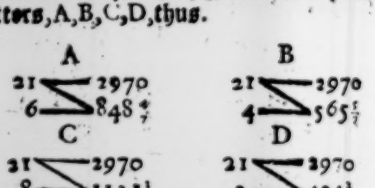
2000
2250
30
5280

but ſhall haue 8 crownes abated of his charge.

Now is the queſtion poſſible to be reſolued, and this is the way to doe it. Marke the proportion of the ſeuerall charges, & ſet out ſmall numbers in that rate, by which you may reduce the worke to the Golden Rule, as here in the firſt forme, the numbers are already named, 6, 4, 8, 3: and in the ſecond forme, (although they be but plainely named) yet they may be the ſame numbers: for 6 is double to 3, and 4 is ⅔ of 6: and againe 8 is double to 4. Now adde theſe together, & they make 21, which 21 muſt be ſet in the number in the Golden Rule: for it, with the ouerplus of each mans charge would make the totall ſumme of the charges: then were thoſe ſeuerall ſums the charges of each man, beſides his ouerplus: but now it is not ſo.

But yet this is true, (ſo excellent are conclusions Arithmetical,) that take what proportion each of their ſeuerall ſummes doth beare to 21, the ſame proportion doth the iuſt charges of euery man (beſides his ouerplus) beare to the totall of the charges, the ouerplus being deducted: wherefore this may you note, that before you doe apply the totall of his charges to the Golden rule, you muſt deduct the ouerplus which is 6, 12, & 20, that is in the whole 38: but then 8 muſt be reſtored for the abatement of the third man, The rule.

man, and then remaineth to be deducted 30. Take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule for the second sum: & for the third sum I must put each of the small numbers before mentioned, which although they be not seuerall charges, yet they represent them in proportion. And so making for euery mans charge a seuerall question, the figures will be 4, which I marke with foure letters, A, B, C, D, thus.



Where I haue set for briefenesse the sum of euery mans charge in the fourth place, presupposing that you can tell how to trie out that fourth summe by so many examples as ye haue had.

Scholar. As I trust that I vnderstand this forme, so I desire much to know what may bee said for them that misstake this question.

Master. You seeme so desirous to know this error, that you haue forgotten to examine, whether this worke be without fault.

Scholar. It seemeth this worke to bee well

wel done, becauſe the Addition of the foure ſeuerall numbers doth make the totall ſum of 2970, which was to be diuided into ſuch foure parts.

Maſter. But then haue you forgotten that the firſt man muſt pay fiue Crownes more beſides his ſhare, and the ſecond man 12 crownes more, the third man 8 crownes leſſe, and the fourth man 10 crownes more, for without theſe your firſt totall of 3000 crownes willnot be made.

Scholar. Then muſt I adde to the firſt mans ſum 6 more, and it will be $854\frac{4}{7}$: and to the ſeconds ſum I muſt adde 12, and it will be $577\frac{5}{7}$: from the thirds ſum I muſt abate 8, and then will the ſumme be $1123\frac{3}{7}$: then adding vnto the fourths ſum 20, it will be $444\frac{7}{7}$, and theſe foure ſummes will make 3000, which is the whole charge, as in this example it may appeare, where firſt I gather the $\frac{4}{7}$, that maketh 2, and ſo procéede I in the Addition to the end.

	$854\frac{4}{7}$
	$577\frac{5}{7}$
	$1123\frac{3}{7}$
	$444\frac{7}{7}$
	3000

Maſter. Now haue you wel done, and this worke in the ſame ſummes is brought of other learned men for the true ſolution of the queſtion as it was firſt proponed, which (as I ſaid) was impoſſible: and now examin it by theſe ſeuerall ſummes, and ſée whether it doth agrée with the ſums in the queſtion proponed.

The

The first man must pay ½ and 6 ouer of the totall summe: how thinke you, is 854 ½ the halfe and 6 more of 3000?

Scholar. No that it is not, for it should be 1506 and for the second man 1012: and for the third man 1992, and for the fourth man 770, whereof not one summe agreeth to this worke. But I marueil, that so wise men could be so much ouer séene.

Master. It is commonly séene that when men will receiue things from elder writers & will not examine the thing, they séeme rather willing to erre with their Ancients for company, then to be bold to examine their workes or writings. Which scrupulositie hath ingendred in it the errors in all kinds of knowledg, & in all ciuill administration, & in euery kind of Art. But these Learned men did not meane any other thing by this question then to find such numbers as should beare the same proportion together, as those numbers in the question proponed did beare one to another: which thing you shall perceiue more plainely by another question of theirs, that is this.

A question of a Testament.

A man lying vpon his death bed bequeathed his goods, (which were worth 3600 crownes) this sort. Because his wife was greate with childe, & he yet vncertaine whether the childe were male or female, he made his request conditionally: that if the wife bare a Daughter, then

then ſhould the wife haue halfe his goods, and the Daughter $\frac{1}{3}$, *but if ſhe were deliuered of a Sonne, then that Sonne ſhould haue* $\frac{1}{2}$ *of the goods, and his wife but* $\frac{1}{4}$. *Now it chanced her to bring forth both a Sonne and a Daughter, the queſtion is: How ſhall they part the goods agreeable to the Teſtator his will?*

Scholar, If ſome cunning Lawiers had this matter in ſcanning, they would determine this Teſtament to be quite voide, and ſo the man to die inteſtate, becauſe the Teſtament was made inſufficient, ſith this condition was not expreſſed in it: and alſo, it might haue chanced, that he ſhould haue brought forth neither Sonne nor Daughter, as often hath beene ſeene: ſo is the Will inſufficient in that point alſo.

Maſter. Such Scanners ſhould ſeeme too cunning and yet not ſo cunning as cruel: for the mind of the Teſtator is to be taken fauorably, for the aide of the Lygatories, when there ariſeth ſuch doubt. But let vs trie this worke not by force of Law, but by proportion Geometricall, ſeeing the Teſtator did mind to prouide for each ſort of them.

Scholar. If the ſonne ſhall haue $\frac{1}{2}$ by force of the Teſtament, ſo muſt the mother haue $\frac{1}{4}$ Againe, becauſe ſhe hath a Daughter alſo. therefore ought ſhe to haue $\frac{1}{3}$, & the Daughter $\frac{1}{3}$, that is both waies $\frac{1}{2}$ $\frac{1}{3}$, and $\frac{1}{4}$ $\frac{1}{3}$, which cometh to to the whole goods, and $\frac{1}{4}$ more.

Where-

Wherefore it seemeth also impossible.

Mast. In this matter, the mind of the Testator is so to be understood, that such proportion should be betweene the portion of the wife & the sonne as is betweene $\frac{1}{2}$ & $\frac{1}{3}$, that is, the sonne must haue $\frac{1}{2}$ for $\frac{1}{3}$ to his mother, so shall he haue 3 to 2 that is, as much as his mother, & halfe as much more, & the mother must haue the like rate in comparison to her daughter. Then must I find out 3 numbers in such proportion that the first may be as much as the second, and halfe as much more (that is) in proportion sesquialtera, and the second to the third in that same proportion: such numbers be 6, 9, 4.

Scholar. I pray you Sir, how shall I find out those numbers.

Master. That will I gladly tell you.

To finde three numbers in any proportion.

Whatsoeuer the proportion be of any three numbers, multiply the Termes of that proportion together, and the number that amounteth, shal be the middle number of the 3: then multiply that middle number by the lesser terme, and diuide that totall by the greater, and the least number of the 3 will amount. So if you multiply that middle number by the greater extreme, and diuide that totall by the lesser extreame, then will the greatest number of that progression amount.

To finde the proportion betweene two numbers.

Scholar. Then in this example, to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must diuide (as you taught me in diuision) $\frac{1}{2}$ by $\frac{1}{3}$, & the quotient will

will be $\frac{1}{2}$, that is $1\frac{1}{2}$, wherby I perceiue that that ỹ proportion in this question as 3. to 2. Therefore as you taught me euen now, I multiplie 3 by 2, and the summe is 6, which must be the middle number: then, I multiplie the middle number 6 by 2, which is the least terme, and the summe is 12: that I deuide by 3. being of the greater terme, and the quotient is 4: so is 4. the least number of the 3. Then I multiplie 6 by 3, whereof cõmeth 18, and that I diuide by 2. & so haue I nine, which is the greatest nũber of the 3.

Master. Another way yet may you finde the third number in any Progression, if you haue two of them: for if the middle number be one of them which you haue, then multiplie it by it selfe (as in this example, 6. by 6. maketh 36) and that totall diuide by the other number which you haue, and the third number will be the quotient.

Scholar. Then I diuide 36 (which cometh of 6. multiplyed by it selfe,) by 4. the quetient will be 9: and if I diuide 36 by 9, the quotient wilbe 4. But what if I know the first number, and the third, and would haue the middle number?

Maister. Multiply the two nũmbers together, & in their totall you must seeke the root of that nũber, & it shalbe the middle nũber: but, because as yet you haue not learnd how to extract rootes, therefore vse ỹ first forme Note.

wyth I haue taught you, till I teach you to extract rootes. And now goe forwards with the answere to the same question.

Scholar. I perceyue then, that the Sonne must not haue $\frac{1}{3}$ of ŷ gœds, neither the Mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{3}$, but yet must ŷ gœds be diuided into such proportion, ŷ the Sonne shall haue 9 crowns for 6 to his mother, and the mother shal haue 6 crowns for euery 4 to her daughter. Then I apply it to the Goldē rule, in 3 examples, thus: where the first number is the Addition of those three numbers, 9, 6, 4 e the third is one of them severally: the second is the totall of the Gœds in that testament: and then by the worke of the Goldē rule I finde out the fourth number in euery worke, that is: for the Sonne, 1705 $\frac{5}{19}$ for the Mother, 1136 $\frac{16}{19}$, and for the Daughter, 757 $\frac{17}{19}$: the which sūmes added together, do make the sum of the whole gœds, as may be seene by this example.

19	9	3600
19	6	3600
19	4	3600

1705.	$\frac{5}{19}$.
1136.	$\frac{16}{19}$.
757	$\frac{17}{19}$.
-3600.	

And this (me thinketh) I do perceiue, that because in this case there is a necessary remedy deuised against an vrgent inconuenience: therefore those Learned men thought they might vse the like liberty in that other question.

Schol.

Scholar. Your geſſe is good, but they had ſo good reaſon for them in the one, as they haue the other: as in another Example of theyrs it may better appeare, as in this.

Another queſtion of a Teſtament.

A man left vnto his three ſonnes 7851 crowns to bee parted in ſuch ſorte, that the firſt Sonne ſhould haue $\frac{1}{2}$, the ſecond ſonne $\frac{1}{3}$, and the third ſonne $\frac{1}{4}$, which is not poſſible: for $\frac{1}{2}$, and $\frac{1}{3}$ & $\frac{1}{4}$, doth make $\frac{26}{24}$, or $1\frac{2}{12}$, that is $1\frac{2}{12}$: ſo is it more then the whole: but reduce theſe fractions into one denomination, the leaſt that they will come to, and they will be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, & ſo may you part the goods into ſuch proportiõ as theſe 3. numerators beare together, that is, the firſt to haue 6 for euery 4 to the ſecõd: & the ſecõd to haue 4 as often as the third hath 3: & ſo their portions wil be for the firſt, 3623 $\frac{7}{13}$, for the ſecond 2415 $\frac{9}{13}$, and for the third 1811 $\frac{10}{13}$, and theſe three ſhares added together, wil make the totall ſumme of the whole goods, as you may eaſily ſee in this example.

	3623 $\frac{7}{13}$
	2415 $\frac{9}{13}$
	1811 $\frac{10}{13}$
	7851.

Another queſtion is there propoponed thus:

Another like queſtion.

There are 450. Crownes to be diuided betweene three men: ſo that the firſt man muſt haue $\frac{1}{2}$, and $\frac{1}{3}$. the ſecond man $\frac{1}{3}$ & $\frac{1}{4}$. the third man ſhall haue $\frac{1}{4}$ and $\frac{1}{5}$.

Scholar. I maruell that any man ſhould be ſo ouerſeene to propound that queſtion, as a thing poſſible, ſith $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{3}$ $\frac{1}{4}$, $\frac{1}{4}$ $\frac{1}{5}$, do make $1\frac{13}{15}$ that is almoſt double the whole ſumme.

But I perceyue it might be thus propo-
ned: that as oftē as the first man did receiue
50. crownes, so often the second man should
receiue 35, and the third man 27: for $\frac{1}{2}$ and $\frac{1}{3}$
is equall to $\frac{50}{60}$: and so is $\frac{1}{3}$ & $\frac{1}{4}$, equall to $\frac{35}{60}$
and $\frac{1}{4}$ and $\frac{1}{5}$ is $\frac{27}{60}$: and so working the que-
stion, the three figures will appeare in this
forme: whereby the first
mans portion is found to
bée $200\frac{50}{56}$: the second
mans part is $140\frac{35}{56}$: the
the third mans share 110
$\frac{27}{56}$: which in the whole
doth make 450 crownes
to be diuided betweene
them.

112	450
50	$200\frac{50}{56}$
112	450
35	$140\frac{35}{56}$
112	450
27	$108\frac{27}{56}$

Master. And thus you are (I thinke) sufficiētly instructed in the rule of Fellowship.

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligaton: which hath his name, for that by it there are diuers parcels of sundry prices, & sundry quantities alligate, bound, or mixed together: whereby also it might be well called the rule of Mixture, and it hath great vse in composition of Medicines, and also in mixtures of Metals,

The rule of Mixture.

tals, & some vse it hath in mixtures of Wines: but I wish it were lesse vsed therein than it is now adayes. The order of this rule is this: Whē any sums are proponed to be mixed, set them in order one ouer another, and the cōmon number (whervnto you will reduce them) set on the left hand: then marke what summes be lesser then that cōmon number, and which be greater, & with a draught of your Penne euermore linke two numbers together, so that one be lesser then the common number, & the other greater then hee: (for two greater or two smaller cannot wel be linked together:) and the reason is this, that one greater and one smaller, may be so mixed that they wil make the meane or cōmon number very well: but two lesse can neuer make so many as the common number, being taken orderly: no more can two summes greater then the meane, euer make the meane in due order, as it shall appeare better to you hereafter. And, as it is of necessity to link euery smaller (once at ý least) with one greater, & euery greater with one smaller; so it is at liberty to linke them oftner then once: & so may there be to one question many solutiōs. When ye haue so linked them, then marke how much each of the lesser nūbers is smaller then the mean or cō mon number, and that difference set against the greater nūbers, which be linked wᵗ those

The reason of this Rule.

smaller, each with his match still on the right hand, & likewise the excesse of the greater nũbers aboue the meane, you shal set before the lesser numbers, which be cõbined with them. Then shall you (by Addition) bring all these differences into one summe, which shall be the first number in the Golden rule: and the second number shall bee the whole masse that you will haue of all those particulars: the third summe shall be euch differẽce by it selfe, & then by them shall be found the fourth number, declaring the iust portion of euery particular in that mixture: As now by these examples I will make it plaine.

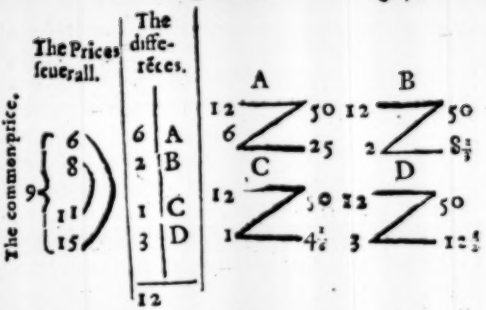
A question of mixing of wines.

There are foure sortes of Wine, of seuerall prices: one of 6 d. a gallon, another of 8 d. the third of 11 d. and the fourth of 15 d. the gallon. Of all these wines would I haue a mixture made, to the summe of fiftie gallons, and so the price of eache gallon may be 9 d. Now demaund I, how much must be taken of euery sort of Wine?

Scholar. If it shall please you to worke the first example, that I may marke the applying of it to the rule: then I trust I shall be able, not onely to doo the like, but also to see the reason in the order of the worke.

Master. Marke then this forme, and the placing euery kinde of number in it.

The



Here (you see) I haue set downe the seuerall prices which be 6, 8, 11, 15, and haue linked together 6 with 15, & 8 with 11. The common price, 9. I haue set on the left side, and the difference betweene it, & euery particular price, I haue set on the right hand, not against the sum, (whose difference it is) but against the sum that it is linked with: also the difference of 15 aboue 9, is 6, which I haue set, not against 15 but 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue set against 15. So likewise, the difference betweene 8 and 9 is but 1 that I haue set against 11, & the difference of 11 aboue 9 (which is 2) I haue set against 8. Then add I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of gallons that I should haue, and the third

ſumme is euery particular difference. Now if you worke by the Golden rule, you ſhall find the number of Gallons that ſhall be taken of eath ſort of wine: For the better diſtinction wherof, I haue ſet theſe letters, A, B, C, D, both againſt the nũbers for which the works do ſerue, and ouer the works alſo, which ſeuerally ſerue for each of them.

The proof of this Rule.

And now (if you liſt to examine the truth of theſe works,) adde theſe foure ſummes together, & they will make 50, that is this totall which I would haue, as by this example you may eaſily perceyue. And (for to prooue how the prices doe agree) doe this: multiplie the totall ſumme 50, by the common price 9, and it will make 450: then kéepe that ſumme by it ſelfe, and afterward multiply euery ſeuerall ſumme of gallons, by the price belonging to the ſame gallons; and if that ſumme do agree with this which you haue kept firſt, then is your worke well done. As here 25 is the number of gallons of 6 pence price, multiplie then 25 by 6, and it maketh 150, which you ſhall ſet downe, then multiplie $8\frac{1}{3}$ by 8, which is the price for the nũber of gallons, & it will make $66\frac{4}{6}$: for againe $4\frac{1}{6}$ multiplied by 11: doth make $45\frac{5}{6}$. And laſt of all $12\frac{1}{2}$ multiplyed by 15, maketh $187\frac{3}{6}$: and theſe

25	—
$8\frac{1}{3}$	
$4\frac{1}{6}$	
$12\frac{1}{2}$	
50	

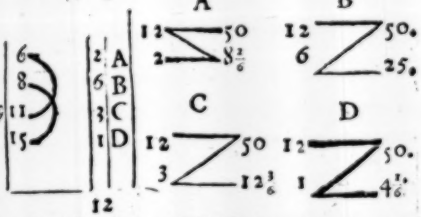
150
$66\frac{4}{6}$
$45\frac{5}{6}$
$187\frac{3}{6}$
450

these added together do make 450, as in the example annexed you may see. wherefore seeing it doth agree with the former summe of 50, multiplied by 9, I may iustly affirme this worke to be good, and well done.

And now to proove how you can doe the like, I propound the same question, onely willing you to vse some other forme of combining or linking the summes.

The variation of this question.

Scholar. That shall I proue with your fauour, and therefore I combine 8 with 15, and 6 with 11, and then the forme will be as followeth.



Where by amounteth the same summe in totall of the differences as did before: and yet now the differences bee altered as the combination is changed, whereof I vnderstand the reason by your former worke. And therefore here appeareth no strange thing, but that now I must haue 8 2/6. of 6.d, and 25 gallons of 8d. and 12 gallons 3/6 of 11 d, & so consequently 4 gallons and 1/6 of 15 d: so that

50	
200	
137 1/2.	
62 1/2.	
450.	

that multiplying 8 ⅛ by 6. it maketh 50. and then 25 multiplied by 8. maketh 200: likewise 12 ⅛ multiplied by 11. yeeld 137. ⅜. and 4 ⅛ multiplied by 15. maketh 62 ⅜. which 4 summes added into one, will yeeld in the totall 450, which agreeth with the multiplication of 50, (being the totall summe of gallons) by 9 the common or meane price.

Master. Seeing you conceiue this worke so well, I will propound another example vnto you of more varietie, in the Alligations or combinings, as thus:

A question of Spices.

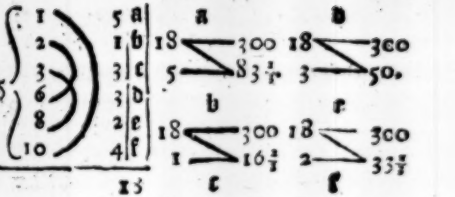
A Marchant beeing minded to make a bargaine for Spices, in a mixt masse (that is to say,) of Cloues, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloues beeing at 6 s. a pound, the Nutmegges, at 8 s. Saffron at 10. s. Pepper at 3 s. Ginger at 2 s. and Almonds at 1 s.

Now would he haue of each sort some, to the value of 300 pound in the whole, & each pound one with another, to beare in price 5 s: How much shall he haue of each sort?

Scholar. That will I trye thus.

First, I set downe those fiue seuerall prices, and at the left hand I set the common price, 5 s. Then, I linke them thus, 1 with 10. 2 with 6. and 3 with 8: as in the example following.

Ma-



Master. I had minded to haue combined them in more variety: but I am content to see your own worke first, and then more varieties in combination may follow anone.

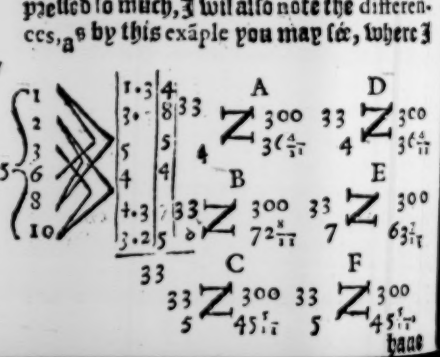
Scholar. Then to continue as I began, I seeke the difference betwéen 1 and 5. (which is 4) and that I set against 10; then against 1 I set 5. which is the excesse of 10 aboue 5. so I gather the difference betwéene 2 and 5, which is 3: and that I set against 6, because it is combined with 2: & likewise the difference of 6, aboue (which is 1) I set against 2. Then take I the difference of 3. from 5. which is 2, and that I set against 8: and before that 3, I set the difference of 8 aboue 5 which is 3. Then gather I all these differences by Addition, & they make 18, which I set for my first nũber in the Golden rule; and so appeareth by those workes, that of Almonds I must take 83 1/3 ℔, of Ginger 16 2/3 ℔, of Pepper 50 pounds, of Cloues, 50 pounds, of Nut-

Nutmegs $33\frac{1}{7}$ pounds, and of Saffron $66\frac{2}{7}$. pounds. $83\frac{1}{3}$ $33\frac{1}{7}$

Then for tryall heereof, I multiplie euery parcell by his seuerall price, as $83\frac{1}{3}$, which is the summe of Almonds, I multiply by one, which is their price. 150. 300 $266\frac{2}{7}$ $666\frac{2}{7}$ 1500.

Also $16\frac{2}{3}$. the summe of Ginger, I multiplie by 2, which is the price of it. And so each other in his kinde, as this Table annexed doth represent, and then adding them altogether, I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300, by the cõmon price 5: wherefore it appeareth well wrought.

Master. Now I will make the alligation, to proove your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I wil also note the differences, as by this exãple you may see, where I



bane

haue alligate 1 with 6 & 8, and therfore haue I set against 1 both their differences, that is 1 and 3: Likewise, because 2 is combined with 8 & 10, I set before him their differences, 3 & 5. Against 3 I haue set only 5, which is the differēce of 10, with whom 3 is combined only. Likewise, 6 is onely alligate to 1 and therfore is the difference of 1 from 5. which is 4 onely set against it: 8 is linked with 1 & 2, and therefore hath set 4 against him, both their differences, 4 and 3: and 10 is ioyned with 2 & 3, therefore hath he their differences, 3 and 2. And because of ease for you, in another columne I haue set the differences reduced into one number, for euery seuerall sort, and haue also added them together, whereby appeareth that they make 33 and so consequently you see the works of the Golden-rule set forth. For the six drugges I haue added letters, A, B, C, &c. as before.

But I would not wish you to cleaue still to these Elementary aydes, but accustome Memorie to trust her selfe: so shall occasion of negligence be best auoyded. And as for the proofe, trie it at more leisure, because the Time now is short, and you sufficiently instructed in that proofe. And there reseth diuers things behinde yet, of which I would gladly giue you some taste, before our departure:

Note.

Scholar. But if it may please you to let me

see

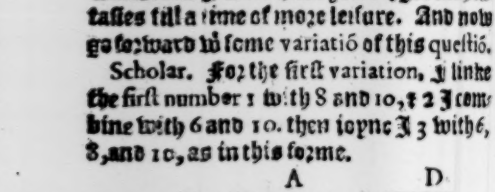
ſée all the varietyes of this queſtion, before you go from it, for me thinketh I could varyit two or three wayes more yet.

Maſter. I am content to ſée you make two or three variations, but I would be loath to stay to ſée all the variations, for it may be varied above 300 waies, although many of them would not well ſerue to this purpoſe.

Scholar. I thought it impoſſible to make ſo many variations.

Maſter. Marrie not therat. for ſome queſtiõs of this rule, may be varied above 1000 waies: but I wold haue you forget ſuch fãtaſies till a time of more leiſure. And now go forward wt ſome variatiõ of this queſtiõ.

Scholar. For the firſt variation, I linke the firſt number 1 with 8 and 10, & 2 I combine with 6 and 10. then ioyne I 3 with 6, 8, and 10, as in this forme.

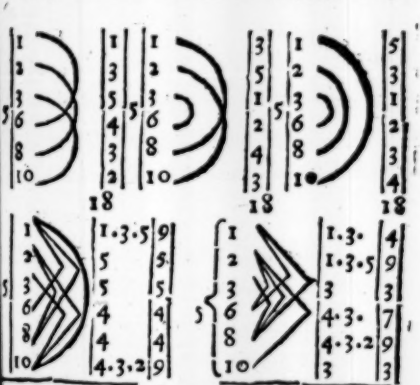


And ſo doth there appeare the proportion of weight for euery kinde of Drugge in this mix-

mixture. Now for the tryall.

Master. Nay, stay there: you shall not need to make tryall in one example so often, or if you list to doe it by your selfe, I am content. But now set forth (for declaration that you conceiue the Rule) two or three examples of seuerall combinations, and then will we passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties: in which, as the



combinations are seuerall, so doth it plainly appeare, that ỹ differẽces by which the proportion of each seuerall kind is takẽ, are also seuerall. And yet I see in the three first of these

fiue

fiue varieties, & in one other before, the totall summe of the differences to be one, that is to say, 18, whereby I perceiue that the variety of their differences seuerall, and not of the variety of their totall summe.

☞

Maister. So is it. And seeing you conceiue it so well. I will make an end of this Rule onely exhibiting vnto you one Question or two of the mixture of Metals, that by it you may deuise others like, & exercise your selfe therin also, because the vse of it serueth often in busines of charge, not so much for Gold smiths, as of coynage in Mynts. First I demand of you this question; *If a Mynt-master haue Gold of 22. karects, and some of 23. karects, some of 24: Againe, some of 15. some 16. & some of 18 karects, & would mixe them, so that hee might haue 100. ounces of 20 karects: How much must hee take of each sort?*

A question of mixing golde.

Scholar. To knowe that I answere in order, thus:

Maister

Master. You haue wrought the question well: but how chanced you made no doubt of that new name Karect?

Scholar. Because I thought it out of time to demand such questions now, seeing you make so much haste to end: and againe in this case the proportion of the number is sufficient for my purpose in this worke: trusting that another time you will instruct mee as well of this, as of sundry other things, which as I haue heard you talke of, so I haue a great desire to them.

Master. Your answere is reasonable, and your request and trust (with Gods helpe) I intend to satisfie. And now to goe forward with this matter, let me see your examination of this last worke.

Scholar. First for the one part I adde together al the particular sums as they appeare in the worke, and they make 100, as here by their Addition doth appeare.

	10
	15
	20
	25
	20
	10
	100

And so it seemeth that the summes are well gathered: but for the further triall of them: I multiply first

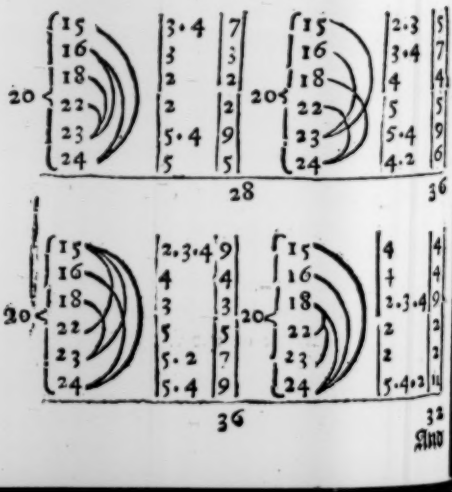
120	20 which is the common or means
240	sum of the Karects by 100, which
360	is the summe of the whole Masse,
550	which I would haue, and it ma-
460	keth 2000. Then I multiply euery
240	particular summe by the Karects
2000	

that it doth containe, as 10 by 15, and that maketh 150.

Likewise I multiply 15 by 16, and it yeeldeth 240: so 20 by 18 maketh 360. And 25 by 22 yeeldeth 550: likewise 20 by 23 bringeth forth 460, and last of all 10 multiplied by 24 yeeldeth 240: which summes all ioined together make 2000, that doth agree with the like summe before, wherefore I may well say, that the worke is good. And now if (it please you) I would set forth some varieties of this question to prove my wit.

Master. Goe to let me see.

Here be foure varieties.



And

And more yet could I make, but not like to the number that you speake of in the variation of the other question.

Master. That will I teach you at more leisure, seeing it is a thing rather of pleasure then of any necessity.

But now for your exercise in this rule, one other question I will propose. A Mint-master hath five ingots of siluer of sundry fineneſſe, some of foure ounces fine, and some of five ounces, some of six, and other of eight, some of 11, and other of 12, and his desire is to mixe 500 pounds weight, so that in the whole Masse euery pound weight should beare nine ounces of fine siluer: How much shall he take (say you) of euery sort of siluer? A question of mixing of siluer.

Scholar. To finde out that, I set the numbers thus in order.

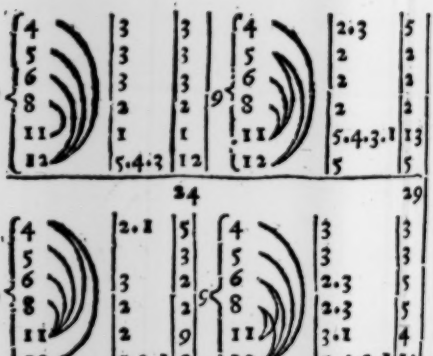
4	2	2
5	2	2
9 6	3	3
8	3	3
11	5.49	
12	3.11	4
		23

And gathering the differences, it will appeare, that, of the first sort there must be $43\frac{11}{23}$: of the second like much: of the third sort $65\frac{5}{23}$: and of the fourth sort as much: of the fift sort $195\frac{15}{23}$: and of the sixt sort $86\frac{22}{23}$, which in the whole will make 500 ℔ weight, and in ounces after 9 ounces fine 4500. that is of the fift sort $173\frac{21}{23}$: and of the second sort

217 $\frac{7}{23}$: of the third sort 391 $\frac{7}{23}$: of the fourth sort 521 $\frac{17}{23}$, of the fift sort 2152 $\frac{4}{23}$, and of the fixt sort 1043 $\frac{11}{23}$, which altogether doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

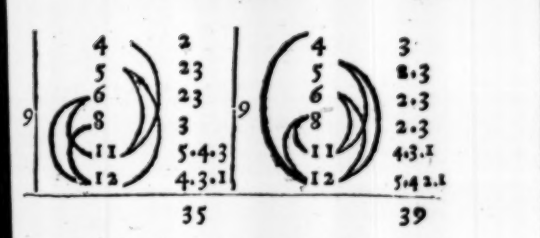
Master. This is well done of you, therefore now make three or foure varieties, and so an end of this Rule.

Scholar. These 4 varieties I set for example.



Master. And by these it appeareth, that you can find out more, with which I will not now meddle, save only (for to shew you an easie helpe in drawing the lines of Combination) I wil set forth two varieties here.

And



And this shall suffice now for the Rule of Alligation or mixture: for by these examples may you easily conjecture such other as dos appertaine to it, as well for the due working, as for variety of drawing the lines of combination.

Scholar. Sir, albeit it pleased you erewhile to put me from my musing at the many varieties that may fall in these combinations and termed them fantasies; yet my fantasy giueth me, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable. Therefore ye may wel think, that at another time conuenient I wil request you to aide me herein.

Master. Truth it is that this consideration may fall in practise as well Politick as Philosophicall, and sundry wayes in them bee applied: Therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hand.

The Rule of Falshood.

The occasion of the name.

NOw will I briefly also teach you somewhat of the Rule of Falsehood, which beareth his name, not for that it teacheth any fraud or Falshood, but for that by false numbers taken at all aduentures, it teacheth how to find those true numbers that you seeke for.

Scholar. So might any other rule bee called the Rule of Falshood, for they worke by wrong numbers, and by them find out the right numbers: so doth the Rule of Alligation, the Rule of Fellowship, and the Golden Rule partly.

Master. In the Golden Rule, the Rule of Fellowship, and the Rule of Alligation, although the numbers that you worke by, be not the true numbers that you seke for, yet are they numbers in iust proportion, and are found by orderly worke: whereas in this Rule the numbers are not taken in any proportion, nor found by orderly work, but taken at all aduentures.

And therefore I sometimes being merry with my friends, and talking of such questions, haue caused them that proponed such questions, to cal vnto them such children or Idiots as happened to be in the place, & to take

take their answere, declaring that I would make them solue those questions, that seemed so doubtfull.

And indeed, I did answer to the question, and worke the trial thereof also by those answeres which they happened at all aduentures to make: which numbers seeing they be taken as manifest false, therefore is this Rule called the Rule of false Positions, and for briefenesse, the Rule of Falshood: which Rule for readinesse of remembrance, I haue comprised in these few verses following, in forme of an obscure Riddle,

Gesse at this worke as hap doth leed,
By chance to truth you may proceed.
And first worke by the question,
Although no truth therein be don.
Such falshood is so good a ground,
That truth by it will soone be found.
From many bate too many, mo,
From too few talke too few also;
With too much ioine too few againe:
To too few adde too many plaine.
In crosse-wise multiply contrary kind,
All truth by falshood for to find.

The sense of these verses, and the summe of this Rule is this:

When any question is proponed appertaining to this rule, first imagine any number that you list, which you shall name the

first position, and put it in stead of the true number, and then worke with it as the question imposeth: and if you haue missed, then is the last number of that worke either too great or too little: that shall you note as here after shall be taught you, and you shall call it the first errour.

Then beginne againe, and take another number, which shal be called the second position, & worke by the question: if you haue missed againe, note the excesse or default as it is, and call that the second errour. Then multiply crosse-wise the first position by the second errour, and againe the second Position by the first error, and note their totals severally by the names of totals. Thē mark whether the 2 errours were both alike, that is to say, both too much, or both too little: or whether they be vnlike, that is, the one too much, and the other too little: for, if they be like, then shall you subtract the one totall from the other (I meane the lesser from the greater) and the remainer shall be your Diuidend: so must you abate the lesser errour out of the greater, and the residue shall be the Diuisor. Now diuide the Diuidend by that Diuisor, & the quotient will shew you the true number that you seeke for. But, and if the errours be vnlike, then must you adde both those totals (whith you noted) together, and take that whole number for the

Diuidend,

Diuidend, so shall you adde both errours together, and that whole number shall be the Diuisor, and the Quotient of that Diuision shall giue you the true number that the quotient seeketh for, & this is the whole rule

Scholar. This rule seemeth so vnlike any other, that without some example I shal not easily vnderstand it.

Master. With a good will: propose halfe a score sundry questions and examples of variety, for the better vnderstanding of the worke hereof: and for the first take this example. *A Mason was bound to build a wall in 40 daies, and it was couenanted so with him, that euery day that he wrought, he should haue for his wages 2 s. 1 d, and euery day that hee wrought not, he should be amerced 2 s, 6 d: so that when the wall was made, and the reckoning taken of the daies that hee wrought, and of the other that he wrought not, the Mason had cleerely but 5 s, 5 d, for the worke. Now doe I demand how many daies did he worke of those 40, and how many did hee not worke?* A question of Masonrie. The first example.

Scholar: I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the rule, be able againe to doe the like.

Master. This order shall you keepe in the worke of this rule: first take some number (as you list) at aduenture, as for example, I say he plaied 12 daies, & wrought 28 daies.

Now

Now call you the wages of euery day, & ſée whether it will agrée with the ſum of 5550.

Scholar. The 28 daies that he wrought after 25 pence the day, yéeldeth 700 pence. Then 12 daies that he wrought not, at 30 pence each day, doth amount to 360 pence which if I abate out of 700 pence, there reſteth 340: but you ſay he had not ſo much.

Maſter. He had but 65 pence, and by this ſuppoſition he ſhould haue had 340: therfore is this ſum too much by 275, which ſumme I muſt ſet downe after this ſort as you ſée héere, where firſt I haue made a croſſe (commonly called S. Andrewes croſſe) at the ouer corner, on the left hand I haue ſet the firſt poſition 12: and at the other corner vnder it I haue ſet 275, which is the firſt error, with this figure +, which betokeneth too much, as this line — plaine without a croſſe-line betokeneth too little. On the right hand of the croſſe I haue left two like rowmes for the ſecond poſition and his error. Therfore to proſecute the worke, I ſuppoſe he plaied 16 daies, & wrought 24.

12
X
275+

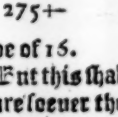
Scholar. I was a while in doubt why you named the daies of his working, ſéeing they be not ſet in the figure: and I doubted how you knew them, or elſe whether that you did ſuppoſe them at all aduentures, as you did the daies that he plaied: but now I gather

ther, that ſeeing 40 daies is the whole time limited, then the daies that he plaied being ſuppoſed, the reſt of 40 muſt needes be the daies that he wrought, and therefore 28 followed 12 of neceſſity, & 24 followeth 16 alſo of neceſſity: but yet I ſcarce perceiue why you ſet not in the figures as well 28 as 12.

Maſter. It forceth not which of them I take, ſo that in the ſecond poſition I take the numbers of the ſame nature that is here both of working daies, or both of idle: but now examine you this ſecond poſition.

Scholar. If he plaied 16 daies, then abating 16 times 30d, the ſumme will be 480d. and for 24 daies that he wrought, euery day yelding 25d, the totall is 600 pence: ſo that abating 480 out of 600 there reſteth 120, and as you ſay, it ſhould be but 65: therefore it is too much by 55: that muſt be ſet on the right hand of the figure at the neather part, and ouer it on the ſame ſide 16, which is the ſecond poſition, thus:

And as I gather by your words, it were all one if I did ſet 28 in ſtead of 12, and 24 inſteade of 16.



Maſter. So were it. But this ſhall you marke, that, of what nature ſoeuer the two poſitions be, of the ſame nature is the quotient. Therefore when the poſitions in this

queſtion

queſtion are 12 and 16, which both being numbers of the playing dayes, the quotient ſhall declare the true number of playing daies: whereas if the poſitions had bœn 28 & 24, which are ſuppoſed to be the working dayes, then would the Quotient declare the true number of the working daies, and not of playing dayes, as it will doe now. And therefore to continue the worke of this queſtion, and to find the true number of playing dayes, I muſt multiply croſſe-wiſe the firſt poſition 12 by 55, that is the ſecond error, & the totall will be 660. Then I multiply 275 & 16, & it yéeldeth 4400. Now becauſe the errors are like, that is to ſay, both too much, I muſt ſubtract 6600 out of 4400 & ſo remaineth 7340, which is the diuidéd, Againe, I muſt ſubtract the leſſer error 55 out of 275, that is the greater error, & there will remaine 220, which ſhal be the diuiſor: then diuiding 3740 by 220, the quotient will be 17. Wherefore I ſay now conſtantly, that 17 is the true number of dayes that the Maſon plaied: and then it followeth that he wrought 23 dayes, and ſo is the queſtion anſwered.

The proof of this rule

Now for the order of triall of this worke, there needeth none other triall but only this, to worke with this number according to the queſtion, and if it agree, then appeareth the number to be it that you would haue. As here

now

now seeing he wrought 23 dayes, and must haue for euery day 25 pence, the whole sum commeth to 575. Then again seeing he plaied 17 dayes, and must abate 30 pence for euery day, the whole sum of the abatement will be 510: Therefore I subtract 510 out of 575, and there will remaine 65, which maketh 5 s, 5 d, the cleere wages of the Mason for his worke according to the queistion.

Scholar. Now I trust I vnderstand the worke and the Rule so well (& the better by this proofe) that I can be able to do the like. And for a proofe, I take the same queistion al saue the last number, where I will suppose that he had 10 s, for his wages cleere. And now to geese at the number of the daies that he wrought, I suppose first that he wrought 20 daies: then say I, if he wrought 20 daies, his wages must be 500 d then did he play other 20 dayes, for which must bee abated 600 d, & then he loseth 100 d. And so am I at a stay, for it is not like vnto your former worke.

Master. You should haue required of me some queistion, & not haue taken a queistion of your owne phantasying, vntill you were more expert in this art: for so might you as well happen on an impossible queistion, as on a possible: but now to goe forward, consider that this number is too little by 220, seeing hee should gaine by your supposition 1200,

120 ℔, and in this position hee loseth 100, those both make 220, which you shall set downe for the first errour with this signe ——, betokening too little, as here in this forme following doth appeare.

And now for the rest goe forward your selfe once againe. 20

Scholar. As my errour hath bttered my folly, so it hath procured me better vnderstanding. 220—

Now therefore considering this position not to solue the question, I take another, supposing that hee wrought 30 dayes: Then for his wages he must be allowed 750 ℔, and for the 10 dayes which hee wrought not, he must abate 300 ℔, and so remaineth clere 450 ℔: but it shall be onely 120 ℔, therefore is it too much by 330 which I set downe in the figure with the former position and his errour, and the figure appeareth thus: 20 30

Now must I multiply in crosse wayes 20 by 330, and it will be 6600. 220—330+

Then againe I multiply 30 by 220, and it will be also 6600. Wherfore if I shall subtract the one out of the other, there will remaine nothing to be the Diuidend.

Master. In this you forget your selfe againe: for in as much as the signes in the errours be vnlike, therefore must you worke

by Addition, adding together thoſe two totals to make the Diuidend, and alſo adding the two errours to make the Diuiſor. And becauſe you ſhall no more forget this part of ỹ rule, take this briefe remembrance.

Vnlike require Addition,
And like deſire Subtraction.

Scholar. You meane, that if the errours haue like ſignes, then muſt the Diuidend & the Diuiſor be made by Subtraction, as is taught before: And if thoſe ſignes be vnlike, (as in this laſt example they bee) then muſt I by Addition gather the Diuidend and the Diuiſor. Therefore muſt I adde 6600 to 6600, and it wil be 13200, which ſhall bee the Diuidend. Then againe I adde 220 to 330, and it will be 550, which muſt be the Diuiſor: wherefore diuiding 13200, by 550, the quotient will be 24, whereby I know that the Maſon wrought 24 dayes, and then it followeth that he plaied 16 dayes.

Maſter. Examine your worke, whether it be agreeable to the queſtion or no.

Scholar. For 24 dayes worke the wages muſt be 600 ♁, and for 16 dayes which the Maſon wrought not, there muſt be abated 480 ♁, and then remaineth cleere to the Maſon 120 ♁, as the queſtion importeth: wherefore it is euident that 24 is the true number of dayes that he wrought.

Maſter. Although you ſeeme now to vnderſtand this worke, yet to acquaint your mind

mind the better with the new Trade of this Rule, I think it good to propone to you 5 or 6 examples more before I make an end of it

Scholar. Sir I thanke you that you do so consider my commodity & profit in knowledge, for undoubtedly it is practice and exercise that maketh men prompt and expert in every kind of knowledge.

Master. You say well, so that they follow some certaine precepts to governe and rule their practice by, else may practice procure custome of errour, and a repugnance to exactnesse of knowledge: namely, as long as the errour is not plainely knowne to the vulgar sort. But to returne to your worke.

A question of wares. *There is a seruant that hath bought of velvet and damaske for his Master 40 yards, the velvet at 20 s, a yard, and the damaske at 12 s and when he commeth home, his Master demandeth of him how much he hath bought of each sort: I cannot tell (saith he) exactly: but this I know, that I paid for damaske 48 s, more then I paid for velvet: now must you gesse how many yards there is of each sort.*

Scholar. Although the gesse seemeth difficult, yet I will prove what I can doe: for I remember your saying, that it forceth not how fond or false the gesse be, so it be somewhat to the question, and not an answere of a contrary matter.

Therefore first I imagine that he bought

20 yards

20 yards of Damaske, for which he should pay after the former price 240 shillings: then must he needs have of veluet other 20 yards (to make vp the 40 yards) & that would cost 400 s. So that the totall of the price of the Damaske is lesse then the sum paid for Veluet 160 s, & should be more by 48. Therefore the first error is 208 to little. Then begin I againe, and suppose he bought of Damaske 30 yards, that cost 360 s: then had he but 10 yards of Veluet, which cost 100 s. and now the price of the Damaske is greater then the price of the veluet by 160 shillings, & should be but 48, therefore is the second error 112 to much, which I set in some of figure as here doth appeare. Then doe I multiply in cresse wayes 208 by 20, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much as the signes of the errours be vnlike, I know I must worke by Addition, therefore adde I those two totals together, and they make 8480, which is the Dividend: then adde I also the two errors together, 208 & 112, and they make 320, which is the Divisor: wherefore dividing 8480 by 320, the quotient will be 26 $\frac{1}{2}$, which is the true sum of yards of Damaske that he bought, & in veluet 13 yards $\frac{1}{2}$, and that appeared by examination,

20 30
208 112

thus: 26 ½ yards of Damaske at 12 s the yard, maketh 318 s: then in Veluet he had but 13 yards and ½, that cost 276 s, at 20 the yard. Now subtract 270 out of 318, and there will remaine 48, which is the number of shillings that the Damaske did cost more then the Veluet.

Master. Now shall you haue a question of another kinde.

A question of debt.

There are three men that doe owe money to mee, and I haue forgotten what the totall sum is, and what the particulars be.

Scholar. Why, then it is impossible to know the debt.

Master. Peace: ye are too hastie: there is more helpe in it then yet you see. I haue three seuerall notes, whereby it apeareth that I did confer their debts together, and found the debt of the first and the second to amount to 47 l, the debt of the first man and the third man did make 71 l, and the second man his debt with the third, did rise to 88 l. Now can you tell what euery man did owe, and what was the whole totall?

Scholar. Nay in good faith: but as I perceiue that it must be found by coniecture, so will I guesse at it, supposing that ỹ first man did owe 20 l, & the second man 30 & ỹ third.

Master. Nay stay there, you are too farre gone already: you may not suppose a seuerall sum for euery man, for it is enough to sup-

suppose one sum for the first man, and let the other rise as the question importeth. Therfore seeing you set the 1 man his debt to bee 20l, the second man cannot owe 30l: for the declaratian is, that their debts added together did make 47l, so must the second man his debt be but 27l, Now the second debt with the third must make 88: therefore subtract 27 out of 88, and there wil remaine 61 as the third man his debt. Then saith the declaration, that the first & third mens debts doe make 71: but by this supposition they make 81, ẏ is 10 too much, which I must set for the first errour Now worke you the second proposition.

Scholar I suppose the first mans debt to be 24l: then must the second mans debt (by you your declaratiõ) be but 23l, seeing both they make but 47l. And the second man his debt with the third. doe make 88l, & the second man oweth but 23: therefore the third man must owe 65 l. Now the third mans debt with the first should make by the declaration 71l, and they doe make 89l, that is 18l too much, and that is the second errour which I set downe with the first, and their position in this forme, and then I doe multiply in crosse waies 20 by 18, and it is 360 And 10 by 24 maketh 240. Also because the signes of the errõurs bee like, I must

worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the diuidend: then doe I subtract 10 out of 18 by the same reason, and so is the Diuisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15ℓ, & then the second man must owe 32ℓ: for those 2 doe make 47ℓ, and the third mans debt is 56, for so much remaineth if I abate 15 out of 71, or if I take 32 out of 88:

The fourth question.

Master. *For the fourth example take this easie question for the variety in worke. Two men hauing seuerall summes, which I know not, doe thus talk together: The first saith to the second, if you giue me 2s of your money, then shall I haue three times so much money as you. The second man answereth, It were more reason that our sums were made equall, and so will it be if you giue mee 3s of your money. Now guesse what each of them had.*

Scholar. I imagine that the first had 9s.

Note.

Master. Consider euermore in your imagination that you take a likely sum, as in this question, take such a sum that hauing 2 added vnto it, may be diuided into 3 parts euen.

Scholar. Why? I remember you said before, it forceth not how fondly soeuer I guessed.

Master. As for the possibility of the solution, it is thruth: but for easines in worke, the aptest

aptest numbers are most conuenient.

Scholar. I thought no lesse, and therefore I take 9 as an apt number to be parted into three: but I perceiue I should haue considered the aptnesse of that partition after the addition of two vnto it, and then 7 had bene more meet.

Master. That is truth, and then should the second man his sum be 5: for although he haue now but the third part of 9, that is 2, yet you must remember that he lent the first man 2, and so had he 5.

Scholar. Then to goe forward: if the second man had three of the first man, then should he haue 8, & the first man but 4: so hath he double to the first man: yet he said in the question they should haue equall: wherefore it appeareth that he hath 4 too much.

Therefore I note that error with his supposition, and gesse againe that he hath 10ſ: whereunto I adde 2ſ borrowed of the second man, and then he hath 12ſ: so the second man hath remaining but foure, whereunto if I adde the 2 that he lent to the first man, so had hee but 6ſ at the beginning. Then take 3ſ from the first man, and giue to the second, then hath the first man but 7, & the second hath 9, which are not equall, but there are two too many wherefore I set downe both

the positions with their errors as here you see and multiply a crosse, so cometh there 40 and 14: t because the signes bee like, I take 14 out of 40, and so resteth 26 to be diuided then likewise I take 2 out of 4, and there resteth 2, by which I diuide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth, that two being added thereto, the sum will be 15, so hath thy second man but 5, and before he had 7: then take 3 from the first, and put to his 7, and so haue each of them 10, & that is equall as the question would.

The fifth example: A question of Lambs.

Master. *For the fifth example take this question. One man said to another, I thinke you had this yeere two thousand Lambes: So had I, said the other: but what with paying the tith of them, and then three seuerall losses they are much abated: for at one time I lost halfe as many as I haue now left, and at another time the third part of so many, and the third time ¼ so many. Now gueesse you how many are left.*

Scholar. Because here is mention made of certaine parts, I must take a number that may haue all these parts, that is to say: ½ ⅓ and ¼ which will be 24, howbeit 12 hath same parts. Therfore I take first 12 to be the number that doth remaine, so hath he lost 6, 4, and 2, that is 13, and the whole 25, but it should be 2000.

Master.

Master. Ye are deceiued yet stil you haue forgotten the 10 part, which must be defalked, that is 200, so there remaineth but 1800 and now go on againe.

Scholar. Then to find the errour I take 25 out of 1800, & there remaineth 1775 to few which I set for the first error. Then for the second position I take 24, whose halfe is 12, the third part 8, and the quarter 6, whereby riseth 50, which is to little by 1750, therefore I set downe both the positions with their errors thus:

And multiply in crosse waies 1775 by 24, where of commeth 42600. Also I multiply 1750 by 12, and there ariseth 21000. And because the signes are like, I doe subtract the one from the other, and so remaineth the Dividend 21600. Then doe I subtract 1750 out of 1775, and there resteth 25, by which I divide 21600, and the quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being added together, will make 1800. And if you adde thereto the tenth which was abated before, then wil the whole summe be 2000.

12	24
1775	1750

864
432
288
216
1800

And now doth there come a question to my memory which was demanded of mee, but I was not a-

able to answere to it: and now me thinketh I could solue it.

Master. Propone your question.

A question of sheepe & tillage. The sixth example.

Scholar. *There is supposed a Law made, that (for furthering of tillage) every man that doth keepe sheepe, shall for every ten Sheepe eare and sow one acre of ground: and for his allowance in sheepe pasture, there is appointed for every foure sheepe one Acre of pasture. Now is there a rich sheepe-master which hath 7000 acres of ground, and would gladly keepe as many sheepe as he might: by that Statute I demand how many sheepe shall he keepe?*

Master. Answer to the question your selfe.

Scholar. First I suppose he may kæpe 500 sheepe, and for them he shall haue in pasture after the rate of foure sheepe to an acre 125 acres, and in arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I guesse again that he may kæpe 1000 sheepe, that is in pasture 250 acres, and in tillage 100 acres, which maketh 350, that is too little by 6650

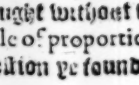
500 1000
6825 6650

Both these errours with their positions I set down as you see, and multiply them crosse 6825 by 1000 and it maketh 6825000, also I multiply 6650 by 500, and thereof commeth 3325000, which summe I subtract out of the former, and

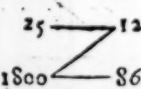
there

there remaineth 3500000 for the dividend; likewise I subtract the lesser error out of the greater, and there resteth 175, by which I diuide 3500000 (the dividend aforesaid) and the quotient will be 20000, so that by this rate, he that hath 7000 acres of ground may kæpe 20000 sheepe.

Master. You haue done well notwithstanding, both this last question, and the next before might be wrought without the second position by the Rule of proportion, as this. When in this question ye found in the first error, that for 500 sheepe, there must bee 175 acres, then might you reduce it to the Golden Rule, thus. Another way of working.



If 175 acres will admit in allowance 500 sheepe, then 7000 will haue 20000. And so by one position, with the helpe of the Golden Rule may you answere that question. Likewise for the question of Lambes, when you had found that 12 came of 25, you might haue set the figure thus as you sée, and haue said:



If 25 doe leaue but 12, what shall 1800 leaue? and it would appeare to be 864.

Scholar. Sir, I thanke you for this aid, for it doth much shorten the worke of this Rule.

Master.

Another way yet.

Master. Yet againe, I will shew you an other way to answere to this last question without the Rule of false position, and that by the Rule of fellowship, for it appeareth in the proponing of the question, that 10 sheepe must haue in pasture two acres & $\frac{1}{2}$, & for them must there be eared but one acre: so it followeth, that for 2 acres eared, there must be 5 set to pasture: and if you put them both into one sum, they wil make 7. Therefore looke what proportion 7 being this totall, doth beare to 5 & to 2, such proportions shall any totall in this question beare to the pasture ground, and the eared ground.

Scholar. This serueth wondrous aptly. Therefore to proue it, I demand this by the former supposition: If a man haue 300 Acres, how much shal he leaue in pasture, and how much shall he turne to tillage? You say that as 7 is to 5, so shal 300 be to the acres of pasture: and as 7 is to 2, so is 300 to the Acres of tillage, whereof for both I haue set examples here following, wherby appeareth that of Pasture, there shall be 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{5}{7}$, which both sums added together doe make 300.

7 Z 5
300 214 $\frac{2}{7}$

7 Z 2
300 85 $\frac{5}{7}$

Another question:

Master. *Now take another example: A man hath three siluer cuppes with one couer, the*

the couer wayeth 18 ounces, the second cuppe wayeth euen halfe the waight of the first and the third. Now if the couer be put to the first Cuppe, they wey iust as much as all the three Cuppes doe wey: and if the couer bee ioyned with the second Cuppe, they wey as much as the second twice, and the third: and if the Couer be put to the third Cuppe, they will make twice as much as the first and the second Cup. Now trie you what was the iust weight of euery Cup.

Scholar. I doe set the weight of the first cup to be 9 ounces: then in as much as these two (that is to say, the couer & the first cup) doe wey the weight of the three cups, I see that the 3 cups must wey 27 ounces for so much is 18 & 9. Also because the first and the third doe wey double so much as the second, therefore is it the third part of that weight, that is 9, and then would it follow, that the third cup also should wey 9 ounces but then the question saith, that the Couer being ioined to the second cup, they wey as much as the second twice, & the third once, that should be 27, and so it doth: then being ioined with the third Cup, they should wey twice as much as the first & the second, that should be 36, and they wey but 27, so is that error 9 to little. Then begin I againe, and say, that the first Cup doth wey 12 ounces, which I ioine with the Couer, & they make

30 ounces

30 ounces: then ſeeing the ſecond is of that weight, it muſt néeds wey 10 ounces, and the third muſt wey 8 ounces, ſeeing the firſt and the third muſt wey 20 ounces. Now put I the couer to the ſecond cup, and they wey 28 ounces, which ſhould be euen ſo: then ioine I the couer with the third Cup, ſo ſhould it wey twice the firſt and the ſecond, that is 44 ounces, and they do wey but 26, that is 18 too litle: thoſe errors with their poſitions I ſet downe, and multiply in croſſe wayes 9 by 12, whereof commeth 108: Alſo 9 by 18, and that yéeldeth 162: and in as much as the ſignes be like, I abate the leſſer out of the greater, and there doth remaine 54. Then doe I alſo abate the leſſer error from the greater, and ſo remaineth 9, by which I diuide 54, and the quotient is 6: which I take for the true weight of the firſt Cup, which being ioined with the Couer, muſt wey as much as the three Cups, ſo doe they wey but 24 ounces. Then ſeeing the ſecond Cup is the third part of that weight, for the other two Cups (you ſay) muſt wey double his weight, the weight of the ſecond Cup is 8 ounces, and ſo the weight of the third muſt be 10 ounces. Now put the Couer to the ſecond Cup, and it will make 26 ounces: that muſt be the weight of the ſecond

cond twice, & the third once, that is twice 8, and once 10, and so is it. Againe, put the cover to the third Cup of 10 ounces, and they must wey twice as much as the first and the second, that is 28, and so is all agréeable.

Master. Then answere to this question. A question of water.

There is a Cesterne with foure Cockes, containing 72 barrels of water: and if the greatest Cocke be opened, the Water will avoid cleane in sixe houres: at the second Cocke it will aske eight houres: at the third Cocke it will avoide in no lesse then nine houres: and at the smallest it will require twelue houres. Now I demand in what space will it avoid, all the Cockes being set open?

Scholar. First I imagine it will avoid in two houres.

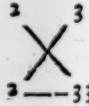
Master. Then must there avoid by the first Cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the second Cocke $\frac{1}{4}$ that is 18, and by the third Cocke $\frac{2}{9}$ that is 16 Barrels, and by the smallest Cocke $\frac{1}{12}$, that is 12 barrels, all which summes put together, doe make 70, as by their Addition it doth appeare, but it should be 72, therefore the error is 2 too few.

Scholar. Then I begin again by your fauour, because I thinke I vnderstand the worke, and put three houres for the due time: so shall there runne out at the greatest Cocke $\frac{1}{3}$,

	24
	18
	16
	12
	70

that

that is 36 barrels, & at the ſecond hole ⅓ that is 27, and at the third Cocke ⅓ that is 24, & at the ſmalleſt hole ⅛, that is 18 barrels, wch altogether doe make 105, and ſhould be but 72, ſo is it too much by 33, therefore do I ſet the errors in order of the figure with their poſitions, and worke by Multiplication, in croſſe. ſaying: 2 times 3 is 6, & two times 33 maketh 66 & becauſe the ſignes are vnlike, I muſt adde thoſe two totals together, which make 72: alſo I adde the two errours, and they make 35, by which I diuide 72, and the quotient riſeth 2 2/7, whereby I ſee that all the Cockes being ſet open, the water will run in two houres and 2/7 of an houre.



Maſter. This exercise maketh you to grow expert in the Rule. Therefore I will inure you somewhat more with a queſtion or two.

A queſtion of partners.

There were two men that had been partners, and had in account between them 300 duckets: whereof the one ſhould haue for his part 180, and the other 120: but in the parting of them they fel at variance, ſo that each of them catched as many as he could: yet afterward being reconciled they agreed that he which had gotten moſt part of them ſhould lay downe ⅓ of them againe, and he that

that had gotten least, should lay downe $\frac{1}{3}$ of those which he had taken, and then parting them into two equall parts, each man to haue halfe thereof, and so had they their iust portions as they ought: now I demand of you what each of them had gotten by the scambling.

Scholar. I suppose he that had least, got 108 Duckets, then the other had 192. wherefore in laying downe again of the 192, there was put downe $\frac{1}{3}$ that is 114, and so had hee left but 48. Also of the 108, there was laid downe 36, that is $\frac{1}{3}$, and so he had left 72. Then I put together 144 & 36, and it maketh 180, which I part into two parts euen, and so commeth 90 to be giuen to each of them: which sum put to 72, maketh 162, and ioined to 148, it maketh 238: and now I doubt how I shall goe forward.

Master. You need not to take but one of them which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other sums, remembring that you make comparison to the same in the second worke: as for example of the first part. If you compare 138 with the lesser summe due, that is 120, so is it 18 too much. and if you compare it with the greater sum, then is it 42 too little. Againe, if you compare 162 to the greater summe, the error wil be 18, as it was in and

Note.

the other: but it will haue a contrary signe: and if you compare it with the lesser sum it, will be 42 too much: so that the error both waies is either 18 or 42 and as for the signes it little forceth, for in them is nothing considered here, but likenesse and vnlikenes which in this case doth either further or hinder: But now goe on with the worke.

Scholar. If it be so, then am I out of my greatest doubt. Then I ioine that 90 (which I found as the halfe of the latter partition) vnto 48, which is left with the one man, and so hath he 138, which (I may say) is 18 too many, for y least should be but 120: that error doe I note, and then make a new position, supposing the one man to haue 204, and the other to haue 96, wherefore of the 204, there must be laid downe 153, and so remaineth with him 51. Also of the 96, there must be laid downe $\frac{1}{3}$, that is 32, and so resteth with that man 64. Now of the 153 and 32 I make one sum as 185, which I must diuide into two equall parts, and so each man shall haue $92\frac{1}{2}$, whereunto if I adde their former portions reserued, then the one shall haue $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherefore take the lesser summe now againe, as I did before, that is $143\frac{1}{2}$, and find that he hath too many by $22\frac{1}{2}$, for he should haue but 120, so haue I for my two positions two errors, which I set downe as here may be

seene,

sæne, each error vnder his position; and then by the Rule I doe multiply in crosse-waies 108 by $23\frac{1}{2}$ and there riseth 2538, which I note, then againe I multiply 96 by 18, & thereof amounteth 1728.

Now because the signes are both like, that is both too many, I must worke by subction, and so abating 1728 out of 2538, there will rest for the Diuidend 810: then for the diuisor I subtract 18 out of $23\frac{1}{2}$, and there remaineth $5\frac{1}{2}$, by which I diuide 810, and the quotient will be $147\frac{3}{11}$, which is the iust portion of him that had the least sum. And if I doe subtract it out of 300, being the totall sum, then will there remaine $152\frac{8}{11}$ as the portion that the other did get.

Master. For the proofe of this worke, you may chuse whether you will examine those numbers according to the forme of the questions, or else worke by other two positions for to find the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each worke confirme other. Note:

Scholar. By your patience, I will proue both waies, not only to seeke their agreement, but also to accustome my minde to those workes, for I perceiue it is exercise

To that

that must bee the chiefe engrauer of these rules in my memory.

Master. You consider it well: then go to.

Scholar. First I will by two other positions try to finde the portion of him which had most,

Master. Although you may do it with any positions, yet to see the agreement of your worke the better, take ỹ same positions that you did before, comparing them now to the greater, as you did before vnto the lesser.

Scholar. Then I suppose that he that had most, had 192, so had the other 108. Now if I take $\frac{1}{4}$ out of 192, that will be 144, & there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: then ioyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to each of these two mens portions remaining with them, the one shall haue 138, and the other 162, of which two I take the greater, (that is 162) and see it to be 18 too few, for it should be 180, that error I note vnder this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{1}{2}$ of 204, and it will be 153, & there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, & there remaineth to him 64: Now put I that 32 to 153, & it yeeldeth 185: which being parted in equall

values,

values, maketh 92 ½ to bee added to each mans remainder, & so the one hath 143 ½, & ỳ other 156 ½: wherfore I take the greatest sum, & it is 23 ½ too little, that do I note also ỳ set both these errors vnder their positions, as in this example following doth appeare.

And then multiplying 192 by 23 ½, there doth arise 4512.

Againe, I multiply 204 by 18, and it maketh 3672, which I doe subtract out of 4512, because the signes be like, and there resteth 840 for the diuidend, the subtracting 18 out of 23 ½ there will remaine 5 ½, which I must take for the Diuisor. And so diuiding 840 by 5 ½, the quotient will be 152 8/11, whereby I haue found an agreeable sum to ỳ which I found by the former Positions, for him that had most, which I doe subtract out of 300, that is the total, there will rest 147 3/11, which was ỳ portion of him that had the least part.

192 204
18 — 23 ½ —

Master. So by diuers positions you see, that one doth confirme the worke of the other. Now examine those two numbers by the forme of the question, and so shall you proue your worke good also.

Scholar. If that he which gate most, had 152 8/11, then must he lay down ¾ of this sum, That is 114 6/11, and so shal remain with him but onely 38 2/11. The other which had least:

that is $147\frac{3}{11}$ must put downe of his sum $\frac{7}{11}$, that is $49\frac{1}{11}$, and so doth there remaine with him yet $98\frac{3}{11}$. Then do I adde together $114\frac{6}{11}$ and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I must part into equal parts, and that will be $81\frac{5}{11}$ to be giuen to each of them: putting $81\frac{5}{11}$ vnto $38\frac{3}{11}$, there doth amount 120 iust, which is the true portion of him that should haue the lesser sum: and adding $81\frac{5}{11}$ $98\frac{3}{11}$, the total wil be 180, the true portion of the other. And so is the worke by this proofe also tried to be good. And this I mark by the way, that in their scambling, hee got most (as it chanceth often) that ought to haue had least by iust partition.

Master. Let your study be to learne truth and iust art of proportion, and to distribute and part according thereunto, as often as occasion shall be ministred. And here would I make an end of this rule, saue that I remember one pleasant question which I cannot ouerpasse, which I will declare somewhat largely, because you shall as well vnderstand some reason in the pleasant inuention, as apt proceeding in the witty working thereof.

An example of mixture of gold and siluer.

Hiero King of the Syracusans in Sicilia, had caused to be made a Crowne of gold of a wonderfull waight, to be offered for his good successe in warres: in making wherof, the Goldsmith fraudulently tok out a certaine por-

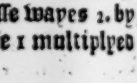
portion of gold, & put in siluer for it, so that there was nothing abated of the ful waight although there was much of ỹ value diminished. Which thing at length being vttered, (as no euill can alwaies lie hid) ỹ King was sore moued: and being desirous to trie ỹ truth without breaking of the crown, proponed ỹ doubt to Archimedes, vnto whose wit nothing seemed vnpossible, wᵒ although presently he could not answere vnto yet hee had good hope to deuise some policy for that inuention, & so musing thereon, as he chanced to enter into a bain ful of water to wash him, he obserued ỹ as his body entred into the baine, the water did run ouer the Tub, whereby his ready wit of such smal effects coniecturing greater works, conceiued by & by a reason of solution to the Kings questiō, & therefore reioycing exceeding more then if he had gotten the Crowne it selfe, forgate that he was naked, and so ran home, crying as he ranne, εὕρηκα εὕρηκα, I haue found, I haue found. And therupon caused two massy pēces, one of gold, and another of siluer, to bee prepared of the same weight that the saide Crowne was of: and considering that gold is heauier of nature then siluer, & therefore gold of like weight with Siluer, must nēds occupy lesse rome by reasō it is more compact and sound in substance, he was assured that putting the masse of gold into a vessell brim full of water, there would not so much

water run ouer, as when he should put in ꝑ siluer masse of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but also the difference or excesse of the one above the other, whereby he learned what proportion in quantity is betweene Gold and Siluer of equall weight. And then putting the crown it selfe into the vessell of water brim full (as before) marked how much water did run out then, and comparing it with the water that ranne out when the Gold was put in, noted how much it did exceed that: and likewise comparing it to the water that ran out of the siluer, marking how much it was lesse then that, and by those proportions found out the iust quantity of gold that was taken out of the Crowne, and how much siluer was put in stead of it: But seeing Vitruuius which writeth this History, doth not declare the particular work of this triall, it shal be no inconuenience to suppose an example for declarations sake, wherein although the true and iust proportions be not expressed, yet the forme of triall shall be truely set forth. And for an example, I suppose the weight of the crown to be 8 pound, and so of each the other two Masses. And when the Masse of Gold was put into the water, I imagine that there ran out two Pound of water; and when the masse of siluer

uer was put in, I suppose there ranne out three pound $\frac{1}{3}$. Againe, when the Crowne was put in, there ran out two pound $\frac{1}{3}$: Now to know what quantity of siluer was in the Crowne, worke by the Rule of false position, and imagine that there was two pound of siluer, then must there be 6 pound of gold, then say thus by the Rule of Proportion: If eight Pound of gold doe expell two pound of water, what shall fiue Pound expell: and it will be 1 Pound $\frac{1}{4}$. Againe for the siluer: if eight pound of siluer expell three pound $\frac{1}{7}$ of water, what shall two pound of siluer put out? it will be $\frac{1}{7}$, now adde those two weights of water together, and they will make two pound $\frac{1}{7}$, and it should be by the supposition two pound $\frac{1}{4}$, so is it too much by $\frac{1}{28}$.

Scholar. Now doe I vnderstand the worke as I thinke, therefore I pray you let me worke the rest of the question. And becase this first supposition did erre, I note that position and his errour, and take a new position, esteeming the Siluer to be but one pound, so must there be in gold seuen pound. Then say I: if eight pound of gold doe yelde two pound of water, what shall seuen Pound yelde: and it will be 1 pound $\frac{3}{4}$. Againe if 8 pound of siluer expell 3 pound $\frac{1}{7}$ of water, what shall 1 pound expell: and it will be $\frac{7}{15}$. Now must I adde those two

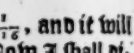
summes together, and they make 2 pound $\frac{3}{4}$ and they should make 2 pound $\frac{1}{4}$, so it is too little, by $\frac{1}{16}$. Therefore I set the positions with their errors, in order as here followeth. And then I multiplie in crosse wayes 2. by $\frac{3}{16}$ and it maketh $\frac{1}{8}$: Likewise 1 multiplyed by $\frac{2}{3}$, maketh $\frac{1}{3}$. And because the signes bee unlike, I muſt adde these two summes, which make $\frac{1}{8}$: and that is the diuidend.



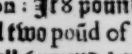
Againe, I muſt adde $\frac{1}{8}$, to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the Diuisor. Now I shall diuide $\frac{1}{8}$, by $\frac{3}{16}$, and the quotient will be $\frac{16}{13}$, that is, $1\frac{3}{13}$: whereby I knowe that there was put $1\frac{3}{13}$ of siluer into the Crowne, and so much Golde taken out for it.

Maſter. Proue it now by examination, according to the queſtion.

Scholar. If there were 1 pound $\frac{1}{3}$ of siluer, then was there of gold 6 pound $\frac{2}{3}$. Now say I by the Rule of proportion: If 8 pound of gold expel two poūd of water, what shall 6 pound $\frac{2}{3}$ expell?



It will be 1 pound $\frac{1}{3}$. Againe, If 8 pound of siluer expell 3. pound $\frac{3}{8}$ of water, what shall $1\frac{1}{3}$ expell? It will be $\frac{7}{11}$.



Now muſt I adde together 1 pound $\frac{1}{3}$, and

$\frac{1}{10}$, and they will make 2 pound $\frac{2}{10}$, that is 2. pound $\frac{1}{5}$, according to the supposition of the question: whereby I perceyue the worke to be well done. And I can not but much reioyce of this excellēt inuention, so my desire is kindled vehemently to be perfectly instructed in euery part thereof, & namely in this point, whether the portion betwēn water & gold be such, that for 8 poūd of gold put into a vessell full of water, there shall run out two pound of water, and for as much siluer, whether 3 pound $\frac{1}{2}$ of water would auoyd?

Master. I perceyue your meaning, & coniecture your imagination to be thus: that if you knewe the exact proportion betwēne golde & siluer, & water, both in their weight and quātities, then could you easily find out the mixtures of them, which thing I haue reserued for another worke that intreateth of such matters especially. And at this time you must consider that you learn Arithmeticke, which intreateth of the maner to solue doubtfull questiōs touching nūber, without regard what matter is signified by that nūber: else were it necessary in Arithmeticke to teach all Artes, seeing in it may be moued questiōs of all Arts. *But seeing you are so desirous to know these things, I will tell it you in such a sort, that you shall practise your Arte in finding it, and propound it in forme of a question. Golde beareth greater proportion to water*

Note, A question of the proportion of gold, siluer and quicksiluer, vnto water,

then

than siluer doth, and their two proportions be in proportion together, as siluer and quicke-siluer vnto water. But to helpe you somewhat in this riddle, you shall note that the proportion of quicke-siluer vnto water, is the iust middle number proportinall in Progression Geometricall betweene the proportion of gold and siluer vnto water. And this proportion is $\frac{19}{17}$. Now if you will know the iust numbers of these three proportions, then must you finde out 3 numbers in Progression Geometricall, whereof the middlemost must be $\frac{19}{17}$, and the first must be vnto the last, as 25 to 48. And thus I will leaue you to finde those numbers when you be at leasure.

Scholar. Yet Sir, I thanke you heartily for thus much, for now I see the possibility to find them out. Howbeit, because this question seemeth strange, if it might please you to instruct mee somewhat in the order of working for it, I should the more easily find the true working.

Master. You desire too much if you will study for nothing: Therefore to occasion you to study the better, I will leaue this doubt wholly to your owne search: But as touching the generality of the Rule, Archimides needed not to take two Masses of Gold and Siluer equall in weight with the Crowne, for the proportion might as well be found in any other waight, yea, although the

the Maſſe of Gold were of one weight, and the Maſſe of Siluer of another. As for example: If the Crowne were of eight Pound waight, as I did ſuppoſe, and I haue not ſo much other fine Gold, but onely one Pound, and trying that by Water, and finding that it doth expell but $\frac{1}{24}$ of an ounce of Water, yet then by it I may inferre, that 8 Pound of Gold would expell 6 Ounces of Water. And likewiſe of Siluer, whereof if I had but two pound, and find that it doth expel three ounces of water, then might I affirme that 8 Pound would expell 12 Ounces, that is, one Pound waight: and ſo is it as good as if the three Maſſes were all of one waight. And thus for this time I will make an end of this other part of Arithmetieke.

Scholar. Although I cannot ſufficiently thanke you for this, yet your promiſe made me to looke for the Art of extraction of Rootes, whereof hitherto I haue learned nothing.

Maſter. I will not breake my promiſe, but intend (God willing) to perfourme it within theſe three or foure moneths, if I perceiue this my paines to be well taken in the meane ſeaſon. And you ſhall not repent the tarrying for it: for it ſhall be increaſed by the tarrying. And in the meane time, you ſhall take this Addition, not for the ſecond Part of Arithmetieke which I promiſed,

but

but for an augmentation of the first part, vnto which I would haue annexed the extraction of Rootes, square and cubike, namely for examples of the Statutes of Assise of wood, but that in the second part I must write of diuers other Roores, and thought it best to reserue those Rules also with their examples vnto the same second part.

Scholar. Sir, although I cannot recompence your goodnesse, yet I shall alwayes doe mine endeuour to occasion you not to repent, your benefit on me thus imployed.

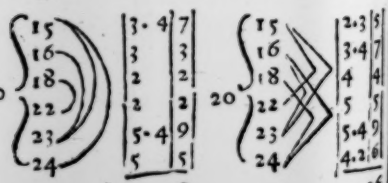
Master. That recompence is sufficient for your part.

FINIS.

COurteous Reader, these figures appertaining to the Rule of Alligation, *being (as it seemeth) long since lost, and diuers impressions past by sundry Correctors, without regard thereof: It now falling to my lot to haue the correction of this Impression, and desirous the Booke should passe according to the worthy Authors first Copy in this Rule, (so neere as I could) I haue for thy sake painfully and carefully laboured to rectifie them, and haue caused them to bee cut and inserted in this leafe (the Impression being past before I knew of their want) desiring thee to place them in their due places according to these directions.*

Page 375, the lower figure leftward, place the numbers thus.

Page 378, these are the two vpper figures:



Page 378, under the nethermost leftward, for 36 set 37.

Page 379, set this figure.

Page 380, place this figure in stead of the upper rightward.

Page 380, place this figure in stead of the lower leftward.

The third part

or Addition to this Booke,

entreateth of briefe Rules, called Rules of Practise, of Rare, Pleasant, and commodious effect, abridged into a briefer method than hitherto hath been published.

With diuers other necessarie Rules, Tables, and Questions, not only profitable for Merchants, but also for Gentlemen, & all other occupiers whatsoeuer, as by the contents of this Booke may appeare.

Set forth by *Iohn Mellis* Schoolemaster.

The first Chapter of this Addition, entreateth of briefe Rules, called Rules of *Practice*, with diuers necessary questions profitable, *not onely for Merchants, but also for all other occupiers whatsoeuer.*

1 Rule.

THe working of Multiplication in Practice, is no other thing then a certain maner of multiplying of one kinde by another: whereupon is brought forth the product of the proponed number, which is accomplished by the meanes of Diuision in taking the *halfe*, the *third*, the *fourth*, the *fifth*, or such other parts of the sum, which is to be multiplied.

And for the better vnderstanding of such conuersions, you shall vnderstand that in the manner and vse of these rules of practise, you ought first to know the euen or aliquot parts of a shilling, which in this table following doth appear.

Item { 6, 4, 3, 2, 1 } *pence is the* { 1/2, 1/3, 1/4, 1/6, 1/12 } *of a shilling*

Wherc-

Wherein as you see according to the order of these rules of Practise at 6 pence the yard of any thing You must take ½ of your number which is to bee multiplied, and the product that commeth thereof shall be shillings, if any vnite doe remaine it is 6 pence.

For 4 ℔ take the ¼ of the number that is to be multiplied, and the product also produceth shillings, if any vnites doth remaine, each one shall be worth in value 4 pence. The like is to be vnderstood of the other 3, &c.

Example.

At 6 ℔ the yard, what 379 yards?

189 ℔ —— 6 ℔

II.

At 4 ℔ the yard, what 104 yards?

34 —— 8 ℔

III.

At 2 ℔ the yard, what 5014 yards?

1253 ℔ —— 6 ℔

IIII

At 2 pence the yard, what 522 yards?

88 ℔ —— 8 ℔

V

At 1 peny the yard, what 409?

34 ℔ —— 1 ℔

Here you may see in the first example, that 379 yards at 6 ℔ the yard, are worth 189 ℔, 6 ℔, in taking the ½ of 379. And in the second example the 104 yards at 4 pence the yard, are

worth 34 s 8 d in taking the $\frac{1}{3}$ of 104. Likewise in the third example 5014 yards at 3 d the yard, bringing forth 1253 s 6 pence, in taking the $\frac{1}{4}$ of 5014. Also in the fourth example at 2 d the yard, maketh 88 s 8 d.

And lastly, in the fifth example: 409 yards at 1 d the yard, amounteth to 34 s 1 d, in taking the $\frac{1}{12}$ of 409: and so is to be done also of all other questions the like, when the number of the pence is any of the euen or aliquot parts of 12 d. Item to bring the products of these shillings, and all other the like into pounds is very easie in diuiding of it in your mindby 20, for it is to be vnderstood that as often as 20, is found in that product, so many pounds doth it containe: which with facilitie to performe, alwaies strike off the figure toward your right hand, with a right downe dash of your pen for the 0 that appertaineth to the 20. And then begin at the left hand, in taking the halfe of the rest. And if that at the last any vnite doe remaine, the same shall be ioined with the figure that is cut off, which shall represent the odde shillings contained in that worke.

As for example, in your third question at 3 d the yard which amounteth to 1253 s 6 pence, the product whereof maketh 62 l 13 s 6 d, as here you may see, is easily performed by this example.

	1	
125	3	
62 —	13 —	6

Also for the working of one penny the yar

yard, it is something harsh and hard to take the $\frac{1}{15}$ of some products: therefore to ease that hard worke, you shall first bring your deliuered summe into groats, by taking $\frac{1}{4}$ part of the product, and if any vnites remaine of that $\frac{1}{4}$ part, as sometimes there may, they are pence: and must be signified with a line from the groates *with their title of* Pence; *and because that* 60 groates *maketh a* Pound *or twenties*, strike *off the first figure toward your right hand, for the* 0 *that appertaineth to* 60 *(as you did euen now for the* 0 *that belongeth to* 20:) *Then in taking the $\frac{1}{6}$ of that product, if there doe remaine any vnites, the same shall you ioyne with the figure that you cut off, esteeming them as* groates, *which keepe in your mind, and by taking the $\frac{1}{3}$ part of them, you shall turne into* shillings, *and so haue you done:* As for example, *by a question or two hereafter proponed, shall more plaine'ly by the worke appeare.*

At 1 *d, the yard, what* 54368 *yards?*

1359 | 2 groates.

$\frac{1}{6}$ £ — 226 — 10 *s*, — 8 *d.*

Here in taking the $\frac{1}{6}$ part of 1359, *in comming to the last worke, the $\frac{1}{6}$ part of* 39 *being taken, the* remainder *is* 3, *which ioyned with the two that was cut off, maketh* 32 groates, *which conuerted into* shillings *by taking the $\frac{1}{3}$ part, maketh as appeareth* 10 *shillings* 8 *d. Many other wayes there are, but none*

more apt for a young learner *to vnderstand then this: wherefore this one way well impressed in memory, is better then 20 wayes doubtfully vnderstood.*

At 1 peny the yard, what 4533 yards?

	313	3 *groats*—1 ♁
½ ℔	18——17——9 ♁	

At 1 peny the yard, what 64768 yards?

	1619	2 *groats*.
℔	269——7——4 ♁.	

2 Rule. NOw followeth also to be vnderstood, that if the number of pence be not an aliquot part of 12, you must reduce them into some aliquot part of 12: and after the aforesaid manner, you shall make of them two or three products, as need shal require, and adde them together into one sum. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.

for pence	5	*take*	3	*and*	2	*or*	4 &	1
	7		4		3		6 &	1
	8		4		4		6 &	2
	9		6		3		4.4	1
	10		6		4		4.4 *and*	2
	11		6	4 & 1	1		4 4	3

Here in the first note of this Table *at* 5 ♁, *you shall first take for* 3 ♁ *the* ¼ *of the number that*

that is to bee multiplied: and likewise for 2 d, the $\frac{1}{2}$ of the same number, adding together both the products: But if you will worke by 4 and 1, you must for 4 d first take the $\frac{1}{4}$ of the number that is to bee multiplied: and for 1 ß take the $\frac{1}{12}$ of the whole sum, or rather, which is more better, for 1 peny you may take the $\frac{1}{4}$ of the product which d d come of the 4 pence: because that 1 ß is the $\frac{1}{4}$ of 4 pence. The totall summes of these two numbers shall be the solution to the question. *And in like manner is to be done of all others as by those* examples following shall appeare.

J

At 5 ß the yard, what 758 yards?

3 ß	189	6 ß
2 ß	126	4 ß
shillings	315	10 ß

Otherwise.

At 5 ß the yard, what 758 yards?

4 ß	252	8 ß
1 ß	63	2 ß
	315 ß,	10 ß

II

At 7 pence the Elle, what 563 Elles?

4 ß	187	8 ß
3 ß	140	9 ß
shillings	328	5 ß

IIII.

At 8 d the pound, what 112 pound?

4 d	37	4 d
4 d	37	4 d
	74 s	8 d

Otherwise.

At 8 pence the pound what 112 pounds?

6 d	56	0
2 d	18	8
	74 s,	8 d.

IIII

At 9 d the Elle, what 356 Elles?

6 d	178	0
3 d	89	0
	267 s	0 d.

V

At 10 pence the peece, what 795 peeces?

6 d	397	6
4 d	265	0
	662	6

VI.

At 11 the pound, what 7576 pound?

6 d	3788	0
4 d	2525	4
1 d	631	4
	6944	8 d.
	l, 347	4 s, 8 d.

Here

1 *Here in this first example, where it is demanded (at 5 d the yard) what will 758 cost? First of 3 d I take the* $\frac{1}{3}$ *of 758: and thereof commeth 189 ℔ 6 d: Then for 2 d I take the* $\frac{1}{2}$ *of the same product, which amounteth to 126 ℔ 4 d, these two summes added together, do make* 315 shillings 10 pence: *and so much are the* 758 yards *worth at 5 d the* yard.

1 *Item also for the same againe: First for 4 d, I take the* $\frac{1}{3}$ *of 758: and thereof commeth 252 ℔ 8 d: Then for 1 peny I take the* $\frac{1}{4}$ *of the same product, that is to say, of 252 ℔, 8 d, and it yeeldeth me 63 ℔, 2 d: which both added together maketh 315 ℔——10 d as before.*

2 Item, for 7 ℔ there is taken the $\frac{1}{3}$ and the $\frac{1}{2}$ *of the whole sum which is to be multiplied, and adde them together, that is to say, first, for 4 pence there is taken* $\frac{1}{3}$ *of 563: which comes to 187 ℔—8 ℔, as appeareth by the worke and for 3 d there is taken the* $\frac{1}{4}$ *of the whole sum, which amounteth to 140 s—9 d. Both which products added together, doe make 328 s—5 d: and so much comes 563 elles to at 7 d the* elle.

3 Item, for the first 8 d there is taken for 4 d *the* $\frac{1}{3}$ *of the whole summe, and another* $\frac{1}{3}$ *for the other 4 d, which added together, as in the example doth evidently appeare, amounteth to 74 s—8 d.*

Againe for the second worke of 112 ℔ *there is taken first the* $\frac{1}{3}$ *of the whole summe for 6 d which comes to 56 s: then for that*

2 d

2 d you haue to take $\frac{3}{6}$ of the whole summe, or if you will, the $\frac{1}{2}$ of the product that came to 6d either of which maketh 16 ß—8 d. These two summes being added together, doe make 74 ß 8 D, as in the third example appeareth.

4 Item, for 9 D there is taken for 6 pence *the $\frac{1}{8}$ of the whole summe: and the $\frac{1}{4}$ of the whole summe for 3 d, or otherwise for the 3 D you may take the $\frac{1}{8}$ of the product that came of 6 D because 3 pence is the $\frac{1}{4}$ of 6 D: which added together, as plainely appeareth in the* fourth example, *amounteth to 267 ß 0 D.*

Item for 10 D, first there is taken for 6 D *the $\frac{1}{8}$ of the whole summe, which amounteth to 397 ß—6 D. Then for 4 D there is found 265 ß both which added together, make 662 shillings 6 D, as appeareth in the* fift example: *It may also be wrought* as appeareth by the second note in the Table, *by 4 D twice taken, and the $\frac{1}{8}$ of the product of 4 pence: or else by the $\frac{1}{8}$ of the whole summe, &c.*

3. Rule. Item, for 11 D, there is first taken the $\frac{1}{8}$ for *6 pence: then the $\frac{1}{4}$ of the whole sum for 4 d: Lastly, that $\frac{1}{4}$ of the last product for 1 d. All which 3 summes added together, maketh in shillings 6944. 8 D, and in pounds 347, 4 ß 8 D.*

Item, likewise by the same reason, when you will multiply (by shillings) any number that is vnder 20 ß, you shall haue in the product pounds, if you know the euen or aliquot parts of 20, which are here in this little

little Table set downe to sight.

Item s̃. $\left\{\begin{matrix}10\\5\\4\\2\\1\end{matrix}\right\}$ *is the* $\left\{\begin{matrix}\frac{1}{2}\\\frac{1}{4}\\\frac{1}{5}\\\frac{1}{10}\\\frac{1}{20}\end{matrix}\right\}$ *of one pound.*

So that for 20 s̃, which in the $\frac{1}{2}$ of a pound, *you may take the* $\frac{1}{2}$ *of the number which is to be multiplied, and you shall haue in your product pounds: if an vnite doe remaine, it shall be worth ten* shillings.

Likewise for 5 shillings you must take the $\frac{1}{4}$ *of the number which is to be multiplied: and if there doe remaine any* vnites, *they shall be fourth parts of a* Pound, *euery vnite being in value* 5 shillings.

For 4 shillings take the $\frac{1}{5}$ of the number *which is to be multiplied: and if there doe remaine any* vnites, *they shall be fift parts of a* Pound, *each vnite being in value* 5 shillings.

For 2 shillings you must take the $\frac{1}{10}$ of the *number to be multiplied, wherefore to take the* $\frac{1}{10}$ *of any number, you must cut off the last figure of the same number (which is neerest your right hand) from all the other figures with a small right downe line or dash with a pen, and so haue you done: for all the other figures which doe remaine toward your left hand from the same figure that you doe separate, shall bee the saide* $\frac{1}{10}$ *of a* Pound: *and that Figure so separated towardes your right hand,*

hand, shall be so many peeces of 2 s the peece, the which figure you must double to make ther of the true number of shillings, as by the example shall appeare.

Finally, for 1 shilling needeth smal worke, *for it is so many shillings as he proponed in the sum, which to bring into pounds, hath been already taught in the* first rule.

Example.

At 10 s the peece,
What are 6543 peeces worth?

1/3 ℔—3271—10 *s.*

At 5 s the Elle,
What comes 4373 Elles to?

1/4 ℔—1093—5 *s.*

At 4 s the yard,
What — 7839 yards?

1/3 ℔—1567—16 *s.*

At 2 s the pound waight
What are 752| 7 pound waight worth.

1/10 ℔—752—14 *s.*

At 1 s the peece
What comes —757|3 peeces to?

1/20 ℔—378—13 *s.*

Nextly

NExtly, now followeth in order to bee vnderstood, that if the number of shillings be not some euen or od aliquot part of 20 you must then conuert the same number of shillings into the aliquot parts of twenty, and thereof make two or three products, as need shal require: which done, adde them together, and bring them into Pounds. And here for thy furtherance I haue set downe a note of the order of their parts, as they are to be taken.

s				s		
3	of	2 & 1	or	5.&1.	13	of	10.2&1
6		4 & 2			14		10.[illegible]4
7		5 & 2			15		[illegible]
8		4 & 4		5.2.1	16		[illegible] 1
9		5 & 4		4.4.1	17		[illegible] 2
11		10 & 1			18		[illegible] 4
12		10 & 2			19		[illegible] 4

For 3 s according to the tenor that you see *is expressed in the Table, you must first take for* 2 shillings *the* $\frac{1}{10}$ *of the number that is to be multiplied. Then for one* shilling *you must take the* $\frac{1}{2}$ *of the product which did come of the same* $\frac{1}{10}$ *part: which two summes added together produceth the effect desired.*

Item, for 6 shillings according to the note *set forth in the Table first for* 4 *s I take the* $\frac{1}{3}$ *of the number that is to be multiplied: Then for* 2 *s the* $\frac{1}{2}$ *of the product that came of* 4 *s, and adde them together.*

Or

Or else as appeareth also in the Table, for 5 shillings *you may take the* $\frac{1}{2}$ *and the* $\frac{1}{5}$ *part of the product that came of* 5 shillings, *and adde them together.*

Item, for 7 s̄, first for - s̄ take $\frac{1}{4}$ of the product *that is to be multiplied, then for* 2s, *take the* $\frac{1}{10}$ *of the number that is to be multiplied, and adde them together &c.*

Item, for 8 s̄, according to reason, and the *intent of the* Table, *for the first* 4 s̄ *take the* $\frac{1}{3}$ *of the product, and the same number againe for the other* 4 *shillings, & adde them together.*

Item, for 9 shillings: first for 5 shillings, *take the* $\frac{1}{4}$; *then for* 4 *shillings take the* $\frac{1}{3}$: *and adde them together.*

Otherwise as you see by the intent of the Table, *worke twice for* 4 *shillings as was taught euen now for* 8: *and then take the* $\frac{1}{4}$ *of the last product for the* 1 *shilling: but* 5 *and* 4 *is the shorter.*

Item for 11 s, first dispatch 10 s: for which *you must take the* $\frac{1}{2}$ *of the product, then lastly for* 1 *shilling take the* $\frac{1}{10}$ *part of the sum produced of the* $\frac{1}{2}$ *of the product, & adde them together.*

Item, for 12 shillings, where I will end with the first part of my Table. *First take the* $\frac{1}{2}$ *for* 10 shillings. *And then for* 2 shillings *take the* $\frac{1}{5}$ *of the sum that came of* 10 *shillings, take & adde them together, or else if you please for* 2 *shillings, you may take the* $\frac{1}{10}$ *of the whole given number.*

To

To write more of the manner of taking the true parts, I omit. The desirous practitioners will no doubt conceyue it. Also the Table *is some ayde to helpe the vnperfect, whereupon by and by I will set downe three or foure of these notes* in Examples, *and the rest I will leaue to thine owne industrie and practise, to labour vpon.*

This is the order most commonly vsed in practise, when the number of shillings is not an aliquot *part of a pound. But* (louing Reader) *after I haue touched the euen or aliquot parts of a pound that falleth out in pence and shillings, I will deliuer two new rules, that shall drowne this common order quite and cleane: wherein shall be comprehended, in one line, or working both euen and odde parts of shillings vnder 20, without regard whether it bee an* aliquot, *or not an* aliquot *part: which two Rules* (when they come in place,) *I committe to thy Friendly iudgement in working.*

Now follow the examples vpon the notes before said.

At 6 shillings the yard.

What	3215	*yards?*
4 *shillings*	643	
2 *shillings*	321	10

£ 964---10 *shillings.*

Other-

Otherwise by Multiplication of 6,

3215 ———

6 shillings	1929	0
l ———	964 —	10 shillings.

At 7 shillings the Elle
What ——— 4563 Elles?

5 *shillings*	1140 —	15
2 *shillings*	456 —	6
l	1597 —	1 *shilling.*

Otherwise by multiplication of 7.

4.563

7 s̄	3194	1

1597—1

At 8 s̄ the peece, what 7562 peeces?

4 s̄	15124 .	12
4 s̄	15120 .	12
l	3025 .	4s

Otherwise by multiplication.

7563

8 s̄	60504	
l	3025 —	4 s̄

At 13 s̄ the peece, what 401 peeces?

10 s̄	200 .	10
2 s̄	40 .	2
1 s̄	20 .	1
l	260 .	13

Other-

Otherwise by Multiplication.

	401
13 s	1203
	401
	521 | 3
	l, 260, – 13 s.

These and such like Questions of compound numbers, which I haue here in this fourth rule for orders sake set downe, for that it hath beene heretofore a common course of work I account but superfluous. For in the eight & ninth rules of this my simple Addition shall appeare, that the giuen price of any euen or odde number of shillings, either vnder or aboue 20, shall bee wrought at one or two workings at the most, how difficult soeuer the question be.

Item there resteth yet a kind of practise, how to bring Pence into Pounds at the first working: whereupon you must vnderstand that 240 pence maketh one pound, or 20 s. In consideration whereof I cut off the last figure or 0, and there remaineth but 24 (of which 24) 8 d is the $\frac{1}{3}$ part thereof, 6 d is the $\frac{1}{4}$ part, 4 d the $\frac{1}{6}$ part, and 2 pence is the $\frac{1}{12}$ part thereof.

Whereupon if it were demaunded what 1486 *yards or pounds of any thing commeth to, at* 8 *pence the yard, in pricking or cutting off the first figure towards your right hand for the* 0 *that appertaineth to* 240: *There is remaining of the saide summe* 148, *whereout I taking the* $\frac{1}{3}$ *part, and it commeth to* 49℔, *and there resteth* 1, *which* 1 *I put to the* 6, *that I pricke or cut off, and it maketh* 16 *peeces of* 8 *pence, which I double to make into groats, & they make* 32, *whereof the* $\frac{1}{3}$ *part maketh* 10 ß, *and there remaineth* $\frac{2}{3}$ ß, *which is* 8 *pence, whereby it followeth, that the* 1486 *yards at* 8 *pence the yard, maketh* 49℔ 10 ß 8 ð, *as by the example shall appeare.*

Item for 6 *pence, take* $\frac{1}{4}$ *part of the number from the prickt figure: and if any vnites doe remaine they are so many sixe pences, whereof taking the* $\frac{1}{3}$ *they are shillings, if there doe remaine yet one, it is in value sixe pence.*

Item for 4 ð *take the* $\frac{1}{4}$ *part of the number from the prickt figure: If any vnites remaine, they are so many groats, which to conuert into shillings, take the* $\frac{1}{3}$ *part: And if any thing yet remaine, they are thirds of shillings, each one in value being worth* 4 *pence.*

Item for three pence, take the $\frac{1}{4}$ *part from the prickt figure, if any vnites remaine, they are so many peeces of* 2 *pence, wherof in taking the* $\frac{1}{3}$ *part, maketh shillings: If any thing yet remaine, they are fourth parts of shillings*

each

each one being in value 3 pence.

Item for 2 pence, as appeareth also by the table, take the $\frac{1}{12}$ part of the number from the prickt figure: if any thing remaine, they are so many peeces of 2 pence, which by taking the $\frac{1}{6}$ part, you shall turne into shillings: and if any vnities remaine, they are so many sixt parts of shillings, or peeces of two pence, whether you will.

If one cost 8 pence,
What are 148 | 6 worth?

maketh pounds 49 — 10 — 8 *pence*

If one cost 6 pence,
What 780 | 5 worth?

maketh pounds 196 — 12 — 6 *pence*

At 4 pence the yard,
Who are 873 | 6 yard worth?

maketh pounds 145 — 12 — 0

If one cost 3 pence,
What are 987 | 4 worth?

maketh pounds 123 — 5 — 6 *pence,*

At 2 pence the Elle
What comes 789 | 4 Elles to?

maketh pounds 65 — 15 — 8

BVt if your number of pence be not an aliquot or euen part of 24: then must you 6. Rule.

6. Rule. Bring them into the aliquot parts of 24, and make thereof diuers products which must be added together, as by the question hereafter following shall appeare.

Item for 5 d. *first take for* 3 d, *then for* 2 d, *and adde them together according to the instruction of the* second Rule: *Or else first take for* 4 d, *then for* 1 d.

Item for 7 d, *first take for* 4 d: *then for* 3 d, *and adde them together.*

Item for 9 d, *first take for* 6 d, *then for* 3 d, *and adde them together.*

Item for 10 d, *first take for* 6 d, *then for* 4 d, *and adde them together.*

Item for 11 d, *first take for* 8 d, *then for* 3 d, *and adde them together: as by these* examples *following doth appeare.*

Examples.

1. *If one yard cost* 5 *pence, what are* 759 | 6 *worth?*

4 *pence*	126—12
1 *peny*	31—13
maketh pounds	158— 5

Otherwise.

1— 5—759 | 6

3 *pence*	94—19
2 *pence*	63— 6
maketh pounds	158— 5 s

2. *If*

2. *If one cost* 7 *pence, what are* 98 | 7 *worth?*

4 *pence*	16—9—
3 *pence*	12—6—9
maketh pounds	28—15—9 *pence.*

Otherwise.

1	7	98 | 7
6 *pence*		24—13—6
1 *peny*		4 — 2—3
maketh pounds		28—15—9 *pence.*

3. *If one cost* 9 *pence, what are* 98 | 7 *worth?*

6 *pence*	24—13—6
3 *pence*	12— 6—9
maketh pounds	37—00—3 *pence.*

Otherwise.

1	9	98 | 7
6 *pence*		24—13—6
3 *pence*		12— 6—9
maketh pounds		37—00—3

4. *If one cost* 10 *pence, what are* 98 | 7 *worth?*

6 *pence*	24—13—6
4 *pence*	16— 9—0
maketh pounds	41— 2—6

5. *If one cost* 11 *pence, what are* 98 | 7 *worth?*

8 *pence*	32—18—0
2 *pence*	12—6—9
maketh pounds ——	45—4—9

But if you haue any shillings and pence to be multiplied together: Then are you to take for the shillings according to the instruction of the third rule: and for the pence according to the first rule before mentioned: vnlesse you can spie the aduantage thereof, and thereby helpe your selfe: as appeareth in this second example, where first I worke for 6 ℔: *which is to be rebated out of the giuen number, and I haue* 719 ℔ 11 ß *my desire.*

At 19 ß—6 ℔ *the yard what* 738 *yards?*

	738	*Otherwise by Rebating.* 728
10 ß	369—	
5 ß	184—10	
4 ß	147—12	6 ℔ 18 —9 ß
6 ℔	18—9	℔ 719—11 ß
℔	719-11 ß	

The like againe is done by rebating, as by these 2 *examples appeareth.*

At 18 ß *the Elle what* 418 *Elles?*		*At* 16 ß *the Elle what* 517 *Elles.*	
2 ß	41 —— 16	4 ß	103—8
℔	376 —— 4 ß	℔	413—12 ß

And

And now I will touch a little the euen parts of a pound, that falleth out in pence & shillings, wherof for those parts you shal take such like part of the giuen number that is to be multiplied, as the price of that giuen number beareth in proportion to a pound, which also for their better aide is here set downe. 7. Rule.

1 s	8 d	} is the	} part of a £
2.	6		
3.	4		
6.	8		

Item, first for 1 shilling 8 pence take the $\frac{1}{12}$ *part of the giuen number, and if any thing doe remaine, they are twelue parts of a pound, each one being in value 1 shilling 8 pence.*

Item for 2 shillings 6 pence take the $\frac{1}{8}$ *part of the number that is to be multiplied. And if any thing doe remaine, they are eight parts of a pound, each one being in value 2 shillings 6 pence.*

Item for 3 shillings 4 pence, as appeareth by the Table, you must take the $\frac{1}{6}$ *part of the giuen number and if any thing doe remaine they are 6 parts of a pound, each one being in value 3 shillings 4 pence.*

Item for 6 shillings 8 pence, take the $\frac{1}{3}$ *part of the number that is to be multiplied: And if any vnities doe remaine, they are thirds of*

a pound, euery one being worth 6 shillings 8 pence.

Other infinite numbers there are, that may be reduced by abreuiation into the proportionate parts of a pound: *as 16 shillings 8 pence maketh $\frac{5}{6}$: which 16 shillings 8 pence is easily reduced into groats by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats: Then set 60 the groates of a pound vnder 50, cutting off the 2 ciphers, as is here performed in the margent. And then haue you brought 16 shillings 8 d into the knowne parts of a* pound, *which maketh $\frac{5}{6}$.*

16	8
3	
5	0
6	0

But yet gentle Reader, for thy further instruction, I haue hereunto annexed in a Table, how pence and shillings beareth proportion to a pound, which I commit to thy friendly beneuolence: it will be some aid vnto the vnground-ded practitioner: but I count him the best workman that can presently reduce his giuen price into the knowne and proportionate parts of a pound.

A Table of the aliquot parts of a pound, or 20 shillings.

s'	d'	l'
0	2	1/120
0	3	1/80
0	4	1/60
0	6	1/40
0	8	1/30
1	0	1/20
1	3	1/16
1	8	1/12
2	0	1/10
2	6	1/8
3	0	3/20
3	4	1/6
3	9	3/16
4	0	1/5
5	0	1/4
6	0	3/10
6	3	5/16
6	8	1/3
7	0	7/20
7	6	3/8
8	0	2/5

s'	d'	l'
8	4	5/12
8	9	7/16
9		9/20
10		1/2
11		11/20
11	3	9/16
12	8	7/20
12		3/5
13		13/20
13	4	2/3
13	9	11/16
14		7/10
15		3/4
16	8	5/6
16		4/5
17	6	7/8
17		17/20
18		9/10
18	4	11/12
18	9	15/16
19		19/20

Here

Heere follow 4 examples vpon the foure notes deliuered.

At 1 s 8 d *the yard*
What are 3884 *yards worth?*

maketh pounds — 323 — 13 — 4 *pence.*

At 2 s — 6 d *the yard*
What are 4563 *yards worth?*

maketh pounds 570 — 7 — 6 *pence.*

At 6 s — 8 *pence the Elle*
What comes 7562 *Elles to?*

maketh pounds 2520 — 13 — 4 *pence.*

Now by custome you are able to worke by all sorts of sums being deliuered in shillings & pence, as one shilling one peny. 2 shillings two pence, 3 shillings 3 pence, & so of all other: wishing you to haue some considerations of your questions, when they are set downe, for there are many subtill abbreuiations and great aduantages to be gotten, and easily to be perceiued.

As 3 s — 8 d *of* 2 s *and* 1 s 8 d.

4 s — 2 d, *of* 3 s — 4 d, *and* 10 d, *which* 10 d *is* $\frac{1}{4}$ *of* 3 s — 4 *pence.*

5 s — 8 d *of* 4 s — 1 s 8 d:

5 s 10 d, *of* 5 s *and* 10 d, *which* 10 d *is* $\frac{1}{6}$ *of* 5 *shillings.*

And by this meane when you haue taken one product, you may oftentimes vpon the same

same take another more briefly then vpon the summe which is to be multiplied, &c.

NOw (gentle Reader) that you haue seen the vertue of the euen or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise hereafter following a briefe and easier method for any euen number of shillings, either vnder or aboue 20, then euer yet hath bin published: Notwithstanding Mr. *Humfrey Baker*, whose trauell is worthy commendation, & whom for knowledge sake I reuerence, hath in some part touched this first part, though not in this method. The worke of the rule both pleasant, ready and briefe, as by the variety of the examples deliuered thereupō shall appeare And first I will set forth a question thereby the better to expresse or teach you the order thereof: which is this. 8. Rule.

If one cost 6 s, what 8574?

1	6	8574
maketh pounds	2572	4 5̄

To the vnderstanding of this example, after you haue set downe your giuen number in forme of the rule of 3, with a line drawne vnder it you shall present y set a pricke vnder your first figure 1, towards your

your right hand drawing from the pricke, as heretofore hath been practised, a little short line, thereto set downe the *shillings* anon, which done, multiply the first figure 4 by 6, the value of your price, (which here you see standeth in sight aboue the line) it maketh 24, which is 1 *pound* 4 *shillings*. The 1 *pound* keep to carry to the next place, and the 4 *shillings* set downe at the end of the prescribed line towards your right hand. Thus haue you done now with 6 aboue the line, and also with 4 in the first place, (for the pricke vnder 4 doth signifie that 4 hath done his office.) Then secondarily for a generall rule take but the $\frac{1}{7}$ of the giuen price, which here is 3, which 3 is the number that shall now continue the rest of the multiplication and end the wor e: whereupon I multiply 3 into 7, standing in the second place it maketh 21, and with the 1 *pound* I kept in mind 22; set down 2 and keepe 2 in mind, working according to the rule of multiplicat ō, deliuering the tenths in mind in their due place, which done, the product from the pricke to your left hand repr senteth the pounds, and the other at the end of the line the shillings, as appeareth by the examples.

Note a generall rule.

If one yard cost 2 s, *what* 7563?

1	2	7563

maketh pounds 753 —— 12 s.

If

If one yard cost 4 s, *what* 8729?
1 4 8729

maketh pounds 1758—8 s

If one peece cost 6 s, *what* 9937
1 6 9537

maketh pounds 2861——2 s

If one cost 8 s, *what* 7509?
1 8 7509

maketh pounds 3003—12 s

If one cost 12 *what* 5794
1 12 5794

maketh pounds 3476— 8 s

If one cost 14 s *what* 30705?
1 14 30705

maketh pounds 2593—10 s

If one cost 18 s, *what* 5703?
1 18 5703

maketh pounds 5132— 14 s

If one cost 22, s *what* 9537
1 22 9537

maketh pounds 1048— 6 s

Let

Let these suffice (gentle Reader) for an entrance into euen numbers. And now I will shew the like rule for any odde or vneuen part of a pound.

9. Rule. TO helpe you to the vnderstanding of these other questiõs that hereafter follow: where in my first example the giuen number is 6487 at 3 s the yard: I multiply 3 aboue the line into 7, it maketh 21. The one shilling is set downe, and the 1 I keep. Now am I to take the ½ of three, which because it is an odde number I cannot. Therefore I shall keepe & continue my multiplication by three still, and worke by the ½ of the rest of the giuen figures or number, to wit, 648. And first the ½ of 8 which is 4 multiplied into 3, make the 12, there to ioyne the 1 I keep in minde, it maketh 13: set down 3, keepe one. Then againe multiply by two the ½ of foure, it maketh sixe, & with one in minde it maketh 7. Then lastly take the ½ of sixe, which is 3, saying 2 times 3 is 9, which 9 set downe, and so is the question answered, as appeareth by the practise, and the examples following.

At 3 s the yard what 6487?

1	3	6487	
maketh pounds —		973 —	1 s

If

If one yard cost 5 s, what 4269? if one yard cost 5 sh

1	5	4269
maketh pounds		1067—5 s

At 7 s the elle, what 6489?

1	7	6489
maketh pounds		2271 ——3 s

If one elle cost 9 s, what 2807?

1	9	2807
maketh pounds		1263 ——3 s

If 11 s the Pistolet, what 8263?

1	11	8263
maketh pounds		4544 – 13 s

If one peece cost 13 s, what 4629

1	13	4629
maketh pounds		3008 ——17 s

But now note (gentle Reader) when the giuen price falleth vpon an odde number, as 3,5,7,11,13, &c. then it is to be presupposed that the giuen sum to be multiplied must be a sum made of euen numbers as 2, 4,6,8,10,&c. else cannot that question be wrought at one line or working.

Prouiding alwaies that it may beare an odde figure in the first place towards your right hand as appeareth in these 6 exãples, which last were wrought, and such like, &c. which

which may beare an odde number for the price, and be done at one line or working very well.

But if the giuen price be an odde number, and the summe to be multiplied odde numbers also: then can it not bee done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall rule. First worke for the euen number, contained in that question: or giuen price according as you haue learned, and then afterwards for the 1 odde shilling, take the $\frac{1}{2}$ of the sum giuen to bee multiplied, omiting the first prickt place, as was taught for the working of one shilling in my first rule of Practise, and adde those 2 together, and you shall haue your desire.

A generall rule.

Example.

At 3 s the yard
What are the 7539 yards worth?

2 s	753 ——	18
1 s	376 ——	19
maketh pounds	1130 —	17 s

At 7 s the elle, what 7539?

1	7	7539	
6 s		2261 ——	14
1 s		376 ——	19
maketh pounds		2638 —	13

At

At 13 s *the yard, what* ——— 7534?

1	13	7534.
12 s.		4520—8
1 s		376—14
maketh pounds		4897—.2

And thus haue I abridged into these two Rules, , how to bring any number of shillings whatsoeuer they be into pounds, with a briefer Method then euer yet hath beene published, which I commend vnto thy friendly censure and iudgement in the vse and practise thereof.

If one cost 6 s 5 d, *what* ——— 1239?

1	—6 s—	5 d	—1231
6 s			369—6
4 d			20—10—4
1 d			5—2—7
maketh pounds,			394—18—11

At 14 s 2 d, ——— *what* 2825?

1	—14—2	——— 2825.
14 s,		1977—10
2 d.		23—10—10
maketh pounds		2001—00—10

At 16 s 4 d, *what* ——— 2531?

1	—16—4	——— 2531.
16 s.	2024	——— 16
4 d.	42	——— 3—8 d
maketh pounds	2060	——— 19—8

At 3 s. the Pistolet, what 8325?

1	3	8325.

maketh pounds —— 1248 —— 15 s.

At 7 s the crowne, what 6529?

1	7	6529

maketh pounds —— 2285 —— 3 s.

At 9 s the peece, what 6567?

1	9	6565.

maketh pounds —— 2955 —— 3 s

These three last questions may seeme some-
10. Rule. *thing harde, yet they are easie anough, if you*
marke them well: if I should explaine them,
then are they too easie. Therefore I leaue them
to whet the mindes of the desirous.

ITem when any one of the summes, which is to be multiplyed, is composed of many denomominations, & the giuen number but of one figure alone: then shal you multiplie all the denominatiõs of the other summe by the same 1 figure, beginning first with that sum which is left in value toward your right hand, & bring the product of those pẽce into shillings, & the prodnct of the shillings nto pounds, as by this example appeareth.

At 2 £ 7 s. a yard, what are 9 worth?

matketh pounds -- 30 —— 6 s̃ —— 0 d.

11. Rule. BVt if in any of the summes that are to be multiplyed ther be a brokẽ number. First worke

work for the whole according to the instructions that you haue learned, and then take such part of the giuen price, as that broken number beareth in proportion to the price, as in the example. After you haue wrought for 3 s and for 6 d, then are you to take the ½ of 3 s. 6 d, for the ½ yard, & adde that to the summe: So adding all the 3 propucts together, which makes 43 l, 2 s, 9 d, the iust price of 246½ ells: & thus must you do of al other.

At 3 s, 6 d, the elle, what 246 ½ ?

1	3	6	246½.
3 s.		6	18
6 d.		6	36
½			1—9
maketh.			43—2—9

At 16 s. 4 d. the peece, what 14¾ ?

1	16.	4	14¾.
16 s		11	4
2 d		0—4	2
¾		12	
maketh pounds		12—0	1

If one peece cost 4 l, —3 d— 6¼ d
What are 12 peeces worth ?

4 l	48	
3 s	1—16	
6 d		
¼		6
maketh pounds	50—2—6 d	

The Proofe.

If 12 *peeces cost* 50 *l.* — 2 *s.* — 6 *d.*
What one peece?

50 — 2 — 6

Maketh pounds 4 — 3 — $6\frac{1}{2}$

12 Rule. ITem, touching the manner how to vnderstand the order of this question, and others the like, first seeke how many times 12 is contained in 50, which is 4 times, and so resteth 2 pound, which 2 pound conuerted into shillings, and ioined with the other 2 shillings, maketh 42 shillings, wherin is found 12, three times, resteth 6 shilings, which turned into Pence, putting thereto the 6 pence in the first place, it maketh 78: wherein 12 is found 6 times, resteth 6 *d.*, which containeth 12, but $\frac{1}{2}$ a time: Put that $\frac{1}{2}$ to the 6 pence: and then the solution is 4 *l.*, 3 *s.*, $6\frac{1}{2}$, as appeareth by the practise thereof.

13 Rule. Item, the like is to be done of any thing that is bought or sold after fiue score to the hundreth, or the Quintall. As for example.

If 100 *pound cost* 27 *l.* — 13 *s.* — 4 *d.*
what one pound?

27*l.*

	27 ℓ — 13 s̃ — 4 d̃		
	20		
s̃	5	53	
		12	
	1	10	
	5	3	
d̃	6	4	0
		10	$6\frac{2}{5}$

Maketh 5 s̃, $6\frac{2}{5}$ d̃.

I haue wrought this at length for the aide of the young learner, because hee should vnderstand how all the Multiplication is set downe.

	27 ℓ — 13 s̃ — 4 d̃		
	20		
s̃	5	53	
		12	
d̃	6	40	
		$100\frac{2}{5}$	

Maketh 5 s̃, $6\frac{2}{5}$ d̃.

But to worke it more neately it is by a little vnderstanding ended thus,

Item, to the vnderstanding of this and such like questions, the right downe line is all the guide, which is pulled downe close by 20, as you see in the example, where 27 pound 13 shillings is reduced all into shillings, and maketh 553 shillings.

The 5 towards the left hand being separated with the hanging or right downe line, is the iust number of shillings that answereth to the question. Nextly, 53 shillings is multiplied by 12, to reduce them to pence, putting to the 4 d̃, it yeeleth for the multi-

plication of the first figure two, too: the one beyond the line towards the left hand, is 1 peny towards the rest of the price: then 53 also multiplied by 1 yeeldeth 53: but the 5 behind the line toward the left hand, is also 5 pence more, towards the price, which 1 and 5 I adde together vnder the line: it maketh 6 d. So is there found now as appeareth by the Titles of shillings and Pence, 5 shillings 6 Pence.

Finally, I come now on this side the line towards the right hand, and vnder 12 I find first 10, and then 3, which added together maketh 40, vnder which 40 you must put the 100, & it maketh $\frac{40}{100}$, which abreuiated cõmeth to $\frac{2}{5}$. So the iust price of one pound after 5 score to the hundred makes 5 s 6 $\frac{2}{5}$ d.

One example more, & so I will leaue this rule.

If 100 *cost* 10 $\frac{1}{4}$ d.

What ——— 9874?

100 — 10$\frac{1}{4}$ — 0874.

6 d	246	17
4 d	164	11—4
$\frac{2}{4}$ d	20	11—5
$\frac{1}{4}$ d	10	5—8$\frac{1}{2}$
	42	5—5$\frac{1}{2}$

Maketh. £ 4 | s 8 | d 5

20

45

12

45$\frac{1}{2}$ | 91

100 | 100 *parts of a peny.*

Alſo

Also the like may bee done of the vsuall weights here in England (which is 112, for euery hundreth weight) in case you know the aliquot parts of a hundreth weight, which are these 56 ℔, 28 ℔, 14 ℔ and 7 ℔. For 56 ℔ is the ½ of 112 pound, 28 ℔ is the ¼ of 112 ℔, 14 ℔ is the ⅛ and 7 ℔ is 1/16 part.

Therefore for 56 ℔ take the ½ of the summe of money that 112 ℔ weight is worth.

For 28 ℔ take the ¼ of the summe of money that 112 ℔ weight is worth.

For 14 ℔ take the ⅛ of the summe that 112 ℔ is worth.

And for 7 ℔ the 1/16 of the summe of money that 112 ℔ is worth.

As for example: At 17 ℔ 19 s the hundreth pounds weight, that is to say, the 112 pound, what shall 3 quarters and 7 pound cost?

1 — 17 ℔ — 19 s — 3 — 7			
2 *quarternes*	8	19	6
1 *quarterne*	4	9	9
7 *pounds*	1	2	¼
Mseth pounds	14	11	¾

The second Chapter intreateth of the Reduction of diuers measures to others value by Rules of Practise.

NOw wil I shew a few examples of practise in reducing of measures as Elles, Yards, Braces, Pawnes of Genes, &c. Much 14 Rule.

more would I haue touched, but that I feare the booke will rise to too great a volume.

In 864 elles of Antwerpe, how many yardes of London?

ITem in these and such like questions of Flemmish measure, to bee brought into yards Englisth: first take the $\frac{1}{3}$ of the giuen number, as appeareth in the first example towards your left hand. Then take the $\frac{1}{2}$ of that product, or the $\frac{1}{4}$ of the giuen number & adde those 2 products together, they shall be yards Englisth: as by the Example you may perceiue.

The second Example toward your right hande, is yet briefer then the first, whose worke is this; Take the $\frac{1}{4}$ of the deliuered number, and that product subtract out of the giuen number, and the rest sheweth your desire. Of these two wayes vse which you thinke best.

The

The proofe.

In 648 yards of London:
How many elles of Antwerpe?

648
216

maketh 864 *Elles of Antwerpe.*

ITem for the vnderstanding of this worke: first take the $\frac{1}{3}$ part of the yards of London; which found, adde that $\frac{1}{3}$ part and the yards together, as appeareth by the practise, and the product shewech the elles of Antwerpe. 15. Rule.

Item, { *In 320 yards of London,* / *How many elles of Antwerpe?*
maketh ——— 426 $\frac{2}{3}$ *Elles.*

320 yards.	Proofe.
$\frac{1}{2}$ 106 $\frac{2}{3}$	426 $\frac{2}{3}$ *Elles.*
426 $\frac{2}{3}$ *Elles.*	$\frac{1}{2}$ 156 $\frac{2}{3}$
	320 *yards.*

Other Productions.

ITem, you shall vnderstād, that forasmuch as six braces of Millan make fiue Elles of Antwerp, whereupon according to the *rules of Practise* you may reduce the one into the other, by the like reasons aforesaide, in taking the $\frac{1}{6}$ part, and then subtract the same, to make Elles of Antwerpe, And againe by the contrary, taking the $\frac{1}{5}$ part with adding the giuen number, to turne the Elles to Braces, as for Example. 16. Rule.

In

In 876 Braces, how many Elles of Antwerp?

876 *Braces*	*The contrary.*
146	730 *Elles Flem.*
Els 730 *Antwerpe.*	146
	876 *Braces.*

Ellet 730 *Antwerpe.*

182½

Yards 547½ *English.*

Thus appeareth, that 876 *Braces by practise, make* 730 *Elles Flemmish: which Elles Flemmish reduce into English yards.*

17 Rule. *So againe vpon the same first question of Braces, I would know how many yards English they make.*

After the rate that 100 Braces are worth 62½ yards.

876 *Braces.*

438

109½

Answere 547½ *yards.*

Item to the vnderstanding of this worke, & such like, first take the ½ *of the giuen braces, and after take the* ¼ *of that halfe, or the* ⅛ *of the giuen number, and adde them together, and the products are also yards English.*

18 Rule. ITem three Elles of Rochell make 5 Elles at Lisbone. So likewise three Elles at Lyons

ons make 5 Elles at Antwerpe.

To worke these and such like, double the Elles of Lyons, and the Elles of Rochel, and from their products subtract the $\frac{1}{10}$: and the rest shall be the Elles of Antwerpe, or the Elles of Lisbone.

Example.

In 63 elles of Lions how many Elles of Antwerpe?

63
63
126
2) 21

Answ. 105 *elles Ant.*

In 100 Elles of Rochell, how many Elles of Lisbone.

100
200
3) 33 $\frac{1}{3}$

Answ. 166 $\frac{2}{3}$ *els of Lisb.*

Touching the proofe or returne of these and such like questions for a generall rule, you shal first take the $\frac{1}{10}$ *of the giuen number: and adde that* $\frac{1}{10}$ *and the giuen number together, and the* $\frac{1}{2}$ *of that product shall be your desire.*

Example.

In 105 Elles of Antwerpe, how many Elles of Lyons?

105
2) 21
2) 126

An. 63 *Els of Lions.*

In 166 $\frac{2}{3}$ *Elles of Lisbone, how many Elles of Rochell?*

166 $\frac{2}{3}$
3) 333 $\frac{1}{3}$
2) 200

Answ. 100 *els of Roch.*

The

The third Chapter teacheth of the order and work of the Rule of 3. in broken number after the trade of Merchants, degressing something from M. *Records*, which is comprehended in three RVLES.

NOw that I haue somewhat intreated of the Rules of Practise, I will giue a few instructions, after my simple order, for the working of the Rule of three in broken numbers, wherein I shall need to say the lesse, because I hope the studious learner, that hath trauelled any thing in the *Grounds of Arts*, is not vnfurnished of knowledge capable to vnderstand me.

But before I deliuer any instructions for broken numbers, I will propone a question, which shall be wrought three sundry waies, thereby to shew as it were three degrees of comparison; how farre the Rule of three in broken, for more speed of worke, differeth from the whole, which I rather set downe for a view, that the studious herein may be more desirous to attaine broken, leauing any more to discourse in dialogue forme, but only to giue instructions where need is: and in the rest to put forth the questions with their answere:

My

My first question is this.

If one yard cost 6 s, 8 d, what are 789 worth at that rate? The first way.

1 — 6 — 8 ——— 789
12 80
——— ———
80 63120 d

Here the product of the summe are pence, according to the nature of the middle number.

11
137 | 1
63120 | 5260
11222 | 2220 (263
111 |

I answere — 263 l. The second way.

1 ——— $6\frac{1}{3}$ ——— 789
——— ——— 20
3 20 ———
15780

Here the product of the summe are s, according to the nature of the middle number.

1
15780 | 5260 | *I answere* 263 l.
3333 | 2220 |

The third way.

1 — $\frac{1}{3}$ — 789
——— ———
3 1 1
———
789

Here

Here the product is pounds, according to the title of the second number.

x
789
333 (263
I answere 263 £.

Now that you haue seene the 3 former vertues of the Rule of three, whose products haue first brought forth d, next s, and lastly pounds, I will deliuer three notes in order following: and with them a dozen questions that shall shew the worke of the Rule of 3 in broken numbers or Fractions.

Note these three Rules.

1 The first foure shall be sundry questions of a fraction comming in the second place.
2 The second foure shall be of two fractions comming in the second or third place.
3 The third foure of Fractions in all three places.

Notes vpon the first Rule for a fraction comming in the second place.

My first question is this.

1 Rule. If one yard cost me 3 s 4 d: what are 756 worth at that price.

The first variety. *In setting downe the question to performe the worke, I turne foure pence into the part of a shilling, which is ⅓, and then the question standeth thus.*

1 ——— 3⅓ ——— 756

To

To the ready working of this Queſtion, and all ſuch other like, my firſt note is this, which take for a generall Rule: that when any one Fraction ſhall come, eyther in the ſecond or third place, that the Denominator of that Fraction or Fractions, muſt alwayes bee brought vnto the Number, or Numerator of the firſt place: and thereby multiplie the one into the other. A generall Rule.

And this beneſite is alwayes gotten by the vertue of bringing the Denominator of the ſecond Numbers Fraction vnto the firſt place: For the Fraction in the middle Number is now releaſed: and the product that commeth of the multiplication, is of the nature and like Denomination of the whole Number in the ſecond place which here are ſhillings. Note this.

Where vpon nowito worke the queſtion, I bring 3, *the Denominator of the Fraction in the ſecond place, vnto my firſt Number* 1, *with a line ſet vnder thus* $\frac{1}{3}$ *thus: and the* 3. *vnder it thus* $\frac{3}{1}$: *ſaying, once* 3, *is* 3 *my Diuiſor: that done, reduce* $3\frac{2}{3}$, *ſaying,* 3 *times* 3 *is* 9, *and the other* 1 *ouer* 3 *make* 10: *my ſecond number in the rule of three: by which* 10 *I doe multiplie my laſt number* 756, *as appeareth by the wirke thereof, and it yeldeth* 7560 *ſhillings, my Dividend.*

Then diuiding 7560 *by* 3 *my Diuiſor, it yeldeth in quotient* 2520 *ſhillings, which maketh* 126 *pounds, as apreareth heere moſt plainely, both*

both by the example and the worke.

At 3 s 4 d the yard, what 756 yards?

1 ——— 3⅓ ——— 756

3 ——— 10

10 ——— 7560 s

r	r	
7560	2520	126.
3333	2220	

I answere 126 *l.*

Yet otherwise vpon the same question, altering the price now into the proportion it beareth to a pound, for the 3 s 4 d is ⅓ part of a Pound: which example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

The second variety.

1 — ⅓ — 756

1 — ⅓ — 756 6 1 1

756 *Pounds.*

As soone as I haue carried 6 the denominator of my middle number vnto my first place, as before hath beene taught, I pull downe one, the numerator of 6, with a line vnder 6 thus, $\frac{1}{6}$, *and that one in custome I pull downe in sight, being the figure that I will multiply my third or last number by, according to the tenor of the rule of three. And because one can neither*

neither multiply nor yet diuide (though here it is set downe in forme of multiplication, the rather for your vnderstanding) the product of the multiplication according to the declaration of this my first Rule or note, is conuerted into the title of my second number, which here are pounds. Now followeth the diuision performed by my Diuisor 6, to make an end of that question. Note.

```
13
756 (126 : which maketh 126 ℓ, as before.
666
```

And thus much for the variety in working that question.

And now followeth another.

My second question.

If one yard of Cotten $8\frac{1}{4}$ ℔, what 859?

```
1 ——— 8¼ ——— 859
4     33      33
              2577
              2577
             28347

        |1
   2(3  |21     |1(1      ℓ    s    ℔
2834 7  |708 (6 |5  9(0(29 — 10 — 6¾
444  4  |122  2 |2  20
        11        Gg
```

This

This question was also wrought like the first, and bringeth forth 29 ℓ—10 s—6¾ d, the price of 859 yards.

My third question.

If 7 pounds of any thing cost 3 ℓ—10 s, what comes 987 pounds to?

	ℓ	
7	$3\frac{1}{2}$	987
2	7	7
14		6909

6909|493 $\frac{7}{14}$|$\frac{1}{2}$. I answere 493 ℓ—10 s.
14.

Notes vpon my second rule for two fractions comming in the second and third place.

My first question is this.

IF one Elle cost 13 s—4 d: what halfe a quarter or $\frac{1}{8}$ of an Elle?

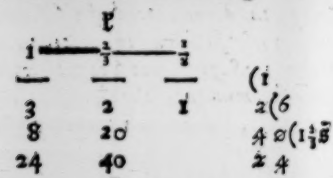
Answer. *First bring 13 s—4 d, into the parts of a pound, which is $\frac{2}{3}$, and then will the question stand thus:*

1—$\frac{2}{3}$ ℓ —$\frac{1}{8}$

ITem for the performance of this worke doe as before was taught in the first Rule: first bring

bring 3 the denominator of the second fraction vnto your first number, setting a line vnder it thus $\frac{1}{}$: Saying, once 3 is 3: that done bring 8 the denominator of the third fraction, setting it vnder 3, and multiply them together, saying, 3 times 8 maketh 24, which 24 is your diuisor. (Now haue you done with the denominator 3, and also with the denominator 8.) Therfore you shall put a line vnder thus $\frac{3}{}$. And the like line also vnder 8, setting or pulling downe vnder them their owne numerators, that is 2 vnder 3, and also 1 vnder 8, as appeareth in the example, which numerators for a generall rule are euermore to be pulled downe of custome in sight, to multiply the one by the other, according to the tenor of the rule of three. Then I multiply the one by the other, saying: once 2 is two, which signifieth 2 ℔ being of the nature and like denomination of the middle number, which 2 ℔ is to be reduced into shillings, otherwise it cannot be diuided by my first number 24. 2 Rule.

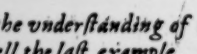
Then diuiding 40 by 24, the quetient bringeth forth $1\frac{2}{3}$ ß. So much is $\frac{2}{3}$ of an elle worth after that rate. Otherwise although 2 pound could not be diuided by 24, yet it might haue beene abreuiated to $\frac{1}{12}$ of a pound: which is worth 1 ß 8 ð, as before.



Second queſtion.

IF one pound of any weight coſt 13 ſhillings 4 pence, what are 7/8 of the pound worth after that rate.

Anſwer. *Reduce the 13 ſhillings 4 pence into the parts of a pound: which is 2/3, and then will the queſtion ſtand thus.*



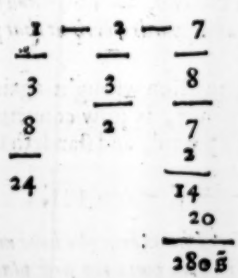
ITem *for the vnderſtanding of this, if you marke well the laſt example, this and the reſt lieth open, and need ſmall inſtruction. For as you did laſt, ſo now againe bring the denominator of the ſecond and third fraction vnto the firſt figure 1, multiplying the one into the other, which maketh alſo 24 your diuiſor.*

Then making a line vnder 2 thus 3 and a line vnder 8 thus 8 and pulling downe their Numerators vnder each figure, that is, 2 vnder 3, and 7 vnder 8, which as I ſaid before for a generall rule I pull downe of cuſtome in fight, to bee the two numbers that of duety ought

Note:

ought to be multiplied together, which done I bring 2 being the lesser figure vnder 7, multiplying them together, it maketh 14: which are of the nature of the middle number: that is to wit pounds: which 14 cannot aptly be diuided among 24, therefore are reduced into shillings, as is plainely to be seene in the example: then 280 shillings parted among 24, yeeldeth for his quotient 11 s, 8 d your desire, and the iust price of $\frac{7}{8}$ of an elle. Otherwise 14, though it could not be diuided by 24, might by mediation or diuision in broken numbers haue beene diuided or abreuiated to $\frac{7}{12}$, which in effect being reduced to his knowne parts, maketh 11 s, 8 d as before. But my good will and meaning is to aid young beginners: therefore haue I reduced the 14 £ into shillings, which is the easier way.

Now followeth the example.



The third Example.

If one yard coſt me 2 ſ—6 d, what 345 ¼ yards?

Anſwer. *Firſt put 6 d into the parts of a ſhilling, and then the queſtion ſtandeth thus.*

1 ——— 2 ½ ——— 345 ¼

Item, to the ready vnderſtanding of this and all ſuch like, according as before hath beeu declared, bring the Denominators of the ſecond and third fractions vnto the firſt place, multiplying them the one into the other, all which make 8 for the diuiſor common. Then next reduce your ſecond number: ſaying, two times 2 is 4, and 1 is 5, as was taught in the example aforeſaid. Laſtly reduce your third number 345 ¼ all into fourths, and they make 1381: which 1381 is to be multiplied by 5, according to the tenor of the Rule of three: which done, maketh 6905 ſ: and diuided by 3 your Diuiſor, yeeldeth in quotient 863 ½ ſ, which maketh in pounds 43 ℔—3 ſ, —1 d ½: and ſo much are the 345 and ¼ yards worth at that price.

The ſame queſtion wrought againe by 2 ſhillings 6 pence, is now conuerted into the parts of a pound, and ſtandeth thus:

1 ——— ⅛ ——— 345 ¼

Item, after I haue brought here my ſecond and third fraction vnto my firſt place, and found 32 to be my diuiſor: hauing thus furniſhed

ſhed my firſt place with al things vnto him belonging (which is meant of bringing and multiplying the Denominators of the ſecond and third fractions into him) I then goe in hand to ſee what is to doe in my ſecond place, where preſently of cuſtome I pull downe my Numerator 1 *vnder* 8: *being the figure in ſight that ſhall multiply my third number.*

Then laſtly I reduce $345\frac{1}{4}$ *all into fourths as afore was practiſed, which maketh* 1381, *the which* 1381 *I am to multiply by* 1 *my ſecond number, they are nothing increaſed, but by the* Metamorphoſis *of my worke they are now* 1381 *pound, being of the nature of the middle number, as I haue often ſhewed you, which diuided by* 32 *my diuiſor, yeeldeth* 43 *pound and* $\frac{5}{32}$ *which* $\frac{5}{32}$ *of a pound reduced into knowne numbers, make* 3 *ſhillings,* 1 $\frac{1}{2}$, *as before.*

Example.

		1		
		105		
1 — 1 — 345 1/4		1381	(43	5
32 1		322		
1381		3		32

NOw follow 4 other queſtions, which are in all three places broken numbers, or whole and broken together. 3 Rule.

Item, first for the finding out of your Diuisor: For you shall take this for a most certaine and generall rule: That you must multiplie the Numerator of the first number in the question, by the Denominator of the second: And that product againe, by the Denominator of the third: And the totall thereof shall bee your Diuisor.

Secondly, for a generall rule to finde out your Dividend, multiplie the Denominator of the first number by the Numerator of the second and the whole thereof by the Numerator of the third. And the totall thereof shall euermore be your Diuidend.

Now for an example, I propone this question, therby to make my meaning the more plaine, and to shewe you as I haue done in the rest, the maner and order of the worke.

If $\frac{2}{3}$ of any weight or measure coſt $\frac{5}{6}$ of a poũd, or 20 ſ, what are $\frac{3}{8}$ of the like weight or measure worth after that rate?

Example.

$\frac{2}{3}$ — $\frac{5}{6}$ — $\frac{3}{8}$

ITem for the more plainer vnderſtanding hereof and all other the like, in broken numbers: First you ſhall pull downe 2, the Numerator of the first Number or Fraction, with a line vnder 2 thus: that done, according as you haue

haue learned before, bring 6. the Denomi-ator of the second Fraction, and set it vnder 2, multiplying the one into the other, which maketh 12. Then lastly bring 8, the Denominator of the third fraction, and set it vnder 12, multiplying that 12 by 8, which amounteth to 96. or else for more briefe, multiplie 6 by 8, saying: 6 times 8, makes 48, which 48 set vnder 2, and multiplie the one into the other, it maketh 96. as before. And this 96. is the first number in the rule of three. That shall alwayes for a most generall rule be your Deuisor.

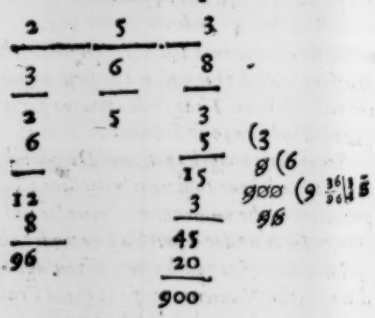
Secondly, to worke for your Dividend, you shall (as hath bene sufficiently declared before) pull down 5, the numerator of your second fraction, & set it vnder 6, with a line vnder thus $\frac{5}{6}$.

That done, (as you knowe,) you are to pull downe 3, the Numerator of the third Fraction, and set it vnder 8, with a line vnder it thus, $\frac{3}{8}$: *multiplying the one into the other, according to the tenour of the Rule of three, which maketh 15. Then according to my note, forget not to bring the Denominator of the first Fraction, which is 2, vnder 15, and multiplie them together, which maketh 45, which 45 is your Dividend. Which 45, are of the nature or denomination of the Middle number, as I haue taught you before. And therefore are 45 ℔. which aptly cannot bee diuided by 96. Therefore you shall reduce the 45 ℔ into* $\frac{3}{8}$: *as you see performed in the Example, which*

amoun-

amounteth to 900 ℔, which diuided by 96 your diuiſor, it yeeldeth 9 ℔, and $\frac{36}{96}$ of a ſhilling, which in leſſer termes is $\frac{3}{8}$, which $\frac{1}{4}$ in money maketh 4$\frac{1}{2}$ d: and ſo much will the aforeſaid $\frac{1}{4}$ coſt, as by the worke following ſhall appeare.

The Example.



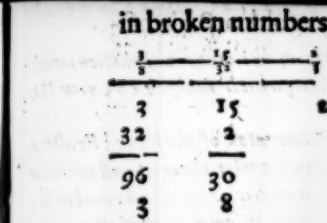
Otherwiſe though 45 could not be diuided by 06, yet by Diuiſion in broken numbers it might haue beene abreuiated to $\frac{15}{32}$ of a ℔, which reduced into knowne parts, will make 9 ℔ 4$\frac{1}{2}$ d, as before.

Now my ſecond example ſhall be the proofes of this queſtion.

If $\frac{1}{4}$ yards coſt $\frac{15}{16}$ of a pound or 20 ſhillings what ſhall $\frac{1}{7}$ coſt?

Anſwer. *Worke as was taught you before, and you ſhall haue your deſire.*

Here



Here as appeareth by the worke, the multiplication being ended, 240 *is to be diuided by* 288, *which to some perchance may seeme hard, yet notwithstanding is the worke good. Therefore abreuiate* 240 *by* 288, *as you see here is practised: and the end of your abreuiation shall come to* $\frac{5}{6}$ *your desire,* $\frac{240}{288}\ \frac{120}{144}\ \frac{60}{72}\ \frac{20}{36}\ \frac{5}{6}$.

Otherwise. 240|120|60|20|5
288|144|72|36|6

Otherwise. 240|40|5
288|48|6

The third question.

If $\frac{1}{4}$ Elles cost 13 s—4 d, what 156 $\frac{3}{4}$ Els?
Answer. *To worke this question the shortest way: reduce* 13 s—4 d *into the parts of a pound, which is* $\frac{3}{4}$.

Then as you did afore, after you haue set downe the question, the numerator of the first fraction 3 *is pulled downe vnder* 4, *and the*

Deno-

Denominators of the other 2 Fractions multiplyed into him, which maketh 18, your Diuisor.

Then the Numerator of the second Fraction two is pulled down, vnder three of custome now in sight, ready to multiplie my third number by: which is performed as soone as the last number 156 $\frac{1}{2}$ is reduced into halfes.

Then lastly I multiply that product by 4. the Denominator of my first Fraction: it yeeldeth 1504, whch I diuide by 18, and my quotient is 139 l, and $\frac{1}{9}$ of a pound remaining, which is worth 2 s 2 $\frac{1}{3}$ d: And so much will 156 $\frac{1}{2}$ ells cost, as by the worke following doth appeare.

3	2	156 $\frac{1}{2}$	1	
4	3	313	47 (2	
3	2	2	176	l.
6		626	250 4 (139 $\frac{1}{3}$	
18		4	188	
		2504	11	

The fourth question.

If 2 $\frac{1}{2}$ Elles cost 1 $\frac{1}{3}$ l, What commeth 29 $\frac{1}{4}$ Elles to?

Item, to the workemanship of this question: first reduce your second Number to one direct Number, in saying, three times 1 is 3. and 2 is 5: Then bring the multiplcation of the Denominators of the second and third Fractions,

ent, which maketh 12: and multiply that 12 by fiue your first numerator, and it maketh 60, which is your diuisor.

Then the reduction of the second number, which is 5 multiplied by 117 the product of the last numbers reduction, make 585, which 585 yet resteth to be multiplied by 2, the denominator of the Fraction in the first place yeeldeth 1170: which diuided by your Diuisor 60, yeeldeth 19 pounds, 10s, as appeareth by the worke thereof.

Thus hauing now touched the 12 questions whereof I first pretended, which with diligence and oft practise, I trust are sufficient to aide the desirous vnto the working of any broken numbers, I will now intreate of diuers necessary Rules incident vnto trafficke, as hereafter followeth.

The 4 Chapter treateth of Losse and gaine in the trade of Merchandise.

IF one yard cost 6 s—8 d, and the same is sold againe for 8 s—6 d: the question is, what is gained in 100 Pounds laying out on such commodities.

Answer. The rule of three direct, applieth two manner of wayes to doe the same, the one is to say: if $6\frac{2}{3}$ giue $8\frac{1}{2}$, what giueth 100?

Multi-

Multiplie and diuide, and looke what your quotient bringeth foorth aboue your laying out, is the neare gaines, and the solution to your question: If you followe the worke, your quotient will bring forth 174 l'—10 s. *which is* 27 l', 10 s *more then your principall, and so much is gained in the* 100 *pounds laying out.*

Item to worke it the other way, which I take the neareſt, ſeeke the difference betwixt the iuſt price and the other price, which is one ſhilling 10 *pence. Then ſay by the rule of three:*

If 6 $\frac{2}{3}$ s. gaine 1-$\frac{5}{6}$-s, what ſhall 100 l', gaine? Multiplie and diuide, and you ſhall finde 27 l'—10 s, and ſo much is gayned in 100 l' laying out.

You may vſe which of theſe two wayes you thinke good.

The proofe.

If a yard of Cloth bee deliuered for 8 s 6 d, wherevpon was gayned after the rate of 27 l' 10 s, in 100 pounds laying out: The queſtion is, what the yard coſt at the firſt hand?

Anſwer. *Put your Gayne to* 100 l', *all maketh* 127 l'--10 s: *Then ſay, If* 127 l'-10 s, *giue but* 100 *pound, what gineth* 8 $\frac{1}{2}$ s? *Worke, and you ſhall finde* 6 s-8 d, *the true ſolution to your queſtion.*

Yet an other Example, or proofe vpon the firſt queſtion.

If one yard cost 6 s 8 d. the question is, what price the same is to be sold again, for to gaine, 27 l'—10 s, in 100 l' laying out?

Answer. *Say by the Rule of three, if* 100 l' *gaine* 27 l'—10 s, *what gaineth* $6\frac{2}{3}$ s? *Multiplie and divide, and you shall finde* 8 s—6 d, *your true solutions.*

If one Elle cost 7 s 8 d, and be sold again for 8 s 6 d: *question.* What is gayned in 20 l' laying out in such commodities?

Answer. *Seeke the difference betwixt the iust price, and the other price, which is* 10. *pence, and then applie the Rule of three, as before is taught, saying: If* $7\frac{2}{3}$ s *gine* $\frac{5}{6}$ *shillings, what gineth* 20 l'? *Multiplie and divide, and you shall finde* 2 l' $3\frac{13}{23}$ s, *and so much is gayned in* 20 l' *laying out.*

The proofe also by an Example of losse.

A Merchant hath bought Holland-cloth at 8 s 6 pence the Elle, which prouerh not to his expectation, wherupon he is content to lose 2 l' 3 s $\frac{11}{23}$ s in 20 pounds laying out. The question is, what price ought to be made of the cloth abating thid losse?

Answer. *Doe as before in Gaynes hath bene taught, putting* 2 l'—3 $\frac{11}{23}$ s *to your* 20 l', *all together beer mak:th* 22 l'—$2\frac{11}{23}$. *Then say by the rule of three: If* 22—3 $\frac{11}{23}$ s, *gine but* 20 l', *what*

what ſhall come of $8\frac{1}{2}$ *s? worke and you ſhall finde* 7 *s*—8 *d*, *the iuſt price that the Elle ought to bee ſolde for after the rate of this loſſe.*

That it appeareth euidently, as in company the Rule is appliable as well to gaine as loſſe.

If $20\frac{1}{4}$ yards coſt 36 *l*—10 *s*, how ſhall I ſell the ſame againe to gaine $\frac{1}{9}$ of the principall, or to make of 34: which is a lone.

Anſwer. *By the Rule of three, if* 3 *doe gine* 4, *what will* $30\frac{1}{2}$ *gine? Multiply and deuide, and you ſhall find* $40\frac{2}{3}$ *l. Then ſay againe, if* $20\frac{1}{4}$ *yards doe gine* $48\frac{1}{3}$ *pounds, as well principall as gaine, what will one yard be worth at that price? Multiply and deuide, and you ſhall find* 2 *l*, $8\frac{16}{24}$ *s.*

If one Elle of cloth coſt me 8 *s*—8 *d*, and afterwards I ſell $10\frac{1}{2}$ Elles thereof for 5 *l*, 13 *s*—4 *d*, I would know whether I winne or loſe: and how much vpon the 100 *l* of money.

Anſwer. *See firſt at* 8 *s*—8 *d the Elle, what* $10\frac{1}{2}$ *elles comes to, and you ſhall find* 4 *l* 11 *s, and I ſold the ſame for* 5 *l*—13 *s*-4 *d, ſo that I did gaine vpon the* $10\frac{1}{2}$ *Elles* 22 *s* 4 *d. Then if you could know how much is gained in the* 100 *l*, *I ſay by the Rule of three, if* 4 *l*-11 *s did gaine* 22 *s* 4 *d, what will* 110 *l*

gaine?

gaine? Multiply and diuide, and you shall find 24 pound — 10 ß — 10 ß $\frac{10}{21}$. And so much is gained in the 100 pound of money.

If 12 $\frac{1}{2}$ yards cost me 11 pound fiue shillings, and I sell the yard againe for 16 shillings, the question is, whether I doe win or lose, and how much in or vpon the pound of money.

Answere. *Looke what the 12 $\frac{1}{2}$ yards come to at 16 ß the yard, and you shall find 10 pound. But they cost 11 pound 5 shillings. So there is lost vpon the whole 1 ℔, 5 ß. Then to know how much is lost in the pound, say by the rule of 3. if 11 $\frac{1}{4}$ pound doe lose 1 $\frac{1}{4}$ pound, what will 1 pound lose? Multiply and diuide, and you shall find 2 ß, 2 ß $\frac{2}{3}$, and so much is lost in the pound of money.*

If I sell the 100 weight of any commodity for 4 pound, whereupon I doe lose after 10 pound in the 100 pound, I demand how much I shal lose or gaine in the 100 ℔? if in case I had sold the same for 4 pound — 10 ß.

Answere. *Say, if 90 pound yeeld 100, how much will 4 gine? multiply and diuide, and you shall finde 4 $\frac{4}{9}$. Then say againe if 4 $\frac{4}{9}$ gine mee 4 $\frac{1}{3}$, what will 100 come to? Multiply and diuide, and you shall find 101 pound $\frac{1}{4}$ which is more then 100 pound by 1 pound 5 shillings. And so much is gained in the 100 pound.*

A Merchant hath sold Currants for the sum of 430 pound, & he hath gained therin

after 10 pound in the 100 pound. The question is to know how much he gained in all.

Answere. *Say by the Rule of three. If* 100 *pound doe gaine* 10 *pound, what will* 430 *pound gaine? Multiply and diuide, and you shall find* 43, *and so much hath he gained in all.*

If one yard be worth 28 ¼ s, for how much shall 10 yards be sold to gaine after 8 l 6 s 8 d, in the 100 Pound.

Answer. *First adde* 8 l – 6 s – 8 d, *to* 100. *Then say, if* 100 l *doe giue* 108 ⅓ *for principall and gaine, what will* 28 ¼ s *principall yeeld? Multiply and diuide, and you shall find* 30 ⅞ s. *Then say againe by the Rule of three, if* 1 *yard doe giue* 30 ⅞ s, (*which is as well the principall as the gaine*) *what shall* 10 *yards giue? Multiply and diuide, and you shall find* 15 l 8 s – 9 d. *And for the same price shall the* 10 *yards be sold, for to gaine after the rate of* 8 l 6 s — 8 d *vpon the* 100.

A branch or proofe out of this Question.

A Merchant hath solde clothes for 15 l 8 s – 9 d, and he hath gained in the whole, the sum of 11 l — 3 s — 9 d. The question is to know how much hee hath gained in the 100 pound.

Answere. *To know this, first rebate the gaines from the price, and there will remaine* 14 l,

14l' 5 s, 0 d. *Then say by the rule of three direct, if* 14l' $\frac{1}{2}$ *giue me* 1 l, 3 $\frac{1}{4}$ s, *what will* 100l' *giue? Multiply and deuide, and you shall find* 81,–6 s–8 d, *the effect desired, the proofe is apparant in the question before.*

Yet another Branch or Proofe of the first Question.

If 10 yards be deliuered for 15l' 8 s 9 d, whereupon was gained after the rate of 8l' 6 s, 8 d, vpon the 100 l, the question is, what the yard did cost at the first hand.

Answer. *First, say by the Rule of three, if* 10 *with principall and gaine yeeld* 15l' 8 $\frac{1}{4}$ s, *what shall* 1 *yeeld? Multiply and deuide and you shall find* 30 $\frac{7}{8}$ s. *Then say againe by the rule of three, if* 108 $\frac{1}{3}$ *principall and gaine giue but* 100, *what shall* 30 $\frac{7}{8}$ s *of principall and gaine yeeld? Worke, and you shall find* 28 $\frac{1}{3}$ s. *And so much did the yard cost at the first Penie.*

If one yard cost 36 s, how much shall 12 yards be solde for to gaine after the rate of 10l' in the 100?

Answer. *First, say, If* 100 *giue* 110 *pound principall and gaine, what will* 36 s, *giue? Multiply and deuide, ad you shall find* 39 $\frac{3}{5}$ s. *Then say againe by the Rule of three. If* 1 *yard of principall and gain yeeld* 39 $\frac{3}{5}$ *shillings,*

what shall 12 yeares gaine? Multiplie and diuide and you shall finde 23 l'—15 $\frac{1}{5}$ s, which $\frac{1}{5}$ s in knowne number is 2 $\frac{1}{5}$ d. And for the same price shall the 12 yards be sold, to gaine after the rate of 10 in the 100.

The proofe.

If 12 yards be sold for 23 l'—15—2 $\frac{1}{5}$ d, whervpon is gained after 10 l' in the 100 l' The question is, what the yard cost at the first pennie?

Answer. *First say, if 12 giue 23 l' 15 $\frac{1}{5}$ s, what one yard? Multiplie and diuide, and you shall finde 49 $\frac{1}{5}$ s. Then say againe by the rule of 3 if 100 pounds giue but 10 $\frac{1}{2}$, what shall 3 $\frac{1}{2}$ s giue? Worke and you shall finde 36 s, the iust price of the yard at the first hand.*

Item, when one Merchant selleth wares to another, and hee giueth to the buyer 11 l'-6 s—8 d, vpon the score, of 20 l': The question is, How much shall the buyer gaine vpon the 100 pound after that rate:

Answer. *First adde 11 l'—6 s—8 d, vnto 20 l', and they are 21 $\frac{1}{3}$. Then say, if 20 pound giue 21 $\frac{1}{3}$ what shall 100 giue? Multiply and diuide, & you shall finde 106 $\frac{2}{3}$. So the buyer getteth after the rate of 6 $\frac{2}{3}$ l', vppon the 100 pound.*

Gentle Reader, other necessary questions appertaining

pertaining to Losse and Gayne, you shall haue in the eight Chapter of this Treatise.

¶ The fifth Chapter entreateth of Losse and Gayne vpon time, wrought by the double Rule of three, or by the Rule of three compounded: which is contayned in 4. *Speciall selected branches or questions of diuers formes; each one of them springing from the first question, and each one of them also, being a proofe to other, &c.*

IF one yard cost me 2 s. — 8 d. ready money, & after I sell the same again for 2 s. 10 d. to be payd for it at the end of 3 moneths: he queſtion is, what I gaine vpon the 100 l. in 12 moneths?

Answer. *First say, if 2 $\frac{2}{3}$ gayne $\frac{1}{6}$, What shall 100 l. gaine? Multiply and diuide, and you shall finde 6 $\frac{1}{2}$ l. Then say againe, by the Rule of three, if three moneths gaine 6 $\frac{1}{2}$ pound, what shall 12 moneths gaine? Worke, and you shall finde 25 l. and so much shall I gayne in 12 moneths after that rate,*

Item, you may also worke it all at one working by the first part of the rule of 3 composed saying

ſaying, if 2 ½ *in three moneths do gaine* ¾ *of a ſhilling (which is* 2 D.*) what will* 100 l. *gaine in* 12 *moneths: which for thy further encouragement the worke of this one example I haue here put downe, to veryfie that I affirme in the firſt part of this* Ground of Arts, *that this rule and ſo all others more reioiceth in* Broken *then in* Whole.

s	moneths	s,	l,	moneths.
2 ½	— 3 —	1 ½	— 100 —	12
8	z			100
3	3			1200
24	x 8 z			3
6	3 6 0 0 (25			
144	x 4 4 4			3600
	x 4			

Where the multiplication and the diuiſion being ended, maketh 25 l. *your deſire.*

If a yard bee deliuered for 2 s.—10 D. to be paid at 3 moneths, whereupon was gained after the rate of 25 l. in the 100 for 12 moneths, the queſtion is now what the yard coſt at the firſt hand?

Anſwere. *Firſt ſay, if* 12 *moneths gaine* 25 l. *what ſhall three moneths gaine? worke and you ſhall finde* 6 ¼ l. *Then ſay againe the ſecond*

second time, if 106 ¼ l, *gaine but* 100, *what shal* 2 s 8 *gaine? Worke, and you shall find* 2 s, 8 d, *which is the iust price that the yard cost at the first hand.*

If one yard of cloth cost mee 2 s — 8 d ready mony, for what terme shal I sell the same againe for 2 s — 10 d, so that I might gaine after the rate of 25 pound vpon the 100 pound in 12 moneths.

Answer. *First say, if* 2 ⅔ *gaine* ⅙ *what shall* 100 *pound gaine? Multiply and diuide, and you shall find* 6 ¼ *pound. Then say againe for the second worke, if* 25 *pound be come of* 12 *moneths, what shall come of* 6 ¼? *Worke, and you shall find three moneths, the iust terme of time that the cloth ought to bee deliuered at* 20 s, 10 d, *to gaine* 25 *pound vpon the* 100 *pound in* 12 *moneths.*

If one yard cost me 2 s 8 d, ready money, for what price shall I sell the same againe to be paid at the end of three moneths, so that I may gaine after the rate of 25 pound in the 100 pound for 12 moneths.

Answere. *First say, if* 12 *gaine* 25 l, *what shal three moneths gaine? Multiply and diuide, and you shall find* 6 ¼ l. *Then say for the second worke, if* 100 l *gaine* 106 ¼, *what gaineth* 2 ⅔ s? *Worke and you shall finde* 2 s — 10 d, *and for that price must the yard be sold to gaine after* 25 *pound in the* 100 *pound for twelue moneths.*

Many other of these questions I might here haue deliuered, but for feare the Booke would rise to too thicke a volume, and so to make the price so much the dearer whereby it might not be so portable to my Country-men as I wish it. But these 4 I haue of purpose framed in this order, hauing relation one to another, assuring you that what question soeuer may be proposed within the compasse of this Rule, you shall find by one of these 4, to make a solution. And moreouer, diuers other are yet to be deliuered: where the Creditor giueth diuers daies of payment, which can neuer be well wrought, nor yet vnderstood, vnlesse you can first find by art the iust times that all those payments, how different soeuer they be ought to be paid at once: whereupon first I thinke good here to giue some instructions into such a rule, for it is the onely aide for the finishing of such questions as hereafter shall follow.

The

The sixth Chapter intreateth of Rules of Payment which is a right necessary Rule, & one of the chiefest hand-maides that attendeth vpon buying & selling, &c.

Example.

A Merchant doth owe a sum of money, whereof the $\frac{1}{3}$ is to be paide at 6 moneths, the $\frac{1}{4}$ at 8 moneths, and the rest at a yeere. If he would pay at one payment, the question is what time ought to bee giuen him.

Answere. *I haue omitted the quantity of the summe, for you shall vnderstand, the Rule is appliable, and yeeldeth a true solution to what sum soeuer shall be proponed. But now for orders sake in teaching, I doe imagine the sum to be 60 pounds whereupon the manner of this worke is to multiply the proportionate part of the money by the time, as in company. Then* 20 *being the first payment, and the* $\frac{1}{3}$ *of* 60, *which* $\frac{1}{3}$ *multiplied in broken numbers by* 6 *his time of payment maketh* $\frac{6}{1}$, *which in whole numbers, as appeareth by the example in the margent, maketh two moneths: Next* 20, *which is the* $\frac{1}{3}$ *multiplied by his terme* 8, *yeeldeth* 4 *moneths, then the rest which*

$\frac{1}{3}$	*by*	$\frac{6}{1}$	2 *Moneths*
$\frac{1}{4}$	*by*	$\frac{8}{1}$	4 *Moneths*
$\frac{1}{6}$	*by*	$\frac{12}{1}$	2 *Moneths*
			8 *Moneths*

is 101, *must needes bee abreuiated into the proportion it beareth to* 60, *which is* $\frac{1}{2}$ *which* $\frac{1}{2}$ *multiplied by his time* 12 *moneths, produceth* $\frac{12}{6}$, *maketh two moneths. All which added together, as appeareth in the Margent, maketh eight moneths, which is the iust time that all those payments ought to bee paide at once.*

A Merchant hath 800 l to pay, the $\frac{1}{6}$ thereof ready mony, the $\frac{1}{4}$ at two moneths, the $\frac{1}{3}$ at foure moneths, and the rest at a yeer. The question is, if hee would pay all at one paiment, what time ought to be giuen him.

Answere. *The ready money is neuer multiplied: then* $\frac{1}{4}$ *multiplied by two moneths as you did before maketh* $\frac{1}{2}$, *then* $\frac{1}{3}$ *by* 4 *produceth* 2 *moneths as appeareth here in the margent. But now for the rest of the mony you cannot multiply it vntil you haue sought what proportion it beareth to* 800 *pounds. Therefore you must subtract the ready mony, the* $\frac{1}{4}$ *and the* $\frac{1}{3}$ *out of the principall. The rest will be* $66\frac{2}{3}$ l, *which you must looke what part it beareth to the principall, which you shall finde to be* $\frac{1}{12}$, *the same you must also multiply by his time* 12 *moneths, and it yeeldeth* 1 *moneth, so all mkt* $3\frac{1}{2}$ *moneths, as appeareth in the margent.*

A Mer-

A Merchant is to pay 1200 l' in three termes, that is to wit, 400 l' at two weekes and 600 l' at foure moneths, laſtly, 200 l, at 5 moneths: The queſtion is, in what time they ought to be paid at once.

Anſwer. *Proportionate the parts, and you ſhall find that 400 is $\frac{1}{3}$ part, and for 600 you ſhall find $\frac{1}{2}$, and likewiſe 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you ſhall haue $\frac{2}{3}$ weekes, more eight weekes, and laſtly three $\frac{1}{3}$ weekes, which together maketh 12 weekes or 3 moneths your deſire.*

A Merchant is to pay 600 l in three termes, whereof 100 l, is paid preſent, more 300 l' at 20 dayes, and the reſt at 5 moneths, accounting 30 dayes to a moneth. The queſtion is, what time ought theſe payments to be paid at once.

Anſwere. *Worke and you ſhall find two moneths.*

The

The seauenth Chapter intreateth of buying and selling in the trade of Merchandise, wherein is taken part ready money, & diuers dayes of payment giuen for the rest, and what is won or loſt in the 100 l. forbearance for 12 moneths more or leſſe, according to the quantity of money, or proportion of time, &c.

A Merchant hath bought Sattens which coſt 8 s. the yard ready money. And he ſelleth the ſame againe to another man for 10 s. the yard, but he giueth two dayes for the payment, that is to ſay, three moneths for the one halfe, and fiue moneths for the other halfe. The queſtion is to know how much the ſeller doth gaine vpon the 100 pound in 12 moneths after that rate.

Anſwere. *Seeke firſt by the Rules of payment, at what time thoſe two payments ought to be paid at once, and you ſhall find foure moneths, at which time the ſecond Merchant ought to haue paid the whole entire payment. And therefore ſay by the firſt part of the Rule of three compoſed:*

If 8 *ſhillings in* 4 *moneths doe gaine* 2 s. *what will* 100 l. *gaine in* 12 *moneths?*

8–4–2–100—12

4 12

32 1200 2400 | 75

2 232

2400

Mul-

Multiply and diuide, and you shall find 75 pounds, as appeareth in the example, and so much doth the first Merchant gaine vpon 100 pounds in 12 moneths.

A Merchant hath sold fifty clothes, at 9½ £ the peece, to be paid the one ⅓ at foure moneths, the ¼ at fiue moneths, and the ⅙ at 7 moneths, and the sellers mind is to take no more but after 8 pounds in the 100 for 12 moneths. The question is now what the first Merchant gaineth in the sale of these cloathes after that rate.

Answere. *First looke what the 50 cloathes come to at that price, and you shall find 475 pounds. Then secondly, according to your direction in the rules of payment, seeke at what time all the payments are to be performed at once. And you shall find 4½ moneths. Then thirdly say by the first part of the Rule of three composed. If 100 £ in 12 moneths gaine 8 £, what will 475 £ gaine in 4½ moneths? Worke and you shall find 15 £ and 5/9 of a pound, which is the neat gaines that the first Merchant hath after the rate aforesaid:*

A Merchant hath bought Holland at 7 s 3 d the elle ready money, and he selleth the same againe for 8 s 4 d the Elle, to be paid ¼ part in ready money, more ⅓ part at 2 moneths, and the rest at 4 moneths. The question

stion is now to know how much the first Merchant doth gaine vpon the 100 pounds in 12 moneths after the rate.

Answere. *According to the direction deliuered you in the rule of paiment the ready mony is not to be multiplied. Then working for the other two paiments, to finde out the true proportion at what time they ought to bee paid at once, you shall finde for $\frac{1}{4}$ at two moneths $\frac{1}{4}$ of a moneth. And the rest of the mony which is $\frac{6}{12}$ multiplied by his terme 4 moneths yeeldeth $1\frac{2}{3}$ moneths, both which added together make $2\frac{1}{3}$ moneths, the inst time that both the paiments ought to be performed at once. And therefore say by the first part of the Rule of three composed, if $7\frac{1}{3}$ in $2\frac{1}{3}$ moneths doe gaine $\frac{13}{240}$ of a £ what shal 100 pounds gaine in 12 moneths after that rate? worke and you shall finde 76 $\frac{13}{20}$ pounds. And so much doth he gaine vpon 100 l, in 12 months.*

A Merchant hath bought 30 cloathes at 6 pounds the peece for ready mony. Afterward he selleth 10 of them for 7 pounds the peece, for 3 moneths terme. And the other 20 he selleth for 8 pounds the peece for 4 moneths terme. The question is now what he gaineth vpon 100 pounds in 12 mouths.

Answere *First finde the value of the 30 clothes, which amount to 180 pounds. Secondly, seeke what the tenne peeces come to at 7 l, and what the twenty peeces come to at 8 l, the*

the one comes to 705, *and the other to* 160: *both which together make* 230, *which is* 50 l. *more then they cost. Thirdly, as I haue taught you in the Rule of payment, proportionate the first and second prices vnto the proportion they beare vnto* 230, *the product of their two prices, and you shall find* $\frac{7}{23}$ *for the first, and* $\frac{16}{23}$ *for the latter. Then fourthly, multiply those parts by their times, and you shall haue* $\frac{21}{23}$ *and* $\frac{64}{23}$: *both which together maketh* 3 *whole moneths, and* $\frac{16}{23}$ *of a moneth, which is the iust time that both those payments ought to be paid at once*

Then say by the first part of the Rule of three composed. If 180 l, *in* $3\frac{16}{23}$ *moneths doe gaine* 50 l, *what shall* 100 *gaine in* 12 *moneths? Multiply and diuide, and you shall find* 90 $\frac{10}{23}$ *pounds, and so much doth he gaine vpon* 100 *pounds in* 12 *moneths.*

A Merchant bath bought cinamon which cost him 9 s the l, ready mony. The questiō is now at what price he ought to sel the 100 waight. To wit 112 pounds to be paid the $\frac{1}{3}$ at two moneths, and the residue at the end of 3 moneths, so that he may gaine after the rate of 10 l, vpon 100 l, for 12 moneths.

Answere. *Seeke first by the rules of payment what terme both the payments ought to be paide at once, where the* $\frac{1}{3}$ *multiplied by his terme two moneths, maketh* $\frac{2}{3}$ *moneths.*

Like-

Likewise the next payment which is $\frac{3}{4}$ *multiplied by the terme 3 moneths maketh* $2\frac{1}{4}$ *moneths, both which added together, maketh* $2\frac{3}{4}$ *moneths, which is the time, that both the paiments ought to be paid at once. Then say by the Rule of three, if 12 moneths doe giue mee 10, pound, what will* $2\frac{3}{4}$ *moneths giue? Multiply and diuide, and you shall find* $2\frac{7}{16}$ *pound. Then say againe by the Rule of three. If one pound cost mee 8s, what will 112 pound cost? Multiply and diuide, and you shall finde 50l—8s. Then say once againe: If 100 pound doe giue* $102\frac{7}{16}$, *what will 50* $\frac{1}{2}$*l giue? Multiply and diuide, and you shal finde 51l—11s—*$1\frac{1}{7}$*d and for that price ought I to sell 112 pound of Cinamon to bee paid at two seuerall payments aforesaid to gaine therby after the rate of 10l vpon the 100l in 12 moneths.*

Briefe Rules for our hundreth waight here at London, which is after 112 pound for the 100.

Item who that mult plieth the pence that one pound waight is worth by 7 and diuideth the product by 15 shall find how many pounds in money the 112 pound weight is worth.

And contrariwise, he that multiplieth the pounds that 112l waight is worth by 15 & diuideth the product by 7, shall find how many

many Pence in money the 112 Pound weight is worth.

Example.

At 10 pence the pound weight, what is 112 pound weight worth?

Anſwere. *Multiply 10 by 7, and thereof commeth 70, the which diuide by 15, and you ſhall find 4⅔ ℔. And thus the 112 ℔ is worth 4 ℔ 13 ß — 4 ϑ, after the rate of 10 pence the Pound aforeſaid.*

At 6 ℔ the 112 ℔ waight, what is one Pound worth?

Anſwer. *Multiply 6 ℔ by 15, and thereof commeth 90: the which diuide by 7, and you ſhall find 12 ß 6/7. So much is one Pound worth when the 112 Pound did coſt 6 ℔.*

The eight Chapter intreateth of Tares and allowances of Merchandiſe ſold by weight, and of loſſes and gaines therein, &c.

AT 16 Pound the 100 ſuttle, what ſhall 795 Pound ſuttle be worth in giuing 4 Pound weight vpon euery 100 for treat?

Anſwer. Adde 4 vnto 100, and you ſhall haue 104. Then ſay by the Rule of three: If

104 bee worth 16 ℓ, what are 895 ℓ worth? Multiply & diuide, and you shall find 237 ℓ 13—10 $\frac{2}{13}$ ʊ, and so much shall the 895 pound weight be worth.

Item at 3 s 4 ʊ the pound weight, what shall 745 $\frac{1}{2}$ be worth, in giuing 4 l weight vpon euery 100 for treat.

Answer *See first by the rule of three what the 100 ℓ is worth, saying: If one cost 3 $\frac{1}{3}$ s, what 100? Multiply and diuide, and you shall find 16 ℓ $\frac{2}{3}$. Then adde 4 vnto 100, and they are 104. Then say againe by the rule of three if 104 be sold for 16 $\frac{2}{3}$ for how much shall 754 $\frac{1}{2}$ be sold for? Multiply and diuide, and you shall find 120 l. 18 s̄ 3 $\frac{3}{13}$ ʊ. And for so much shall the 754 $\frac{1}{2}$ be sold for at 3 s. 4 ʊ the ℓ in giuing 4 vpon the 100.*

Other necessary briefe rules there are for the finding of treats, or casting vp of chests of Suger, &c. which for that it is a mystery, I omit: If any lacke instruction that way, they shall find me ready to pleasure them.

Item, if 100 l be worth 36 s 8 ʊ, what shall 800 ℓ bee worth in rebating 4 ℓ vpon euery 100 for tare and cloffe.

Answer. *Multiply 860 by 4, and thereof commeth 3440, the which diuide by 100, and you shall haue 34 $\frac{2}{5}$ ℓ, abate 34 $\frac{2}{5}$ from 860, and there*

there will remaine 825 $\frac{3}{3}$. *Then say by the rule of three. If* 100 *l cost* 36 $\frac{2}{3}$ *s, what wil* 825 $\frac{3}{3}$ *cost after that rate? Multiply and diuide, & you shal find* 15–2–6 $\frac{16}{31}$. *And so much shall the* 860 *cost, in rebating* 4 *l vpon euery* 100, *for tare and cloffe.*

Item, whether doth he loſe more that giueth 4 *l* vpon the 100: or he that rebateth 4 *l* vpon the 100.

Anſwer. *First note that hee that giueth* 4 *on* 100, *giueth* 104 *for* 100. *And he which rebateth* 4 *l vpon the* 100, *giueth the* 100 *for* 96. *Therefore say by the rule of three, If* 104 *be deliuered for* 100, *for how much shall the* 100 *be deliuered? Multiply and diuide, and you shal find* 96 $\frac{2}{13}$ *and hee which rebateth* 4 *in the* 100, *maketh but* 96 *of* 100, *so that he looſeth* 4 *in the* 100, *and the other which giueth* 4 *vpon the* 100 *looſeth but* 3 $\frac{11}{13}$ *vpon the* 100. *Thus you may ſee that he which rebateth* 4 *in the* 100, *looſeth more by* $\frac{11}{13}$ *in the* 100 *l, then the other which gaue* 4 *vpon the* 100, *for tare and cloffe.*

If 100 *l* of any thing coſt me 23 *s* — 4 *d*: the queſtion is, how I ſhal ſell the pound to gaine after the rate of 10 *l*, vpon the 100 *l*.

Anſwere. *Say by the rule of three, if* 100 *l* giue 110 *l*, *what ſhall* 23 $\frac{1}{3}$ *s* giue? *Multiply and diuide, and you ſhal find* 1 $\frac{17}{60}$ *l*. *Then say*

againe, If 100 *pound be worth* $1\frac{12}{20}$ *l, what is one Pound worth? Multiply and diuide, and you shall find* $3\ s\ \frac{2}{3}\ d$. *And so much is the Pound worth in gaining,* 10 *l vpon the* 100.

Item, a Grocer hath bought 100 weight of commodity, for 6 *l* 10 *s*. The queſtion is now to know how many Pounds thereof he ſhall ſell for 32 *s* 4 *d*, to gaine 20 *s* in the 100 weight.

Anſwer. *Adde* 20 *s vnto* 6 *l*—10 *s*, *and they make* 7 *l*—10 *s*. *Then ſay, if* $7\frac{1}{3}$ *yeeld me* 112 *pound, what ſhall* $1\frac{1}{3}$ *l yeeld. Multiply and diuide and you ſhall find* 4 *Pound* $\frac{8}{8}$. *And ſo many Pound ought hee to ſell to gaine* 20 *s in his* 100 *weight.*

If one pound weight coſt 2 *s* 4 *d*, and I ſell the ſame againe for 4 *s*, which is gained 100 pound of money laid out in that commodity?

Anſwere. *You may ſay. If* $3\frac{1}{3}$ *s gine* 4 *s, what will* 100 *l gine? But then when you haue found, you muſt ſubtract* 100 *l out of the product, the reſt is your neat againe, or elſe to produce the neat againe in your worke at the firſt. Then ſubtract the iuſt price out of the ouer price, as I taught before in the firſt beginning of Loſſe and Gaine, and your concluſion ſhall be all one. Multiply and diuide by which of the two waies you thinke good, and you ſhall find*

find that hee gaineth 20 l, *in the* 100 *pound.*

Item, if the pound weight which cost 4 s be sold againe for 3 s—4 d, I demand what is lost in the 100 l' of money.

Answer. *Say, if* 4 s *loose* 3/4 s, *what shall* 100 *loose? Multiply and diuide, and you shall find* 16 l' 13 s, 4 d, *and so much is lost vpon the* 100 l, *of money.*

Item, if 100 weight of any commodity cost 45 l, and the buyer repenting, would loose 5 l, in the 100 l, of money, I demand how the l, may be sold, his losse to be neither more nor lesse then after the rate aforesaid of 5 by the hundreth?

Answer. *By the Rule of 3, if* 100 l' *loose* 5 l, *what shall* 45 l *loose? Worke and you shall find* 2 1/2 *pound, which rebated from the principall* 45 *resteth* 42 l, 15 s. *Lastly say, if* 112 l, *yeeldeth but* 42 l. 15 s *what one pound? Multiply and diuide, and shall find* 7 s, 7 d 17/21. *And so much is the pound worth after that losse.*

A Grocer hath bought 7 peeces of raisins, weying 175 1/2 l, 182 1/4 l: 191 l: tare for each frayle 2 1/2 l, at 25 1/2 s, the 100 weight. The question is, what they amount to in money.

I answere 61—3 s—4 d 23/28.

A Grocer hath bought three ſackes of Almonds waying 267 ¼ pound, Tare 2 pound at 257 ⅕ l, tare 2 ⅕ pound 252 pound tare 3 pound at 2 ß, —10 ½ the pound, what amount they to in money?

I anſwere 110 ℔ — 12 ß — 3 ¾ ð.

The ninth chapter intreateth of lengths and breadths of Arras & other clothes, with other queſtions incident vnto length and breadth.

IF a peece of Arras be 7 els and ½ long, and 5 Els and ⅓ broad, how many Els ſquare doth the ſame peece containe?

Anſwer. *Multiply the length by the bredth, that is to ſay* 7 ½ *by* 5 ⅓. *And thereof will come* 43 ½⅓ *Elles: ſo many Els ſquare doth the ſame peece containe.*

Item more, a peece of Arras doe containe 22 els ſquare, and if the ſame were in length 3 ¼ Els, I demand how many Els in breadth the ſame peece doth containe?

Anſwer. *Dinide* 22 *Els by* 3 ¼ *and thereof commeth* 6 ⁸⁰⁄₁₃. *So many Els doth the ſame containe in breadth.*

Item more, a Merchant hath 3 ¼ Elles of Arras,

Arras, at $1\frac{3}{4}$ els broad, which he wil change with another man for a peece of Arras, that is $\frac{7}{8}$ Els square. The question is how many Els of that squarenesse ought the first Merchant to haue?

Answere. *Multiply the first Merchants peece his length by the breadth, and you shall find it containeth* $5\frac{5}{12}$ *Els, which* $5\frac{5}{12}$ *Elles you shall diuide by* $\frac{7}{8}$, *and you shall find* $6\frac{4}{11}$ *Elles, and so many Els of that squarenesse ought the latter Merchant to giue the first.*

Item a Student hath bought $3\frac{1}{3}$ yards of broad cloth, at 7 quarters broad, to make a gowne, and should line the same throughout with lambe, at a foot square each skin: The question is now how many skinnes he ought to haue.

Answere *Seeke first the number of yards square that his cloth containeth, which to doe multiply* $2\frac{1}{3}$ *his length, by* $1\frac{3}{4}$ *his breadth, and you shall find* $6\frac{1}{4}$ *yards square: then say by the rule of three, if one yard square giue 9 foote, what shall* $6\frac{1}{4}$? *worke and you shall find* $55\frac{1}{2}$ *skinnes.*

Item more, a Lawyer hath a rich peece of seeling come home, which is 24 foot & three inches long, and 7 foot and $2\frac{1}{2}$ inches high, the ioyner is to be paid by the yard square: The question is how many yards this peece containeth.

Answere. *Multiply his length by his breadth, that is to wit, 24 $\frac{1}{2}$ foote by 7 $\frac{5}{24}$ foote, and you shall finde 174 $\frac{77}{96}$ foote square, which 174 you shall diuide by 9 (for so many foote make a yard square) and you shal find 19 yards three foote, and $\frac{77}{96}$ of a foote, and so many yards doth this peece hold.*

Item, bought a peece of Holland cloth containing 36 elles $\frac{1}{3}$ Flemish. The question is, how many elles English it makes.

Answere. *You must note, that fiue Elles Flemish doe make but three Elles English.*

Therefore say by the Rule of three, if 5 elles Flemmish make but three Elles English, how many Elles English will 36 $\frac{1}{3}$ Elles Flemmish make? Multiply and diuide, and you shall finde 21 $\frac{4}{5}$, and so many elles English doth 36 $\frac{1}{3}$ elles Flemish containe. The like is to be done of all others.

Item more I haue bought 342 els Flemmish, of Arras worke at two Elles broad Flemmish, and I would line the same with Elle broad Canuas of English measure. The question is, how many Elles English will serue my turne?

Answere. *For as much as three Elles English are worth fiue Elles Flemmish, therefore put 3 Elles English into his square, in multiplying 3 by himselfe, which maketh 9. Likewise*

wise multiply the Englisſh Elle, which is fiue quarters, euery way into himſelfe ſquarely, and you ſhal find 25. Then multiply 342 which is the length of the peece, by 2, which is the breadth, and thereof commeth 684, then ſay by the Rule of three, as before: if 25 Elles ſquare of Flemiſh meaſure be worth 9 Elles ſquare of Engliſh meaſure, what are 684 of Flemmiſh meaſure? Multiply and diuide, and you ſhall finde 246 $\frac{6}{25}$ Elles Engliſh.

The ſame is alſo wrought by the Backer Rule of three in ſeeking the ſquares contained in the Flemiſh Elle of two Elles broad (which are 18:) and alſo in ſeeking the ſquares contained in the Engliſh Elle (which are 25) then ſay by the Rule of three backward, if 18 quarters require 342 Elles, what ſhal 25 quarters giue? multiply and diuide by the Rule of thee Reuerſe, and you ſhall finde as before 246 $\frac{6}{25}$ Elles Engliſh.

Item more, at three ſhillings foure pence the Flemiſh Elle, what is the Engliſh Elle worth after the rate?

Anſwere. *Say if three quarters giue 3 $\frac{1}{4}$ s, what giueth fiue quarters? Multiply and diuide, and you ſhall finde 5 ſhillings 6 $\frac{1}{4}$ d.*

Item more, at 8s,4d, the Flemiſh Elle ſquare, what is the Engliſh elle worth after that rate.

Anſwere.

Answere. *According to the reason of the last question, consider that a Flemmish Elle square is equal to nine quarters of a yard English, and an English elle square is equall to 25 quarters of a yard. Therefore say by the rule of three, if 9 quarters giue 8 1/2 s, what 25 quarters? Worke and find 23 s 1 1/2 pence. And so much is the English Elle worth.*

Item more, at 6 s 8 d the Elle square: what shall a peece of cloath cost that is 7 1/4 Elles long, and three 1/4 Elles broad.

Answere. *Multiply the breadth by the length, and you shall find 24 3/4 Elles square. Then say by the Rule of three, if 1 Elle square cost 6 2/3 s, what 24 3/4. Multiply and diuide, and you shall find 8 pounds, two shillings 6 pence, and so much the same peece of cloath cost.*

Item more, a Mercer sold three peeces of silke. To wit 24 1/4, 23 1/4, and 25 yards, at 9 1/4 s the yard, and was glad to receiue in part of payment againe, a cloth containing 34 1/2 yards at 7 1/2 shillings the yard. *The question is now, what the Debitor is in the Creditors debt. Worke, and you shall find hee oweth the Mercer two and twenty pounds 3 shillings 2 1/2 pence.*

The

The tenth Chapter intreateth of reducing of Pawnes of Geanes into Englisſh yards.

NOte, that 100 Pawnes doe make 26 yards, whereupon three Pawnes $\frac{11}{13}$ do make one yard & one pawne after that rate and proportion is $\frac{13}{100}$ of a yard.

In 4563 pawnes of Geanes, how many yards Englisſh.

Anſwere. *Say by the Rule of three, if 100 Pawnes doe make 26 yards, what will 4563 Pawnes make? Multiply and diuide, and you ſhall find 1186 yards $\frac{19}{50}$. So many yards doe 4563 Pawnes make.*

Otherwiſe, take ſome other number at your pleaſure, as 10 Pawnes, which is the $\frac{1}{10}$ parte of 100, then to find his proportion, take the $\frac{2}{25}$ part of 26, which is $2\frac{3}{5}$, and then ſay alſo by the Rule of three, if 10 pawnes giue $2\frac{3}{5}$ yards, what will 4567 pawnes giue? Worke, and you ſhall find 1186 $\frac{19}{50}$ yards as before.

More, at 2ſ 6d the pawne of Geanes, what will the Englisſh yard be worth after the rate?

Anſwer. *Say by the rule of three, if $\frac{13}{100}$ of a yard coſt $2\frac{1}{2}$ ſ, what one yard? Multiply and*

and diuide, and you shall finde 9 s, 7 $\frac{4}{11}$ d.

More, if 246 $\frac{1}{3}$ Pawnes cost 20 l, 12 s, 4 d sterling, what is that the English yard after the rate.

Answere. *Say by the Rule of* 3, *If* 346 $\frac{1}{3}$ *Pawnes cost* 20 $\frac{1}{3}$ *pounds, what are* 3 $\frac{11}{11}$ *Pawnes worth (for so many Pawnes make a yard): Multiply and diuide, and you shall find* $\frac{92}{3}\frac{0}{7017}$ *parts of a Pound, which in knowne numbers is worth* 6 s 9 d $\frac{6272}{3009}$.

The eleuenth Chapter intreateth of Rules of Loane and Interest, with certaine necessary Questions and Proofes incident th reunto, &c.

ITem lent my friend 226 l for 5 $\frac{1}{2}$ moneths simply without any Interest, vpon condition, to haue the like curtesie againe when I need. But when I came to borrow he could spare me but 149 l, 8 s, 4 d. The question is now how long time I ought to haue the vse thereof, to counteruaile my friendship before time shewed him.

Answere. *Say by the backer Rule of three, if* 226 *giue* 5 $\frac{1}{2}$ *moneths, what time will* 149 *pounds* $\frac{5}{12}$ *giue? Multiply and diuide and you shall find twelue moneths, and so long time ought*

ought I to vse his Money.

The Proofe.

Item lent my friend 149 l. 4 s. 4 d. for 12 moneths. The question is now, how much money hee ought to lend me againe for 5 ½ moneths to recompence my friendshippe shewed him.

Answer. *Say by the backer or reuerse Rule of three: 12 moneths gine 149 ½, what shall 5 ½ moneths giue? Worke and you shall find 326 pounds, and so much ought he to lend me to requite my gentlenesse or good turne.*

Two other Branches yet more for proofe out of the same question.

Item, lent my friend 149 l. 8 s. 8 d. for 12 moneths, to haue the like friendship againe when I need. And comming to borrow of him, hee very curteously tooke me 326 l. (for that he could well then spare the same.) The question is now, how long I ought to occupy it, not vsurping friendship, but in his due time to restore it againe.

Answere. *Say by the Rule of three Reuerse, if 149 ½ gine 12 moneths, what shall 326 pounds gine? Multiply and deuide, and you shall find, that at 5 ½ moneths terme, I ought to restore it againe.*

Proofe

Proofe.

Item, lent my friend 326 l' for 5 1/3 moneths. The queſtion is now, how many pounds he ought to lend mee for 12 moneths to recompence this pleaſure againe.

Scholar. *Worke by the Rule of three Reuerſe, as you haue done before, and you ſhall finde* 149 l'—8 ſ,—4 d.

Againe, foure other ſelected queſtions of Loane and intereſt, all out of one branch, and each one alſo a neceſſary Queſtion, and a particular proofe to other.

Item, lent my friend 430 l' at intereſt for three months to receiue after the rate of 8 l, in the 100 for 12 moneths. The queſtion is what the intereſt commeth to.

You may if you pleaſe worke it at two workings by the Rule of three direct, in ſaying: If 12 moneths giue 8 l, *what giueth three moneths? Multiply and diuide, and it giueth* 2 *pound.*

Then for the ſecond worke ſay: If 100 *pound yeelde two l, what yeeldeth* 43 *pound? Multiply and diuide: and you ſhall finde* 8 l, 12 ſ, *and ſo much comes the loane of* 430 l, *to, for* 3 *moneths*

neths, after the rate of 8 pounds in the 100 pounds of 12 moneths.

Otherwise wrought thus by the Rule of three at twice also.

If 100 ℔ giue 8 ℔, what gineth 430 ℔? multiply and diuide, and you shall find 34 pounds $\frac{2}{7}$, Then againe for the second worke say: If 12 moneths giue 34 pounds $\frac{2}{7}$, what gineth three moneths? Worke, and find 8 ℔ 12 ß as before.

Otherwise yet at one working: By the first part of the rule of fiue numbers forward, in saying, if 100 pounds in 12 moneths gaine 8 ℔, what shall 43 pounds gaine in three moneths? Multiply the first by the second for your Diuisor, and the other three the one into the other for the Diuidend, and you shall find eight pounds 12 shillings, as aforesaid.

Proofe.

Item a friend of mine receiued of mee 8 pounds 12 shillings for the Interest & vse of 430 pounds for three moneths termes: The question is now, what he tooke in the 100 pounds for 12 moneths after that rate.

Answer. *For most briefe, say by the first part or rule of fiue numbers forward: If 430 pounds in three moneths did pay 8 ℔, 12 ß, what doth 100 pounds in 12 moneths take after the rate. Worke, and you shall find 8 pounds, and so much he tooke vpon the 100 pounds for twelue moneths.*

A third queſtion and proofe alſo by the Backer Rule of 5 Numbers.

Item, lent my friend 430 pound to receiue for the intereſt thereof, after the rate of 8 l in the 100 for 12 moneths. The queſtion is now how long time my friend ought to giue the vſe thereof, that it may be returned with 8 l — 12 s̄ gaines.

You may worke it if you pleaſe by the Rule of three direct at twice, in ſaying: If 100 l *yeeld* 8 l, *what yeeldeth* 430 l? *Multiply and diuide, and find* 34 *pound and* $\frac{2}{7}$.

Then againe for the ſecond worke ſay, if $34\frac{2}{7}$ *gine* 12 *moneths, what giueth* $8\frac{1}{2}$ *pounds. Multiply and diuide, and you ſhall find three moneths, and ſo long time ought my friend to vſe it to returne with* 8 l — 12 s̄ *gaine.*

Otherwiſe at one working by the Backer Rule of 5 *numbers, in ſaying: If* 100 l *in* 12 *moneths dee gaine* 8 l, *how long time ſhall* 430 l *be a gaining of* 8 l — 12 s̄? *Multiply the firſt and the ſecond into the laſt for your diuidend, and the third and fourth multiply together for your diuiſor, and then diuide, and you ſhall find* 3 *moneths, the iuſt time that my friend ought to vſe it, to returne it with* 8 l, 12 s̄ *gaine.*

A fourth deriued queſtion out of this Branch, which is a proofe of this laſt and alſo of the other two going before.

ITem, how much money ought a Merchant to deliuer after 8 ℔ in the 100 for 12 moneths, that in three moneths hee may gaine 8 ℔ 12 ß.

Anſwer. *You may alſo if you pleaſe worke it by the Golden Rule of three at twice, firſt ſaying, if three moneths gaine 8 ℔, what 12 moneths gaine? you ſhall find 34 l. Then ſay again if 8 l. be come of 100 ℔, what ſhall come of 34 ℔ 8 ß? Worke, and you ſhall find the anſwer to the queſtion, which is 430 l, and ſo much ought the Merchant to deliuer.*

But moſt briefly it is anſwered by the Backer Rule of fiue numbers, where I argue thus, ſaying: If 100 l, be 12 moneths a gaining of 8 l, then but for three moneths terme onely to take 8 l, 12 ß, muſt needes bee a good round ſumme, to worke it, ſet your numbers thus: 100—12 8—3—8 ½: multiplying the firſt into the ſecond, and alſo by 43 the product of the fifth, for your diuidend, and the third and fourth together with 5 the Denominator of your fraction for your Diuiſor: then diuide and you ſhall find as before, 430 l: the true ſolutions to your queſtion.

The twelfth Chapter intreateth of the making of Factors, which is taking in two ſorts.

THe firſt is, when the eſtimation of the factor is taken vpon the ſending of the merchant, as if the eſtimation of his perſon be $\frac{1}{4}$ it is vnderſtood that he ſhall haue $\frac{1}{4}$ of the gaine, and the merchant the other $\frac{3}{4}$.

The other ſort is when the eſtimation of his making is out of the ſending of the merchant, as if the order & agreement between them were ſuch that the merchant ſhal put in 800l, and the factor for his making ſhall haue $\frac{1}{4}$ neuertheles he ſhall haue but $\frac{1}{5}$ of the gaine or profit, for the $\frac{1}{4}$ of 800 is 200 (for the eſtimation of his making) which with the 800l make 1000l, wherof the 200 is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, vnder ſuch condition, that the ſaid Factor ſhall haue $\frac{1}{4}$. And after certaine time, they finde in profit 124l, 6s 8d, I demand how much the merchant ſhal haue hereof, and how much ought the Factor to haue?

Anſwere *When the eſtimation of the Factor is out of the ſending of the Merchant it maketh.*

l	s	d	
99	— 9	— 4	*for the Merchant:*
24	— 17	4	*for the Factor.*

But if that his eſtimation be at the ſending of the Merchant, then it maketh but,

l	s	d	
93	— 5	— 0	*for the Merchant.*
31	— 1	— 8	*for the Factor.*

For the Merchant is then to haue $\frac{3}{4}$, *and the Factor* $\frac{1}{4}$.

A Merchant doth put into the hands of his Factor 800l, & the Factor 400l to haue the ½ part of the profit: I demand now for how much his perſon is eſteemed when the ſame is counted vpon the ſending of the Merchant?

Anſwere. *According to the tenor and order before preſcribed in the firſt rule, that is, if his eſtimate be* $\frac{1}{4}$ *hee ſhall haue the* $\frac{1}{4}$ *of the gaine. Therefore ſay by the Rule of three direct, If* $\frac{1}{4}$ *taken put in 400l, what is the eſtimate, or putting in of* $\frac{1}{2}$ *taking? Multiply and diuide, and you ſhall finde 320l, and ſo much is the perſon of the Factor eſtimated.*

Otherwiſe.

To finde the eſtimation of the perſon of

the Factor, you shal consider, that seeing it was agreed betweene them, that the Factor should take the $\frac{1}{5}$, then the Merchant shall haue the residue, which are $\frac{4}{5}$: wherefore the gaine of the Merchant vnto that of the Factor is in such proportion as 5 vnto 4. Then if you will know the estimation of the person of the Factor: say, if 5 gine 4, what will 400 gine? Multiply and deuide, and you shall find 320 pound. And so much is the person of the Factor esteemed to be worth.

Other conditions then these aforesaid, may also bee betweene Merchants and Factors without respect either of sending or not sending of the Merchant, where most commonly the estimation of the body of the Factor is in such proportion of the stocke which the Merchant layeth in, as the gaine of the said Factor is vnto the gaine of the Merchant. As thus, If a Merchant doe deliuer into the hands of his Factor 400 pound and he to haue halfe the profit. The person of the saide Factor shall be esteemed to be worth 400 pound, and if the Factor doe take but $\frac{1}{3}$ of the gaine, he should haue but $\frac{1}{3}$ so much of the gaine as the Merchant taketh, which must haue $\frac{1}{4}$, wherefore the person of the Factor is esteemed but the $\frac{1}{3}$ of that which the Merchant layeth in, that is to say, 200 pound.

And if the Factor did take the $\frac{1}{2}$ of the gain,

then

then the Merchant ſhall take the reſidue which are $\frac{2}{7}$, wherfore the gaine of the Merchant vnto the Factor is then in ſuch proportion as 3 vnto 2 :whereupon if you will then know the eſtimation of the perſon of the Factor, ſay, if 3 giue 2, what ſhall 400 giue? Work, & you ſhall find $266\frac{2}{3}$ pounds. And ſo much is the perſon of the Factor eſteemed to be worth.

And if the Merchant ſhould deliuer vnto his Factor 400 pound, & the Factor would lay in 80, and his perſon, to the end hee might haue the $\frac{1}{3}$ of the gaine, I demand how much ſhall his perſon be eſteemed?

Anſwere. *Abate* 80 *from* 400, *and there will remaine* 320. *And at ſo much ſhall his perſon be eſteemed.*

A Merchant hath deliuered vnto his Factor 900l. to gouerne in the trade of Merchandiſe, vpon condition that he ſhall haue the $\frac{1}{3}$ of the gaine, if any thing be gained, and alſo to beare the $\frac{1}{3}$ of the loſſe, if any thing be loſt. Now I demand how much his perſon was eſteemed at?

Anſwer. *Seeing that the Factor taketh the* $\frac{1}{3}$ *of the gaine, his perſon ought to be eſteemed as much as* $\frac{1}{2}$ *of the ſtocke, which the Merchant layeth in: That is to ſay the* $\frac{1}{2}$ *of* 900 *pound, which is* 450. *The reaſon is, becauſe* $\frac{1}{3}$ *of the gaine that the Factor taketh, is the* $\frac{1}{2}$ *of the* $\frac{1}{3}$ *of the gaine that the Merchant taketh, and*

so the Factor his person is esteemed to be worth 450 pounds.

A Merchant hath deliuered vnto his Factor 600 pound, and the Factor laieth in 250 pound and his person. Now because he laieth in 250 pounds and his person, it is agreed betweene them, that hee shall take the $\frac{3}{5}$ of the gaine. I demand for how much his person was esteemed.

Answer. *For as much as the Factor taketh $\frac{3}{5}$ of the gaine hee taketh $\frac{3}{5}$ of that which the Merchant taketh, for $\frac{3}{5}$ are the $\frac{3}{5}$ of $\frac{1}{1}$. And therefore the Factors laying in ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the Merchant laied in. Then subtract 250, which the Factor did lay in from 400 pound which should haue beene his whole stocke, and there remaineth 150 pound for the estimation of his person.*

More, a Merchant hath deliuered vnto his Factor 800 pound vpon condition that the Factor shall haue the gaine of 160 pound, as though he laid in so much ready money: I demand what portion of the gaine the said Factor shall take?

Answere. *See what part the 160 (which the Factor laid in) is of 960, which is the whole stocke of their company, and you shall find $\frac{1}{6}$. And such part of the gaine shall the Factor take.*

But in case, that in making their couenants,

it

it were so agreed betweene them, that the Factor should haue the gaine of 160 *pound of the whole stocke which the Merchant laieth in, that is to say of the* 800 *pound, then should the Factor take* $\frac{1}{5}$ *of the gaine: for* 160 *is* $\frac{1}{5}$ *of* 800 *pound.*

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandise, which is distinct into seuen Rules with diuers other necessary questions incident thereunto.

The first Rule.

Two Merchants willing to change their merchandise the one with the other. The one hath 24 broad cloathes at 10*l*, 10*s*. the peece. The other hath mace at 12 $\bar{s}$ the poũd. The question is, how many pounds of Mace he ought to giue for his Cloth, to saue himselfe harmelesse and be no loser.

Answere. *Seeke first by the Rule of three, what the* 24 *clothes cost at* 10*l*, 10 *shillings the peece, and you shall find* 252 *pound. Then to find the quantity of Mace, say againe by the Rule of three, if* 12 *shillings buy one pound what shall* 252 *pound buy me? Worke, and you shall find* 420 *pound of Mace: and so many pound ought he to giue for his Cloathes.*

The proofe.

Two barter, the one hath 420 pounds of Mace at 12ß the pound, to barter or change broad cloths at 10 pounds 10 ſhillings the peece. The queſtion is, how many broad clothes he ought to giue for all his Mace.

Anſwere. *Firſt ſay, if one coſt* 12ß, *what* 420? *you ſhall finde* 5040ß. *Then ſay againe, if* $10\frac{1}{2}$ *pounds giue* 1 *cloth, what ſhall* 5040 *ſhillings giue? Worke and you ſhall finde* 24 *clothes your deſire.*

The ſecond Rule.

Two change merchandiſe for merchandiſe: The one hath Pepper at 2 ſhillings 4 pence the pound, to ſell for ready mony. But in barter hee will haue no leſſe then three ſhillings the pound. And the other hath Holland at fiue ſhillings 6 pence the elle ready mony. The queſtion is now at what price he ought to deliuer the elle in the barter to ſaue himſelfe harmeleſſe.

Anſwere. *Say by the Rule of three direct: if* $2\frac{1}{3}$ ß *ready mony giue* 3ß, *in barter what ſhall* $5\frac{1}{2}$ *giue in barter? you ſhall finde* $7\frac{1}{14}$ ß, *and at that price ought the ſecond merchant to ſell his Holland in barter.*

The

The proofe.

Two barter. The one hath Holland at 5s 6 pence the elle to sell for ready mony. And in barter he will haue $7\frac{1}{14}$s. The other hath Pepper at 2s–4d, the pound to sel for ready mony. The question is now how hee ought to sell in barter.

Answere. *Say by the Rule of thee direct, if* $5\frac{1}{2}$ *ready mony giue* $7\frac{1}{14}$ *in barter, what ought* $2\frac{1}{4}$s *to take in barter: Multiply and diuide, and you shall finde* 3s *your desire.*

The third Rule.

Two barter. The one hath cloth of Arras at 20s the elle ready mony, but in barter he will haue $35\frac{1}{2}$s. And the other hath white wines, which he deliuered in barter for 16l, the Tunne. The question is now what his wines cost the Tunne in ready mony.

Answere *Say by the Rule of three direct, If* $35\frac{1}{2}$s, *in barter giue but thirty ready money, what did* 19l *in barter cost? Worke and you shall finde* 13l, 10s $\frac{30}{71}$. *And so much cost his wines for a Tunne ready mony.*

The proofe.

Two barter merchandise for merchandise: The one hath wines white at 13l,—10s $\frac{30}{71}$

the

the Tunne to sell for ready money. But in barter he deliuered it for 16l. The other to make his match good & saue himself harmlesse, deliuereth Arras at 35 ½ s the Elle The question is now, what an Elle of his Arras cost in ready money.

Answer. *Say by the Rule of three direct: If* 16 l *in barter giue but* 13 l — 10 $\frac{12}{21}$ s *in ready mony, what shall* 35 ½ s *yeeld in barter: work, and you shall find* 30 s *your desire.*

The fourth Rule.

Two barter. The one hath Kersеis at 14 l the peece ready mony. But in barter he will haue 18 l. And yet he will haue the ⅓ part of his ouerprice in ready mony. And the other hath Ginger at eight groates the pound to sell for ready money. The question is how he ought to deliuer the Ginger by the poũd in barter to saue himselfe harmelesse, and make the barter equall.

Answere. *Item for the working of this question, and such other the like, you must vnderstand if the party ouerselling his wares, require to haue also some portion in ready money, as* ½ : ⅓ : ¼, *&c. Then shall you first rebate the same demanded part whatsoeuer it bee from the ouerprice, and also from the iust price. And those two numbers that shall remaine after the subtraction is made, shall be*

the

the two first numbers in the Rule of three. And the iust price of the same Merchandise shall be the third number, which by the operation of the Rule of three direct, shall yeeld you a true solution how and at what price you shall ouersell that your Merchandise, to saue your selfe harmelesse, and make the barter equall.

Example.

Take the ⅓ (of eighteene) which is the ouer price of his cloath, which ⅓ of eighteene is sixe, which you must subtract from 18, *there rest* 12. *And also abate it from* 14, *which is the iust price of the cloath, and there remaineth* 8, *which* 8 *and* 12 *are the two first numbers in the Rule of three. Then take eight groates or two ⅓ shillings for the third number. Then say by the Rule of three direct: If* 8 *℔ giue* 12 *pounds, what shall* 2 ⅓ 8 *giue? Multiply and deuide, and you shall find* 4 8. *And for so much shall the second Merchant sell his Ginger, or his commodity in barter, to ballance the same equall.*

	14 —	18
	6	6
	8 —	12

The proofe.

Two barter: The one hath fine Kerseyes at 14 pound: the peece ready mony. But in bar-

barter he will haue 18 pounds, and yet he will haue the $\frac{1}{3}$ part of his ouerprice in ready mony: And the other hath ginger, which hee hauing cunning enough to make the barter equall, deliuered in barter for 4 s the pound. The queſtion is now, what his Ginger coſt him in ready money?

Anſwer. *After you haue made the ſubtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14, (as before was taught you): then will there remaine 8 and 12 for your two firſt numbers in the Rule of three. Then ſay, if 12 giue 8, what ſhall come of 4 the ouerprice of the £ of Ginger? Multiply and diuide, and you ſhall find two ſhillings 8 d your deſire.*

Two Merchants barter Merchandiſe for merchandiſe. The one hath *Denſhire* whites at 7 £–13 s–4 d the peece ready money: but in barter he doth them away for 8 £ 3 s 4 d, and yet he will haue the $\frac{1}{3}$ part of his price in ready money. And the other hath cottens at 3 l, the peece ready money. The queſtion is now, at what price he ought to ſell or exchange his Cottens in barter to ſaue himſelfe harmeleſſe, and make the barter equal?

7	13	4	8	3	4
2	14	$5\frac{1}{3}$	2	14	$5\frac{1}{3}$
4	18	$10\frac{2}{3}$	5	8	$10\frac{2}{3}$

An-

Anſwer. *Firſt ſeeke the* $\frac{1}{3}$ *part of* 8—3 s 4 d, *which is*, 2 l—14—5 $\frac{1}{3}$ d, *which rebated from* 8–3–4 d, *there reſteth, as appeareth by the example aboueſaid*, 5—8—10 $\frac{2}{3}$ *pence, which* $\frac{1}{3}$ *of* 8—4 d, *alſo rebated from* 7—13—4 d, *there reſteth* 4—18—10 $\frac{2}{3}$, *the two firſt numbers in the Rule of three, and the* 3 l, *which is the next price of the peece of Cotten, is the third number. Then ſay by the rule of three direct as was taught before: If* 4—18—10 $\frac{2}{3}$ d *giue* 5 l—8 s—10 $\frac{2}{3}$ d, *what ſhall* 3 l, *giue? Multiply and diuide, and you ſhall find* 3—6—0 $\frac{72}{80}$ *pence, the iuſt price that he ought to deliuer his Cottens in barter.*

The fift Rule.

Two Merchants will change merchandiſe for merchandiſe. The one hath Kerſeis at 40 s the peece to ſell for ready money. And in barter he will ſell them for 56 s 8 d, and hee will gaine after 10 l vpon the 100 l. And yet he will haue the $\frac{1}{3}$ of his ouerprice in ready money. The other hath flaxe at 3 d the pound ready money. The queſtion is now, how hee ſhall ſell the pound of his flaxe in Barter?

Anſwer. *See firſt at* 10 l *vpon the* 100 l, *what the* 56 $\frac{2}{3}$ *commeth to, in ſaying (by the Rule of three direct) if* 100 l *giue* 110 l, *what* 56 $\frac{2}{3}$ s? *Multiply and diuide, and you ſhall find* 3 l–2 s–4 d, *of which the* $\frac{1}{3}$ *that he demandeth*

deth in ready mony, is 1l, ——— 11s ——2d *the same* 31s ——2d *abated from* 40s, *and also from* 56s—8d, *there will remaine* 8 *shillings*: 10d, *and* 25s—6d, *for the two first numbers in the Rule of three, and* 3 *pence the price of the pound of flax for the third number. Then multiply and diuide, and you shall finde* $8\frac{3}{5}$d, *and for so much shall he sell the pound of flax in barter.*

The sixth Rule.

Two are willing to exchange merchandise: the one hath Norwich Grograns at 25 s the peece ready mony: and in barter he will haue 30s and he will haue the $\frac{1}{4}$ part of his ouer price in ready mony. The other hath Norwich stockings at 40 s the dozen to sell for ready mony. But in as much as the first Merchants Grogranes are no better, he would deliuer them so to ballance the barter, that he may gaine after 10 l, in the 100l. The question is now, how hee shall sell his Hose the dozen in barter according to his request.

Answere. *Say, if* 100 *giue* 110 *what shall* 40s *giue, which is the iust price of the dozen of stockings? Multiply and diuide, and you shall finde* 44s. *Then take the* $\frac{1}{4}$ *of* 30s, *which is* 7s 6d, *and subtract it from* 25s, *and also from* 30s, *and there will remaine* 17s ——6d, *and*

22s

22s——6d, *for the two first numbers in the Rule of three, and 44 shillings, which is the iust price (with his gaine in the dozen of stockings) for the third number. Then multiply and diuide, and you shall finde 56 s 6 $\frac{6}{7}$ d and for so much he is to sel his dozen of stockings in barter.*

The seuenth Rule.

Two Merchants will change theit merchandise one with the other. The one hath 720 elles of Cambricke at 5s the elle to sell for ready mony, but in barter hee requireth 6s 8d. And yet notwithstanding he looseth by it after 10 pounds vpon the 100l, wherupon he requirethone halfe of his ouerprice in ready mony, and the other Merchant hauing skill enough to make the barter equal, deliuereth English Saffrons at 30s the pound. The question is now, what his Saffrons cost the pound in ready mony;

Answere. *You must first seeke what is lost vpon the 100l, which to doe you may say (If you please) If 100l, loose 10, what shall 6 $\frac{2}{3}$ loose? Worke, and you shall finde $\frac{2}{3}$s (or 8d) which must be rebated from 6s—8d so resteth 6s still. Or you may say, if 100l giue me but 90 pounds, what shall 6s——8d giue? Worke this way either, and you shall finde also as before directly in your quotiēt 6 shillings your desire*

fire. Then are you next to caſt vp what the 720 Elles of Cambricke commeth to at 6ſ 8d the Elle, and you ſhall find 240 pounds: the ⅓ whereof the Cambricke Merchant will haue in ready money, (which is 120l): Nextly you muſt caſt what the Cambricke commeth to after his loſſe in the 100l, which as you found is but 6ſ an Elle, and you ſhall find 216 pounds. Now muſt you ſubtract his ready money (which is 120l,) out of 240l, and alſo out of 216l, and there will remaine 120 pounds, and 96 pounds for your two firſt numbers in the Rule of three. and 20 ſhillings is the ouerprice of your Saffron for the third nnmber. Then multiply and diuide, and you ſhall find 24 ſhillings. And ſo much did his Saffron coſt in ready money.

Two Merchants barter. The one hath 50 Cloathes to put away for ready money at 11l, the cloth, and in barter putteth them away for 12 pounds, taking Holland cloth at 20 the Flemmiſh Elle, which was worth no more but 18d. The queſtion is now, what Holland payeth for the Cloth, and what he winneth or looſeth by the bargain?

Anſwer. 50 *Clothes at* 11l *the cloth commeth to* 550l, *and put away at* 12l, *the peece, maketh* 600 *pound. Then to find what Holland payeth for the Cloth, ſay by the Rule of three direct, If* 20d *buy* 1 *Elle, what* 600l? *Worke*

and

and you ſhall find 7200 Els. Now to find the eſtate of his gaine or loſſe, you muſt ſeeke what his 7200 Elles commeth to at 18 ß the Elle Worke by the Rule of proportion direct, and you ſhall find 540 pounds, which is not ſo much as his cloathes were worth in ready money by 10 ℔: and ſo much loſt the firſt Merchant by his exchange.

A Venetian hath in *London* 100 peeces of ſilke, to put away for ready money at 3 ℔ the peece. But in barter he deliuered them for 4 ℔ the peece, taking Wools of a Felmonger at 7 ℔ 10 ß the C weight, which was worth no more but 6 l, the C ready money. The queſtion is now, what wools payeth for the ſilkes, and which of them winneth or loſeth by the barter.

Anſwere. *100 peeces of ſilke at 3 pound, is* 300 l, *and* 4 l, *is* 400 l. *Then to find what woolles payeth for the ſilke, ſay by the Rule of three direct: If* 7 ½ *buy me* C *waight, what* 400 l? *Worke, and find* 53 ⅓ C *weight of wooll. Now to find the eſtate of their gaine, or loſſe, caſt vp his wooll at* 6 l, *the* C (*for ſo much they were worth ready money*) *and you ſhall find* 320 *pound, which is* 20 *pound more then the ſilkes were to be ſold for ready money, whereupon the Venetian gained* 20 *pounds by the barter.*

A Merchant hath 5 $\frac{1}{3}$ C waight of wooll at 6 l. the C, to sell for ready money, but in barter he will haue 7 pounds 10 s. and another doth barter with him for silkes, which are worth 3 l. a peece ready money. The question is now, how he ought to deliuer his silkes the peece in barter, and how many payeth for the wools?

Answere. *Say by the Rule of Proportion, (or by the Rule of three direct) If 6 l. for C weight ready money yeeld me 7 l. 10 s. what will 3 pound yeeld, which is the iust price of a peece of silke in barter, to make the Trucke equall? Worke, and find 3 pounds 15 shillings, the price of a peece of silke in barter: Then say if 3 pound 15 shillings require 1 peece of silke, how many peeces of silkes are bought with 40 pound, which is the value of the 5 $\frac{1}{3}$ C weight of Wooll at 7 l. 10 s? Worke by the rule of three direct, and you shall find 106 peeces of silke and $\frac{2}{3}$ of a peece, and so many peeces of silke payeth for the wooll, and neither party hath aduantage of other.*

Two men will change merchandise the one with the other. The one of them hath Beere at 4 s 8 d the barrell to sell for ready mony, but in barter he will sell the barrell for 8 s, and yet hee will gaine moreouer after 10 l. vpon the 100 l. And the other hath white Spanish wooll at 20 s the Roue, to sel for ready money. The question is now, how he shall

shall deliuer the Roue of wooll in barter to saue himselfe harmeleſſe.

Answere. *Say, if* $6\frac{2}{3}$ *s, which is the iuſt price of the barrell of Beere, be ſold in barter for* 8 *ſhillings: for how much ſhall* 20 *ſhillings (which is the iuſt price of the Roue of Wooll) be ſold in barter? Worke by the Rule of three direct, and you ſhall find* 24 *s. Then for becauſe the firſt Merchant will gaine after* 10 *l, vpon the* 100 *l, he maketh his* 100 *l,* —110 *l. And therefore ſay by the Rule of three, if the ſecond Merchant of* 110 *doe make but* 100 *l, how much ſhall he make of* 24 *s? Multiply and diuide, and you ſhall find* 21 *s* 9 *d* $\frac{2}{11}$ *of a peny. Add for ſo much ſhall he ſell the Roue of wooll to be deliuered in barter, to the end the firſt Merchant may gaine* 10 *in the* 100.

Two Merchants will change their commodities the one with the other The one of them hath white Paper at 4 s the reame to ſell for ready money. And in barter he will doe it away for 5 s, and yet he will gaine moreouer after the rate of 10 l vpon the 100 l. And the other hath Mace at 14 s 6 d the pound weight to ſell in barter. Now I demand what the pound did coſt in ready money?

Anſwere. *Say if* 5 s, *(which is the ouer price of the Paper in barter) be come of* 4 s *the iuſt price, of how much ſhall come* 14 *ſhillings, which is the ſurpriſe of the pound of Mace in*

barter? Multiply and diuide, and you shal find 11 $\frac{3}{5}$ *s. Then for because the first Merchant of Paper will gaine after* 10 *vpon the* 100. *Say, if* 100 *doe giue* 110, *what shall* 11 $\frac{3}{5}$ *shillings giue? Worke, and you shall find* 1 *li.* 2 *s.* — 9 $\frac{17}{21}$ *d. and so much did the pound of Mace cost in ready mony.*

The fourteenth Chapter intreateth of exchanging of mony from one place to the other.

Exchange is no other thing then take or receiue money in one City to render or pay the value therof in another City: or else to giue mony in one place, and receiue the value thereof in another, at tearme of certaine dayes, moneths, or faires, according to the diuersity of the place.

But this pratice chiefly consisteth in the knowledge of the mony or Coines in diuers places, of which for thy benefit (after a few examples giuen to the introduction of this worke) I wil set down certaine notes of the diuersity of the common and vsuall coines in most places of Christendome for traffique.

And first I will begin at Antwerpe, *where they vse to make their accounts by* Deniers de gras,

de groſſe, *that ir to ſay pence Flemmiſh, wherof* 12 *doe make* 1ß, *Flemmiſh, and* 20ß *doe make one ponds* de gros.

Item, a Merchant deliuered at *Antwerpe* 400 dounds flemmiſh to receiue in *London* 20ß ſterling, for euery 23ß—4d, Flemmiſh The queſtion is now, how much ſterling money is to be receiued at *London* for the ſaid 400 pounds of flemiſh?

Anſwere. *Say by the Rule of three, if* 23 $\frac{5}{7}$ *Flemmiſh giue* 20ß *ſterling, what* 400l' *Flemmiſh? Worke, and you ſhall finde* 342l' 17ß 1 $\frac{5}{7}$ *pence, and ſo much ſterling ſhall I receiue in* London *for the ſaid* 400l' *flemmiſh.*

Otherwiſe alſo wrought by Rules of Practiſe in taking the $\frac{1}{7}$ *of the flemmiſh mony deliuered, and abating the ſame from the principall, the reſt in Engliſh mony ſterling, as before.*

400—0—0

57—2—10 $\frac{3}{7}$

342—17—1 $\frac{5}{7}$ *ſterling.*

A Merchant at *London* deliuered 200l' ſterling for *Antwerpe* at 23ß—5d flemmiſh the pounds ſterling. The queſtion is, how much he muſt receiue at *Antwerpe*?

Anſwere. *Say by the Rule of three, if* 1l'

ſterling giue 23 *s* 5 *d Flemmiſh, what* 200 *l ſterling? Worke, and you ſhall find* 234 *l*—3 *s* 4 *d. So many pounds Flemmiſh ſhall he receiue at* Antwerpe *for the ſaid* 200 *pounds ſterling.*

Otherwiſe by practiſe.

1	23	5	200
3 s–4 d	33	—6—	8
1 d		—16—	8

Maketh 234 *l*, —3 *s*—4 *d ſterling.*

In *London* 200 pound ſterling is deliuered by Exchange, for *Antwerpe* at 23 *s* 9 *d* Flemmiſh the pound ſterling. The queſtion is at what rate the Flemmiſh money ought to be returned to gaine 4 pound vpon the 100 *l* ſterling at *London*?

Anſwere. *Firſt ſay by the Rule of three direct: If* 1 *pound ſterling giue* 23 *s* 9 *d Flemmiſh, what* 200 *pounds ſterling? Multiply & deuide and you ſhall find* 237 *pounds* 10 *ſhillings. The which to returne to gaine* 8 *pounds ſterling in* London, *ſay by the backer rule, if* 200 *pounds ſterling require the exchange* 23 *s* 9 *d Flemmiſh, what the exchange to make* 208 *l, ſterling? Worke by the Rule, and find* 22 *s* 10 $\frac{1}{20}$ *d Flemmiſh, the effect in the queſtion required.*

If

If I take vp money at *Antwerpe* after 19 s 4 d Flemmish, to pay for the same at *London* 20 shillings sterling, and when the day of payment is come, I am forced to returne the same money againe in *London*, to pay my bill of Exchange: so that for 20 shillings which I take vp here at *London*, I must pay 19 s 6 d at *Antwerpe*: I demand whether I doe win or loose, and how much in or vpon the 100 l of money?

Answer. *Say by the Rule of three: If* 19½ *giue* 19⅓, *what will* 100 l *giue? Multiply and diuide, and you shall find* 99 l, 2 1/17 s, *which being abated from* 100 l, *there will remaine* 17 *shillings* 10/17, *and so much I doe loose vpon the* 100 l *of money.*

If I take vp at *London* 20 shillings sterling to pay at *Antwerpe* 22 s, 4 d, and when the day of payment is come, my Factor is constrained to take vp money againe at *Antwerpe* wherewith to pay the aforesaid sum, & there he doth receiue 23 s, 4 d Flemmish, for the which I must pay 20 s, at *London*: The question is now, whether I doe win or loose, and how much vpon the 100 l, of money after that rate?

Answere. *Say by the Rule of Proportion. If* 22⅓ s *giue* 23⅓ s, *what will* 200 *pounds giue?*

Multiply and diuide, and you ſhall find 104 *pounds* 9 *ſhillings* $\frac{3}{67}$, *from the which abate* 100 *pound, and there will remaine* 4 *pounds* 9 *ſhillings* $\frac{3}{67}$ *and ſo much is there gained vpon the* 100 *pounds of money.*

In *Antwerp* is deliuered 200 pounds Flemmiſh by exchãge for *London* at 20 ſhillings ſterling for euery 23 ſhillings 4d Flemmiſh. The queſtion is, at what rate the ſame is to be returned to gaine 10 pounds vpon the 100 pounds Flemmiſh in *Antwerpe.*

Anſwere. *Firſt, ſay by the Rule of three, if* 23 $\frac{1}{3}$ *Flemmiſh giue* 20 ß, *what ſhall* 200 *pounds gaine? Worke and you ſhall finde* 171 *pounds* 8 ß, 6 $\frac{6}{7}$ d. *Then ſay againe by the Rule of three direct, if* 171 l', 8 ß, 6 $\frac{6}{7}$ ß *ſterling, giue me* 210l, *Flemiſh, what ſhall* 20 ß *ſterl. giue? Worke, and you ſhall find* 24 ß 6 d *Flemmiſh. And at the ſame rate ought the ſame to bee returned at* Antwerpe *to gaine* 10 *pound vpon* 100 *Flemmiſh.*

A Merchant of *Antwerpe* deliuereth 234 pound 7 ß, 4 d Flemmiſh, to receiue at *London* 200 pounds ſterling: The queſtion is now how the exchange goeth after this rate?

Anſwere *Say by the Rule of three direct, If* 200 *giue* 20, *what* 234 $\frac{1}{6}$? *Multiply and diuide, and you ſhall find* 23 ß——5——d. *And for ſo much goeth the exchange.*

Item, the Exchange from *London* into *France*

France is not like as it is in *Flaunders*, but is deliuered by the French Crowne, which is worth 50 ſoulx Turnois the peece.

Whereupon alſo you muſt note, that in France *they make their accounts by Francks, Soulx, and Deniers Turnois, whereof 12 Deniers maketh one Soulx Turnois, and 20 Soulx maketh 1 pound Turnois, which they call a Liure or Franke. But the Merchants to make their accounts doe vſe French Crownes, which is currant among them for 51 Soulx Turnois. But by exchange it is otherwiſe, for it is deliuered but for 50 Soulx Turnois the Crowne, or as the taker vp of the money can agree with the deliuerer. And note that this ∆ Character repreſenteth the Crowne by exchange, and is euer 50 Soulx Turnois for French money.*

A Merchant deliuereth at *London* 240 l. ſterling after 5 ſhillings 6 pence the crown, to receiue at *Paris* 50 Soulx Turnois for euery Crowne. I demand how much Turnois or French money payeth the bils for the ſaid 240 pounds ſterling.

Anſwer. *Say by the Rule of three, if* 5 s. 6 d. *ſterling giue me* 50 s *Turnois, what ſhal* 240 l. *ſterling giue? Reduce the pounds into ſhillings, then multiply and diuide, and you ſhall finde* 2181 *Liures, ſixteene, Soulx, foure Deniers,*

niers, and $\frac{4}{17}$ Turnois, and so much payeth the bils at *Paris* for the 240 ℔ sterling.

A Merchant deliuereth at *Roan*, or elsewhere in *France* 1430 pounds or franks, the which franck or pound is 20 soulx, or a pound Turnois, to receiue in *London* 6s,4d sterling for euery △ of 50 soulx Turnois. The question is, how much sterling money I ought to receiue at *London* for my 1430 pound Turnois.

Answere. *Say if two* $\frac{1}{3}$ *pounds giue me* $6\frac{1}{3}$ s, *what will* 1430 *giue mee? Worke, and you shall finde* $3622\frac{3}{7}$ *sterling, which maketh* 181 *pounds* 2s 8 *pence, and so much money is to be receiued at* London *for the said* 1430 *Liuers Turnois, after* 6s-4 *pence euery* △ *of* 50 *soulx.*

In *London* is deliuered 200, pound sterling by exchange for *Paris*, at 5 s—9d the △ of 50 soulx Turnois. The question is, at what price the said △ is to be returned to gaine 6 pounds vpon the 100 pound sterling at *London*.

Answere *First say (by the Rule of three direct) if* $5\frac{1}{4}$ s *sterling giue* 50 *soulx Turnois, what shall* 200 *pound sterling giue? Worke, and you shall find* 1739 *franks or liuers, two soulx* $\frac{14}{31}$. *Then, the which to returne and gaine 6 pounds vpon the* 100 ℔ *in* London, *say by the rule of three direct, if* 1739 *franks, two soulx* $\frac{14}{31}$ *yeeld*

yeeld. 212 *pounds, what the* △ *of* 50 *Soulx? worke and finde* 6s—1$\frac{1}{3}$d, *the effect required in the question.*

A Merchant deliuered in *London* 160l sterling to receiue in *Biskay* for euery 5s 6d one Ducket of 374 Maruides The question is, how many Maruides ought I to receiue at *Briskay?*

Answere. *Say, if* 5$\frac{1}{2}$s *sterling giue* 374 *Maruides, What shall* 160 *pounds sterling giue? Multiply and diuide, and you shall finde* 217600 *Maruides, and so many I ought to receiue at* Briskay *for my* 160 *pounds sterling.*

A Merchant deliuered in *Baion* 40000 Maruides to receiue in *London* 5s–8d sterlig for euery Duckat of 374 Marueids. The question is now, how much sterling money paieth the Bils of Exchange for the said 40000 Marueids?

Answere. *Say, if* 374 *Maruides make one Ducket, what* 40000 *Maruides? Multiply and diuide, and finde* 106 *Duckets* $\frac{178}{187}$.

Then say againe, if one Ducket giue 5$\frac{2}{3}$s, *what giueth* 106$\frac{178}{187}$ *Duckets? Worke, and find* 30 *pounds* ——— 6s, *and* $\frac{34}{561}$s.

Otherwise it is wrought more briefe at one working, as in the last question before, in con-

considering that 5 ß 8 d *containeth one Ducket, or* 374 *Marueides. Therefore say by the Rule of three, if* 374 *Marueides giue* 5 $\frac{2}{3}$ ß, *what* 40000 *Marueides worke, and you shall also finde in your quotient* 30 *pound:* 6ß $\frac{34}{187}$. *And so many pounds sterling is to be receiued for the* 40000 *Marueids.*

In *London* 200 l. deliuered by Exchange for *Vigo*, 374 Marueids the Ducket of 5 ß, 10 d sterling, maketh 256457 $\frac{1}{7}$ Marueids: the which to returne and gaine 10 l. vpon the 100 pounds in *London*, say by the Rule of three direct, if 220 Pounds require 256457 $\frac{1}{7}$ Marueids, what 5 ß 10 d? Worke, and find 340 $\frac{25}{3280}$ Marueids, the price of euery Ducket in returne, which is the effect in the question required.

These may seeme sufficient for instructions.

NOtwithstanding for the further aid & benefit hereafter follow 6 speciall and most briefe Rules of Practise for English, French, and Flemmish money.

1	teacheth	*How to turne Flem. to Eng. sterlin.*
2		*How to turne Eng. sterlin. to Flem.*
3		*How to turne Flemmish to French.*
4		*How to turne French into Flemish.*
5		*How to turne sterling into French.*
6		*How to turne French into sterling.*

The

The fifteenth Chapter intreateth of the said fixe Rules of Breuity, and of valuation of Fnglish, Flemish, and French mony, & how each of them may easily be brought to others value.

How briefely to reduce poũds, shillings and pence Flemish into pounds, shillings, and pence English sterling.

IT is to be noted, that 7l' Flemish maketh but 6 pounds sterling: 7s flemish maketh 6s sterling, and 7d, flemish 6d, sterling, so that 7 yeeldeth but 6. Wherein is euident that then is lost $\frac{1}{7}$, (if it may be so called) when it is reduced into English money. wherfore toknow how much 233l' 13s,4d, flemish maketh English, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c, & that which resteth after this subtraction is the sum required: so that 233l' 13s,4d, flemish maketh 200l',5s,$8\frac{4}{7}$d,sterling.

Example			Another example.		
l'	s	d	l'	s	d.
233	13	4	311	0	0
$\frac{1}{7}$ 33	7	$7\frac{1}{7}$	$\frac{1}{7}$ 44	8	$6\frac{6}{7}$
200	4	$8\frac{4}{7}$ ster.	264	11	$5\frac{1}{7}$

To

To reduce pounds, shillings, and pence sterling, into pounds, Shillings, and pence Flemmish.

Note that a pound sterling maketh 1 l. 3 s, 4 d Flemmish: that is, 1 $\frac{1}{6}$ l 1 s sterling maketh 1 $\frac{1}{6}$ s Flem. and 1 ster. maketh 1 $\frac{1}{6}$ Flem. So that there is gained (if it may be so called) $\frac{1}{6}$ of the sum being thus reduced to Flē. for of $\frac{6}{6}$ is made $\frac{7}{6}$ which is one whole and $\frac{1}{6}$. Then to know how much 237 l, 7 s. 6 d, sterl. maketh Flem. subtract from your ster. the $\frac{1}{6}$ of the whole sum, and adde it to the same summe, and it maketh 276 l, 18 s, 4 d, which is the sum required.

Example.			Another Example.		
l,	s	d	l	s	d
337	7	6	237		
$\frac{1}{6}$ 39	11	3	$\frac{1}{6}$ 56	3	4
276	18	9 *ster.*	393	3	4

Ye shall note that the equality of Flemmish and French money is this, that is to say, the pound Flemmish maketh 7 pound $\frac{1}{7}$ French, or Turnois, 1 s Flemmish maketh 7 $\frac{1}{7}$ s French, and a groat Flemmish, maketh 7 $\frac{1}{7}$ d French.

Wherefore to know how much 142 l, 4 s 9 d, Flemmish maketh French ye must multiply the whole number twice by 6, beginning

ning at 8 d, and ſo forward, and the product of your ſecond multiplication diuide by 5, ſo the worke is finiſhed. Or multiply the ſaid ſum by 7 and take out of it $\frac{1}{5}$, adding it to the product of your multiplication by 7, and that is your number required. So that as wel by the one as by the other, 143 l. 4 s 9 d. Flemmiſh, maketh 1031 l. – 6 s. – $2\frac{2}{5}$ d French or Turnois.

Example.			The ſame otherwiſe.		
l.	s	d	l.	s	d
143	— 4	— 9 *Flem.*	143	— 4	— 9
		6			7
859	— 8	— 6	1002	— 13	— 3
		6	$\frac{1}{5}$ 28	— 17	— $11\frac{2}{5}$
5156	— 11	— 0	1031	— 6	— $2\frac{2}{5}$ *Frẽ.*
$\frac{1}{5}$ 1031	—	— $2\frac{2}{5}$ *Frẽ.*			

Another example.		Or thus.		
143 l. *Flem.*		143		
6		7		
858		1001		
6		$\frac{1}{5}$ 28	— 12	
$\frac{1}{5}$ 5148		1029 l.	— 12 *French*	
1029 l. $\frac{1}{5}$, *or* 12 s $\frac{1}{5}$ *Fre*				

A briefe reduction of ℔, ſ, and ð, French, into ℔, ſ, and ð, Flemmiſh.

Multiply 233 l, –8 ſ–4 ð French by 5, and diuide the Product twice by 6, that is, the ſaid number by 6, and the product againe by 6, and the quotient of this ſecond Diuiſion is the thing required. So that 233 ℔ 8 ſ̃ 4 ð French, maketh 32 l, 8 ſ, 4 $\frac{5}{9}$ ð Flemmiſh.

Example.			Another.		
℔	ſ̃	ð	℔	ſ	ð
233	–8—	4 *Fren.*	753		*Fren.*
		5	5		
1167	–1—	8	2756		
$\frac{1}{3}$ 194	–10–	$3\frac{2}{6}$	$\frac{1}{6}$ 627	—10	
$\frac{1}{6}$ 32	—8—	$4\frac{5}{9}$ *Flẽ.*	$\frac{1}{6}$ 104	—11—8	*Flẽ.*

To reduce l, ſ, and ð, ſterling into ℔, ſ, and ð, French, or Turnois.

The ℔ ſterling maketh 8 l, 8 ſ, French, that is to ſay 8 $\frac{1}{3}$ l: the ſ, maketh 8 $\frac{1}{3}$ ſ, and the peny 8 $\frac{1}{3}$ ð French. Wherefore to know what 231 ℔, 13 ſ, 4 ð ſterling maketh French, ye muſt multiply your whole ſum by 4 $\frac{2}{3}$, that is, by 7, & the product of it by 6, and diuide this

this second product by 5, and that is the summe required.

Otherwise, multiply the sum sterling by 8, and adde twice to the product $\frac{2}{5}$, and it shall produce the summe required. So that both wayes 232 ℔,13 ß,4 ð sterling maketh 1946 ℔ French, as hereunder followeth.

Example.

℔	ß	ð	
231	13	4	*ster.*
		6	
1390	0	0	
		7	
$\frac{1}{1}$ 9730	0	0	
$\frac{1}{1}$ 1946	0	0	*Fren.*

the same otherwise.

℔	ß	ð	
231	13	4	*ster.*
		8	
1853	6	8	
46	6	8	
46	6	8	
1946	0	0	*Fren.*

Another example.

753	*Ster.*
6	
4518	
7	
31626	
$\frac{1}{1}$ 6325	4 *Fren.*

The same.

	753	*Ster.*
	8	
	6024	
$\frac{2}{5}$	150	12
$\frac{1}{1}$	150	12
	6325	4 *Frē.*

To reduce pounds, shillings, and pence, French, into pounds, shillings, and pence sterling.

To know how much 1256 l, 12 s, 6 d, *Frē.* maketh in sterling money: multiply the sum by 5, and diuide the product by 7 and 6 at twice & the last quotient shall be the thing required, that is to say, 1256 l, 12 s, 6 d, maketh 140 pounds, 11 s, $1\frac{4}{7}$ d sterling.

Example.			Another Example.		
l,	s	d	l,	s	d
1256	12	6 *frē.*	2531		*Fren.*
		5		5	
6283	2	6	12755		
$\frac{1}{6}$ 1047	3	9	$\frac{1}{6}$ 2109	3	4
$\frac{1}{7}$ 149	11	$11\frac{4}{7}$ *ste.*	$\frac{1}{7}$ 301	6	$2\frac{2}{7}$ *ster.*

Note, that when any money is giuen by exchange at *London* for *Roan* at $7\frac{1}{2}$ d. or rather $71\frac{1}{7}$ for the Crowne of 50 s French, there is neither gaine nor losse: for it is one money for another, accounting 8 l, 8 s French for two pound sterling. So the giuer looseth the time of payment, which is about 15 dayes. and he that taketh it, hath the gaine of the same.

They of *Roan* that put forth or take money by exchange for *London*, ought to haue like consideration.

Item,

Item, when any man giueth at *London* 64 pence $\frac{1}{3}$, or rather 64$\frac{1}{3}$ 0, to haue at one of the Fayres of *Lyons* a Crowne *de Marc*, he that so giueth the money looseth the time, and he that taketh it, gaineth the same: for 62 pence $\frac{2}{7}$ is equall in value to 45 s French. He that putteth or taketh money at *Lyons* for *London*, ought to consider the same.

Item, when any deliuer in *Antwerpe* 75 pence, to receiue at *Lyons* a Crowne of *Marc*, he that putteth it forth looseth the time, and he that taketh it gaineth the same For 75 groates Flemmish is equall in value to 45 s French.

Thus for this time I make an end of the practise of Exchange and the instructions thereunto belonging: and according to my promise, yet further to gratifie such as are desirous to know the common Coynes vsed for trafficke among Merchants in these Cities following. Here followeth a briefe declaration of their Moneyes, and the reckonings, and accounts of them.

The sixteenth Chapter containeth a declaration of the valuation and diuersity of Coines of most places of Christendom for traffick? And the maner of exchange in those places from one City or Towne to an other: which knowne is right necessary for Merchants, by meanes whereof they doe find the gaine or losse vpon the exchange.

ITem for as much as the greatest diuersity of mony of exchange is at *Lyons*, therfore I will begin duly of the mony of that place.

At *Lyons* they vse Franckes, Soulx, and Deniers Tournois. A Francke maketh 20 Soulx, and one Soulx 12 Deniers. But the Merchants to keepe their bookes of accounts doe vse French Crownes of the marke at 45 soulx the peece and doe diuide it into 20 ß 1 ß and 12 ð.

Item, a marke of Gold maketh 65 ♎ of the marke, which serueth for exchange, and diuide it into 8 ounces. The ounce into 24 pence or deniers, the denier into 24 graines and at the sum or whole by imagination or gesse.

Also at *Lyons* there are 4 faiers in a yeere at the which they doe commonly exchance which are from three months to 3 months.

At

At *Geanes* they vſe the ſoulx: one Ducket maketh 3 pound.

At *Naples* they vſe Ducket, Taries, and Graines, the Ducket maketh 5 Taries, and one Tarie 20 graines: but they take 6 Duckets (which maketh thirty Taries) for the ounce.

A Duket maketh ten Carlins, and a Carlin tenne Graines, ſo that 2 Carlins make a Tarie, and 100 Graines make a Ducket.

At *Rome* they vſe the duckets of the Chamber: one ducket is worth 12 Guilis, and one Guilis 10 Soulx.

At *Venice* they vſe duckets currant at 124 ſoulx a peece or 24 deniers and one denier maketh 32 picolis,

At *Palerme* and *Meſſine* they write, after ounce, tarie, and graines, and one ounce is worth 6 duckets or 30 taries, and 1 tarie is 20 graines, and 1 graine 6 picolis, 1 ducket is alſo worth 24 Carlins.

At *Millan* they vſe ℔, ſ, d, of ducket Imperials, and Δ of exchange is worth 4 ℔.

At *Lucques*, *Florence* and *Ancone*, they vſe the Δ of Gold: in Gold the french Crowne is worth 71, but at *Boloigne* 31, 10 ſ.

At *Barſelone* they vſe the ſoulx: the ducket of exchange is worth 22 ſoulx.

At *Valence* and *Saragoſſe* they vſe the Liuer, Soulx and denier: the French Crowne of exchange is worth 20 ſoulx, and 1

soulx is twelue Deniers.

At the Faires of *Castill* they vse the Marueides, the Ducket is worth 375 Marueids.

At *Lisbone* they vse the Rayes, one Ducket of exchange is worth 400 Rayes.

At *Noremburge*, *Franckford*, and *Augusta* in *Germany*, they vse the *Krentzars*, wherof 60 make a Floren.

At *Antwerpe* they vse s, and d, de Gros, and they exchange into the denier *de Gros*, to wit, our English Peny.

At *London* they vse the 1 l sterling, and 1 d sterling, and they exchange it in 1 d sterling.

The exchange of Lyons at sundry places.

Item at *Lyons* there is exchange in three sorts, at the Cities and Townes following.

First they deliuer at *Lyons* one Mark to haue or receiue at *Naples* almost 41½ Duckets at *Venice* 70 Duckets currant, at *Rome* 63 Duckets of the Chamber; at *Lucques* and *Florence* 65 Δ of Gold at *Millan* 82 Δ.

And contrariwise, at the said Cities aforesaid they doe giue so much of mony to haue a marke at *Lyons*.

Secondly, they giue at *Lisbone* one Δ of Marke of 45 soulx Turnois a peece, to haue at *Geanes* almost 68 soulx, at *Palerme* and *Messine* almost 24 Carlins, at *Barselone* 22 soulx, at *Valence* or *Saragosse* 10 soulx, at the faire at *Castill* 350 Marueides, at *Lisbone*

bone 360 *Rayes*, in *Antwerpe* 57 Deniers *de Gros*, and at *London* 70 ß sterling.

And contrariwise they giue in the said Cities almost as much of their money to haue a French crown of the mark of *Lyons*.

Thirdly they doe giue at *Lyons* a △ of the Sun, to haue almost 93 *Krentzers* at *Franckford*, *Ausburg*, *Noremberge*, or other Cities in *Almaine*.

Also at *Lyons* only they do pay the change the ½ in Gold, and ½ in Money, or else all in Money, in giuing 1½ for the hundreth.

Changes at Naples and other Townes.

Item at *Naples* they giue or deliuer almost 112 Duckets, to receiue at *Rome* 100 Duckets of the Chamber at the old value.

Through *Lucques* and *Florence* they deliuer 100 duckets Carlins, to receiue there almost 86 △ of Gold.

Through *Palerm* & *Messine*, one ducket of 5 Tary, to receiue there almost 164 graines.

Through *Millan*, one ducket, to receiue there almost 90 Soulx.

Through *Geaus* one ducket, to receiue there almost 65 soulx. The whole summe to be paide within 10 dayes after the sight of the bill of Exchange.

Also at *Naples* they deliuer one ducket to receiue in *Antwerpe* almost 67 ß, or deniers *de gros*, within two moneths At *London* al-

most 60 ℔ sterling in three moneths. At *Barselone* almost 20 soulx within two moneths. At *Valence* almost 18 soulx within two moneths. At *Lisbone* 333 Rayes, within three moneths, and at the Fayre at *Castill*, almost 340 Marueyds at the same Faire.

Change of Venice to other places.

At *Venice* they deliuer 100 duckets currant to receiue in *Almaine* almost 140 *Florenes* at *Krentzers* the peece.

At *Lucques* and *Florence* almost 108 △ of Gold in 10 dayes.

Likewise at *Venice* they deliuer a ducket currant to receiue at *Palerme* and *Messine* almost 21 Carlins, at *Millan* almost 93 soulx, at *Geanes* almost 62 soulx, the whole at ten dayes end.

Of the Pair or Pari.

As touching the Exchange, it is necessary to vnderstand or know the *Pair*, which the Italians call *Pari*, which is no other thing then to make the money of the change of one City or Towne, to or with the money of another, by meanes whereof they doe find the gaines or losse vpon the Exchange.

Example.

Item haning receiued letters of credit of one of *Antwerpe*, that the △ of the Sunne is there worth 7 soulx: The question is, what the same is worth at *London*, when the *Pair*

or

or Exchange goeth for 63 ſhillings.

Anſwer. *Say if* 23 *giue but* 20, *what gineth* 7? *Worke, and find* 6 *s*, 1 $\frac{1}{23}$ *li*, *and ſo much is the* △ *of the Sunne worth at* London.

The ſeuenteenth Chapter containeth *alſo a declaratiõ of the diuerſity of the waights and meaſures of moſt places of Chriſtendome for trafficke. At the end of which diſcourſe are two Tables, the one for waight, and the other for meaſure, proportionate and reduced to an equality of our Engliſh meaſure & waight, by the aide whereof the ingenious may eaſily by the Rule of three, conuert the one into the other at pleaſure, &c.*

AT *London*, and ſo all *England* through, are vſed two kinds of waight & meaſures, as the *Troy* waite & the *Haberdepoiſe* frõ the *Troy* waight is deriued the proportion and quantity of all kind of dry and liquid meaſures, as peckes, buſhels quarters, &c. wherewith is bought & ſold all kind of graine & other cõmodities met by the buſh. And in liquid ale, beere, wine, oyle, butter, hony, &c, vpon theſe grounds & ſtatutes is bread made & ſold by the *Troy* waight: and ſo is gold, ſiluer, pearle, precious ſtones, and iewels. The leaſt quantity of this *Troy* waight

weight is a graine: 24 of these grains make a penny waight, 20 peny weights an ounce, and 12 ounces a pound; 2 pounds or pintes of this weight maketh a quart. And so ascending into bigger quantities is produced the measures wherby are sold our other naturall sustenance: viz. Ale or Beere, with all other necessarie commodities, as Butter, Hony, Herrings, Eeles, Sope, &c. All which last before rehersed, though their measures (wherein they are contained) bee framed and deriued from the *Troy weight*, yet are they in traffique with duers commodities, as Lead, Tinne, Flax, Wax, with all other commodities both of this Realme, and of other forraigne Countries whatsoeuer, bought and sold by the *Haderdepoise weight* after 16 ounces to the pound and 112 pound to the C. weight. And to euery C. is allowed but 1 pound weight at the common beame. From hence is also deriued the weight of *Suffolk* Cheese, which containeth 32 cloues, 8 pound to a clue, and waieth in all 256 pounds. And also the barrell of *Suffolke* Butter is or should be of like wait with the weight of cheese, viz. 256 pounds. More, 14 of these pounds make a stone and 26 stone containeth a sacke of English wooll. Forraigne wools, to wit, French Spanish, and Estrich is also sold by the pound or C. weight, but most commonly by the

See further of these waits and measures in reduction, beginning pa. 126.

Roue

Roue, 25 pound to a Roue: other commodities of Tale are bought and sold by the C, fiuescore to the C. Except headed ware, to wit, Cattel, Nayles and Fish, which are sold after sixescore to the 100 There is also two other sorts of measures, to wit, the Elle & the Yard. By the Elle is vsually met Linnen Cloth, as Canuas, &c. And by the yard silkes, woollen clothes, &c.

Antwerpe.

At *Antwerpe* are also 2 sorts of weights, their gold and siluer weight, and their common waight Gold and siluer is weighed by the *Mark*, the *Marke* is 8 ounces, the ounce 20 Esterlings and the Esterling 32, as our graines The Goldsmiths diuide that into smaller but not the Merchants: The proofe of gold is made by Caracts, whereof 24 maketh a *Marke* of fine Gold, the Caract is 24 graynes: the proofe of the mony is made by *Deniers*, 12 deniers is 1 £ fine, that is, a *mark* of fine siluer; the denier also is diuided into 24 graines, and the graine into 4 quarters.

Item, 100 *Markes in* Antwerpe, Troy *waight, maketh at* Lyons 101 *Markes two* $\frac{1}{2}$ *ounces, and* 20 *graines*, 231. *At* Noremberge 102 *markes* $\frac{1}{2}$ *ounces*, 2 *quints*, 2 *Deniers*; *at* Frankford 105 *markes*, *at* Ausburge 104 *markes, three ounces*, 1 *quint*. *At* Venice 103 *markes*, 1 *ounce*, 7 *deniers* 18 *Graines*. *At* London 66 *pounds*.

The marke of Gold or siluer at Antwerpe

Troy

Troy *weight, which is* 8 *ounces, maketh* 7½ *ounces common weight, with which all other merchandise is weighed. So that the* Troy *waight is greater then the common waight by* 6 ¼ *in the* C. *By this weight of* Troy, *they also waigh* Musk, Amber, Pearle *&c,*

All silkes are bought at Antwerpe, *by the* Burges elle, *which is greater then the common measure, by which they retaile by two in the hundred. Their common elle is* ¾ *of our yard, and* ⅘ *of our elle.*

Lyons.

At *Lyons* is vsed 3 sorts of waight, whereof the firſt is the common towne waight, with which they weigh all kinde of Spicery and diuers other merchandise. The second is called *Geneua* waight, which is 8 in the C. greater then the common weight, with which they weigh silkes, &c. The third is french waight, called commonly the *Mark weight*, and 100 pounds thereof maketh 106 ¼ ℔ *Geneua*, and 114 ½ of their common weight: with which french waight, is weighed all things that paied custome or tole.

At *Lyons* is also vsed two sorts of elles or *Aulnes*. The one wherewith they measure grosse clothes, as canuas, and such like. The other is called the french elle or *Aulne*, with which they measure all other kinde of merchandise, whereof seuen common town

Elles

Elles maketh 11 ordinarily french elles,

Roan.

At *Roan* 6 ½ Muides of salt being the measure of the place, make a C. at *Armuiden* in *Zeland*; and the C. of *Bronage* measure of *Armuiden* maketh at *Roan* 11 Muides, 30 Muides make a last of corne, and 16 a last of Oates, 100 pound weight there, maketh at *London* 114 ½, and 190 ⅔ at *Antwerpe*. And 100 elles make at *London* 115 ¼.

Noremburge

A 100 pound weight at *Noremburge* maketh at *London* 111 ¾, at *Antwerpe* 107 ½, and 110 elles at *Noremburge* make at *London* 75 ½, at *Antwerpe* 95 ½. &c,

Lisbone.

The 100 waight at *Lisbone* maketh foure Roues, euery Roue 32 pounds, so that their C. waight is 128 pounds, and their pound containeth 14 ounces, and a 100 pounds of their waight maketh at *London* 113 ½.

Their silke, cloth of Gold, and Woollen is measured with a measure which they call a cubite, conteining about ¾ of a Varre of *Castile*. Howbeit, their common measure is called a Varre, which maketh 5 Palmes, and containeth 1 ½ of a Varre of *Castile*, our elle of *London* is equall with the Varre of *Lisbone*.

All

All kind of Merchandise brought from *Flanders*, *Roan*, or *Britaine* payeth at *Lisbone*, as a ducty or custome to the King, 20 in the 100, which they call the tenth in merchandise, and the other tenth in money.

Note also that all kind of Merchandise comming to *Lisbone* by land, payeth lesse in custome then that that commeth by water.

Ciuill.

The Roue of *Ciuill* is 30 pound, 4 Roues make their C weight, which is 120 pounds. The 100 pounds of *Ciuill* maketh at *London* 102 pounds. Their other common measure is a Varre, whereof 100 maketh at *London* 74 Elles, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* be two sorts of waights, the one called *La Grosse*, the other *La Suttle*, with the grosse is weighed all kind of great wares, and with the small all kind of spicery, and such like: 96 poūds of grosse weight there maketh at *London* 100 pounds & 100 pounds of spicery there without any tare or allowance, make at *London* 64, and with tare 65.

Their owne common measure are Braces whereof 100 make at *London* 55 ½ Elles, at *Antwerpe* 92 ½ &c.

Florence.

At *Florence* the 100 l. waight maketh at

Aqui-

Aquila, for Saffron 110, and 145 pounds of *Florence*, make at *Roane* but 100 pounds, the weight of *Florence* and that of *Luke* is all one.

Their other measures are braces, whereof 100 maketh at *Antwerp Burges* measure, 81 ½ Els, 100 Braces there make at *London* 49 Elles, &c.

Lucque.

The *Lucque* Sattens are commonly sold at *Lyons* by weight, and 132½ pounds maketh at *Lyons* 100 pound, so that 1 pound ⅝ maketh at *Lions* but one pound.

Their other measures are braces, whereof 100 of them make at *London* 50 Els, at *Antwerpe* 83⅓ Els, &c.

Aquila.

At *Aquila* their 100 pounds maketh at *London* 71¼, their 136 ¾ pounds of Saffron maketh at *Geneua* but 100, and 111 ℔ of *Geneua* maketh 151 ℔ at *Aquila*.

Valentia.

At *Valentia* be 2 sorts of waights, a great and a small. The C weight of great waight containeth 4 Roues the Roue 36 ℔, so the C great weight is 144 ℔, & the C weight small containeth but 120 poũds, & is also parted into 4 Roues, which is 30 poũds to a Roue. By the smal is sold the scarlet grain with all other kind of spicery, & by the great is sold wool, with all such like grosse wares. The 1⅓

pounds

pounds of silk at *Valentia*, maketh at *Lyons* one pound *Geneua* waight. The charge of great merchandise at *Valentia* containeth 432 ℔, and in small wares 360 pounds. The weight here and at *Barsellone* is all one. Their 100 ℔ weight maketh at *London* 78 ℔, and at at *Antwerde* 75.

Dansicke.

At *Dansicke* or *Spruce-land* the rule is that whosoeuer buieth any merchandise there buieth it by the ship-pound, which is 320 ℔, 20 Lispounds make a ship pound and the Lispound containeth 16 ℔, which ship-pound of *Dansicke* maketh at *Antwerpe* 260 ½. Their 100 l, waight maketh at *London* 86 ⅓ &c.

Their other common measures are elles, whereof 100 make at *London* 72 ¾, and at *Antwerpe* 120 ¼ Elles.

Toulhouse.

At *Toulhouse* 6 Cabes of wood maketh a Charge, two Cesternes of corne-measure, and all kind of graine maketh a Charge the Cesterne weigheth 160 l, weight of that place, Their C. l, in weight, maketh at *London* but 91 ¼ pound.

Geanes.

At *Genua* or *Geanes*, 100 l, of their weight maketh at *London* 71 ¼, and at *Antwerpe* 67 ½, a 100 l weight at *Genua* maketh at *Venice* to wit, Suttle 106 ℔.

Their

Their other common Measures are *Palmes*, whereof 100 make at *London* $20\frac{4}{5}$ Elles: and at *Antwerpe* $34\frac{3}{5}$.

The rest are supplied in two Tables, which hereafter followeth: whereby the ingenious may gather his desire.

The agreement of the Waights of diuers Countries, the one with the other, being reduced to an equality, and drawne into a Table as followeth.

112 pound waight at *London*, make at		112 pound waight at *London*, make at	
Antwerpe.	$107\frac{5}{8}$	*Venice grosse weight.*	$105\frac{3}{8}$
Franckford.	099	*Venice suttle waight.*	$166\frac{7}{8}$
Collen and Ausburg.	$102\frac{1}{4}$	*Aquila.*	$157\frac{1}{4}$
Noremburg.	$100\frac{1}{8}$	*Vienna.*	$089\frac{3}{8}$
Roan.	098	*Preslaw.*	$134\frac{5}{8}$
Paris.	$102\frac{1}{4}$	*Leipsig.*	$101\frac{1}{4}$
Lyons.	$118\frac{1}{2}$	*Dansicke.*	$129\frac{1}{4}$
Diepe.	$100\frac{1}{4}$	*Lubecke.*	$097\frac{3}{8}$
Geneua.	$090\frac{1}{8}$	*Barcellon.*	$144\frac{1}{2}$
Towlouse.	$122\frac{3}{4}$	*Lisbone.*	099
Rochell.	$124\frac{7}{8}$	*Geanes.*	$157\frac{1}{4}$
Marcellis.	$124\frac{1}{4}$		
Ciuill, &c.	$109\frac{3}{4}$		

The other Table of the agreement of Measures of diuers Countries reduced vnto an equality, by the aide whereof you may with the vse of the Rule of three conuert either more or lesse of any one measure vnto other.

60 Elles or 75 yards at *London* make at	*Antwerpe*	100	*Elles.*
	Norenburg	104 1/2	
	Franckford	125	
	Leibsig	125	
	Preslaw	125	
	Dansicke	83	
	Vienne in Austi	87	
	Lyons in France	60 4/42	*Aulnes.*
	Paris in France	57	
	Roan in Normandy	52	
	Lisbone	60	*Varres.*
	Cisill in Spaine	81	
	Castile in Spaine	81	
	Metbera Iles	62	
	Venice	108	*Braces.*
	Luques	130	
	Florence	122 1/3	
	Millan	138	
	Rome	90	*Canes.*
	Geanes	288 67/75	*Palmes.*

The

The eighteenth Chapter treateth of Sports and Paſtimes, done by Number.

IF you would know the number that any man doth thinke or imagine in his mind, as though you could diuine, bid them triple it, or put twice ſo much more to it as it is; which done, aske him whether it be euen or odde: if he ſay odde, bid him take one to it, to make it euen, and for that one keepe one in your mind. Now after he hath taken one to it, to make it euen, bid him giue away halfe, and keep the other halfe for himſelfe, which when he hath done, bid him triple that halfe, and againe after he hath tripled it, aske him whether it be euen or odde: if he ſay odde, then bid him take one to make it euen againe, and for that laſt one keepe two in your mind, now after he hath made his number euen, bid him caſt away the one halfe, and keep the other ſtill, from which halfe that he keepeth, cauſe him ſubtilly to put away or giue you 9 out of his number, and for each 9 that he giueth you, keepe 4 in your mind, and thereunto ioyne the 3 which I bad you keepe, and you ſhall haue your deſire.

Example.

Imagine he thought 7, the triple whereof is

is 21, *and because it is odde, hee is to take* 1 *to make it euen, which first* 1 *giuen is for you to keepe in minde. Then the halfe of his* 22 *being cast away, hee reserueth still* 11, *which after you haue bid him triple, it maketh* 33, *then in giuing of him one againe to make it euen: vpon that last* 1 *reserue* 2 *in your minde, then his halfe of* 34 *maketh* 17, *from whence hee can giue you* 9 *but once. Therefore that yeelding to you* 4 *and the* 3 *that you keepe make* 7 *your desire.*

Another kinde of Diuination, to tell your friend how many pence or single peeces, reckoning them one with another, he hath in his purse, or should thinke in his minde.

Which to doe, first bid him double the peeces hee hath in his purse, or the number hee thinketh, (if he participate his number or secrecy vnto some one friend that sitteth by him that can but multiply, and adde nauer so little: if their number be great, then shall they worke as you bid them so much the surer).

Now after hee hath doubled his number, bid him adde thereunto 5 more, which done, bid him multiply that his number by 5 also: which done, bid him tell you the iust sum of his last multiplication, which sum the giuer thinking it nothing auaileable, because it is so great aboue his pretended imagination: yet thereby shall you presently with the helpe of Subtraction tell his proposed

numbers.

number. The Rule is this.

Imagine he thought 17 *Double* 17, *and it maketh* 34, *whereunto if you ad* 5, *it maketh* 39: *which multiplied by* 5, *as here is practiſed in the margent, it yeeldeth* 195, *which* 195 *is the ſumme deliuered you in the worke: then for a generall rule, you ſhall euermore cut off the laſt figure towards your right hand, with a daſh of your Pen, as here is performed, as a figure nothing auaileable vnto your worke, and then rebate* 2 *from your firſt figure, after* 5 *is cut off, and the reſt ſhall euermore be your deſire, as by this example doth appeare.*

17	
2	
34	
5	
39	
5	
19	5
2	
17	

Another of a Ring.

If in any company you are diſpoſed to make them merry by manner of diuining, in deliuering a Ring vnto any one of them, which after you haue deliuered it vnto them, that you will abſent your ſelfe from them, and they to deuiſe after you are gone, which of them ſhall haue the keeping thereof, and that you at your returne will tell them what perſon hath it, vpon what hand, vpon what finger, and what ioynt. Which to doe, cauſe the perſons to ſit downe all on a row, & to keepe likewiſe an order of their fingers: now, after ye are gone out frō them to ſome other place, ſay vnto one of the

lookers on, that he double the numbers of him that hath the Ring, and vnto the double bid him adde 5, and then cause him to multiply the Addition by 5, and vnto the product, bid him adde the number of the finger of the person that hath the Ring. And lastly, to end the worke, beyond that number towards his right hand, let him set downe a figure signifying vpon which of the ioints he hath the Ring, as if it be vpon the second ioint let him put downe 2. Then demaund of him what number he keepeth, from the which you shall abate 250, and you shall haue three figures remaining at the least. The first towards your left hand, shal signify the number of the person which hath the ring, the second or middle number shall declare the number of the finger, and the last figure towards your right hand shal betoken the number of the ioint.

Example.

Imagine the seuenth person is determined to keepe the Ring vpon the fifth finger, and the third ioint: first double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplied by 5 yeeldeth 95, vnto which 95 adde the number of the finger, and it maketh 100: and beyond 100 toward the right hand, I set downe 3 the number of the ioint, all maketh 1003, which is the number that is to bee deliuered you, from which abating 250, there resteth 753,

which

which prefigureth vnto you the seuenth person, the fifth finger, and the third ioint.

But note that when you haue made your subtraction, if there doe remaine 0: in the place of tennes, that is to say, in the second place, you must then abate 1, from that figure which is in the place of hundreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth finger, as if there should remaine 803, you must say, that the seuenth person vpon his tenth finger, and vpon his third ioint, hath the Ring.

Another of three Dice casting.

And after the same maner if a man doe cast 3 Dice, you may know the points of euery one of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same sum to multiply by 5, and vnto the product ad the points of one of the other Dice. And behind the number towards the right hand, to put the figure which signifieth the points of the last Die, and then to aske what number he keepeth, from which abate 250, and there will remaine 3 figures, which doe note vnto you the points of euery Die.

Another of things hidden.

If three diuers things are to be hidden of three diuers persons, & you to diuine, which of the three persons hath the three diuers

things, do thus: imagine the three things to be represented by *A*, *B*, *C*. Then secondly keepe well in your mind which of the persons you meane to be the first, second, and third. Then take 24 counters or stones, and your three things, and giue *A* to the party whom you imagine to be your first man, & therewithall giue him one of your 24 counters in his hand, and *B*, vnto your second man, & therewithall 2 Counters. And *C*, vnto your third man, and therewithal 3 counters: and leaue the rest which are 18 still among them, which done separate your selfe from them, & afterwards bid them change the things among them as they shall thinke good: which done, after they are agreed, bid him that hath such a thing, as before you haue represented by *A*, for euery counter that he hath in his hand, to take vp as many more. And for him that hath *B*, for euery one in his hand to take vp two. And for him that hath *C*, for euery one in his hand to take vp 4, and the rest of them to leaue still vpon the boord. These 3 things & the three persons being fully printed in your mind, come to the Table, & you shall euermore find one of these 6 numbers, 1, 2, 3, 5, 6, or 7. If therefore one remaine still vpon the boord, then haue they made no exchange, but keepe them still, as they were deliuered vnto them. So that the first man hath

hath *A*, the second *B*, and the third *C*. But if 2 remaine, then the first man hath *B*, your second man *A*, and your third man *C*. The rest of the worke and the order thereof are here apparant by the Table following.

	1	*A*			1	*B*
1	2	*B*		5	2	*C*
	3	*C*			3	*A*
	1	*B*			1	*C*
2	2	*A*		6	2	*A*
	3	*C*			3	*B*
	1	*A*			1	*C*
3	2	*C*		7	2	*B*
	2	*B*			3	*A*

Another Diuination of a number vpon the casting of two Dice.

First let the Caster cast both the Dice, & marke well the number: then let him take vp one of them, it maketh no matter which, and looke what number it hath in the bottome, and adde all together: then cast the Dye againe, and keepe in his mind what all together maketh: then let the Dyce stand, and bring seuen with you, and thereunto adde the rest of the pits that you see vpon the vpper side of the side of the Dyce, and so many did the Caster cast in all.

FINIS.

A briefe Addition of Interest, with a Table of Board and Timber measure, calculated by *Robert Norton* Gentleman.

Denfinitions of interest.

1 PRincipall, *is the summe from which the* Interest *is reckoned.*

2 Interest *is the summe reckoned for the lending or forbearance of the* Principall *for any termes or time.*

3 *The proportion betweene the* Interest, *and the* Principall *is the reason of* Interest.

4 Interest simple, *is that which is counted from the* Principall *only.*

5 Interest compound, *is that which is counted for the* Principall, *together with the* Arriage *of* Interest *and* Principall.

6 Interest profitable, *is that which is added to the* Principall.

7 Interest damageable, is that which is *to be subtracted from the* Principall.

Scholar. *Sir, I haue beene much troubled with resoluing questions of* Interest: *I pray you shew mee some way to ease the tediousnes, which I find breedth errors also.*

Master. *Although vsury ought not to bee found amongst* Christians: *yet seeing our hearts are growne to that hardnes, that Chari-*

ty

ty ſhall haue nothing but what ſhee paies for: therefore moſt Common-weales haue, although not allowed, you winked at this vſury, onely curbing the horrible exceſſes, which elſe would grow, and tolerating the vſe thereof for auoiding worſe extremities.

And becauſe the *compound Intereſt*, yea and the *ſimple Intereſt*, alſo in many queſtions of *profit and loſſe*: otherwiſe aske much labour, and ſo becomes ſubiect (as you ſay) to much error, I haue thought good to ſet downe this Table containing *foure Columns.*

The firſt whereof is for Intereſt ſimple damageable, *where loſſe commeth, by receiuing before the day due.*

The ſecond is, for the number of yeeres, quarters, moneths, *or other equall tearmes mentioned in each queſtion.*

The third is, for compound Intereſt, *when a* yeerely, quarterly, and continued paiment *is not in the queſtion.*

The laſt is, for yeeres continued.

As the queſtions following ſhall ſhewe vnto you, wherein I would haue you to marke diligently the reaſon of them, that you may apply any other queſtion, for that this will ſerue for any number of yeeres of equall termes, whatſoeuer vnder 21, and for any ſumme of mony propounded.

Scholar.

The Table for intereſt, after 10, for 100 taken out of SYMON STEVENS Arithmetick.

Inter. ſimp. dom.	*yeeres*	*Compound Intereſt.*	*Intereſt continued.*
9090909	1	9090909	9090909
17424242	2	8264463	17355372
25116549	3	7513148	24868520
32359406	4	6830135	31698655
38926072	5	6209214	37907869
45176072	6	5644740	43552610
51058425	7	5131582	48684191
56613980	8	4665075	53349266
61877138	9	4240977	57590143
66877138	10	3855434	61445677
71639043	11	3504940	64950617
76184497	12	3186309	68136926
80532323	13	2896645	71033571
84695090	14	2633314	73666885
88699090	15	2393922	76060807
92545244	16	2176293	78237100
96248948	17	1978448	80215548
99820377	18	1798589	82014137
103268653	19	1635081	83649218
106601986	20	1486437	85135655
109827792	21	1351306	86486961

The roote of theſe three Tables is alwaies 10000000.

Scholer.

Scholar. I pray you proceed as plainely and briefely as you can for my memories ſake, and I ſhall be euer bound to you for your paines.

Maſter. *There is* 324*l* *to be paid in ſixe yeeres: namely,* 54 *each yeere: the queſtion is, what it is worth ready mony, abating* ſimple Intereſt *after* 10 *for a* 100? The firſt queſtion.

Heerein you may perceiue it will come vnto leſſe mony, and therefore it muſt bee *domageable*, and belongeth vnto the *firſt columne*, whereby you ſhall find the ſolution. Thus, Multiply the roote of theſe Tables which is 1000000 by the *yeeres* of the *queſtion* 6, and will produce 60000000. And then ſay 60000000 giue 45176072 (which is the number of the *firſt collumn* anſwering to the ſixt yeare) how much will 324*l* giue, which if you worke by the *golden Rule* will anſwere the queſtion.

Scholar. *This can I ſoone doe, and finde it giueth* 243*l*' *and* $\frac{91471128}{100000000}$ *parts of a pound, which I alſo can reduce into ſhillings and* D, *by your former inſtructions. But I pray you proceed to ſome queſtions of other kindes of Intereſt.*

Maſter. *A man oweth* 224 *l to be paide by* 56 *l a yeere for* 4 *yeares, what ſhall hee pay by ſimple intereſt* 10 *pro* 100. *if he pay all the parts together at the end of the* 4*th yeere?* The ſecond queſtion.

To

To answer this & such like, you must consider that hee holdeth (which by the first conditions hee ought not) part of the principall each yeare, and seeke the interest of ech principall for ech yeere.

100 100 100 100 giueth, 0 10 20 30

400 60

Or thus, 400 giues 60 l' what wil 224 l' giue.

400l' 60l' 224l'

Answer. 33 l' 12 s, which added to the whole debt 224 l' maketh 257 l' 12 s.

3 Questiō. *Another Question is, if a man be to pay 324 l: namely 55 l' per An. for 6 yeares, how much shall he pay ready money, abating compound interest, after 10 for 100?*

For this seeke in the fourth columne of the Table which is for continued paiments of compound interest) for the number against the 6 yeare, which is 43552610, and multiply the roote of the Table 10000000 by 6, and it will produce 60000000. And then say, if 60000000 giue 43552610, what shall 324 giue.

Scholar. That I finde by the rule of three direct to giue 235 l' $\frac{31457664}{60000000}$ part of a poūd: yet I pray you propound a question or two, and prooue if I can worke it right.

Master

Maſter. *What commeth 380 ℔ to with his compound intereſt profitable for 8 yeeres after 10 for 100?* 4 queſtion.

Scholar. The queſtion being of compound intereſt, profitable, and not for continued payments, I muſt ſeeke in the third colum for the number againſt the 8 yeere, where I find 4665075: Therefore I ſay, if that number giue the 10000000, the roote of the Table, what will 380 giue 814 poũd $\frac{162000}{10000000}$ parts of a pound.

Maſter. You conceiue the reaſon well, I will trie you with one queſtion more, and ſo leaue you to your practiſe.

There is one oweth 700 ℔ to be paid at the end of 10 yeeres, the queſtion is what it is worth in ready money, abating intereſt compound damageable? 5 queſtion

Scholar. Now ye had almoſt ſpoſed me, but that I conſidered that it was no payment continued and being damageable it muſt decreaſe the ſumme, I therefore know the queſtion belongeth vnto the third columne of the Table, and muſt herein ſay if the roote of the Table 10000000 giue 3855434 (the number therin againſt the 10 yeere or terme,) what ſhall 700 ℔ giue. So that by multiplying and diuiding, I find 269 ℔, and $\frac{3803800}{10000000}$ parts of a pound.

Maſter. If your queſtion be to ſeeke the reaſon of your intereſt, multiply the roote

of

of the Table by the yeeres or termes of the queſtion, and the product ſhall be in ſuch reaſon to the ſaid roote, either as the principall to his Intereſt, or as the Intereſt vnto the principall according to the nature of the queſtion, which reaſon it ſelfe will ſufficiently direct, and if you deſire to haue it after 12, 16, 20, or any other ſum for 100, if you multiply the root of the Table, by 100 the Principall, and diuide the product by the principall and his Intereſt, the quotient will be the number anſwering the ſame reaſon required. And thus much ſhall be ſufficient to haue ſpoken of the calculation of Tables and ſolutions of Queſtions of Intereſt. And ſo I will leaue you to your owne practiſe, or my further leaſure.

Scholar. Sir, I muſt thanke you for your louing paines, and deſire I may be able to requite this your curteſie: And leſt I ſhould be troubleſome to your greater buſineſſe, I will take my leaue at this time, and attend your better leaſure.

The vſe of this Table following.

IF vpon a ſcale or ruler you diuide one inch into 10 equall parts or primes, and againe by diagonals and paralell lines if you ſubdiuide each of them into 10 equall parts or ſeconds, as my *Trigonal Scales* are diuided with your compaſſes, you may take of more exact running meaſure for boord and timber then by any other means whatſoeuer, and ſo place the ſame, or this Table if you will vpon any rule.

Alſo by meanes of the column of ſquare rootes, you may readily find a ſquare equal to any paralelipipedon or peece of timber which is thicker then it is broad. As for example, ſuppoſe a peece of timber to be 10. inches thicke and 20 inches broad: if I multiply thoſe ſides one by another, they will produce 290, then ſeeking in the columne of ſquare roots for 290 which I finde not; but I finde 289 the neereſt number to 290 to ſtand againſt 17: therefore I ſay 17 inches *fere* will make a ſquare equall to ſuch an vnlike ſquared peece: then looking in the columne of Timber meaſure againſt 17, you ſhall finde that 5 inches, 9 prime or $\frac{9}{10}$, and 8 ſeconds, or $\frac{8}{100}$ of an inch in length of that peece will make a foote of Timber.

Likewise for boord measure, you may finde how much in length of any breadth of boord must be in one foote.

By the like meanes suppose for example that a boord appointed to bee measured is 15 inches $\frac{1}{4}$ broad, if I desire to know how much in length thereof will make a foote. I seeke in the columnes that stand vnder the title of vnites and parts for 15 $\frac{1}{4}$, and also ouer against the samee in the columne vnder the title of boord measure, where I finde 9 inches 1 prime or tenth of an inch, and 4 seconds or hundreths of an inch will make a foot at that breadth: the like may be practised for any other bredth of boord whatsoeuer.

A Table of Boord and Timber measure, more perfect then euer hath beene made; shewing also the Square roote, betweene 4 and 37 from quarter to quarter, Calculated by *R. N.*

Boarde measure.	Vnits and parts.	Square rootes.	Timber measure.
36 0 0	4	16	108.0.0
33 7 8	1	18	98 7 4
32 0 0	2	20	85 3 8
30 1 0	3	22	76 5 8
28 0 8	5	25	69 1 2
27 4 2	1	27	62 3 3
26 1 1	2	30	57 1 7
24 9 9	3	32	52 2 6
24 0 0	6	36	48 6 0
23 0 4	1	39	44 2 4
22 1 4	2	42	40 8 9
21 3 3	3	45	37 7 1
20 5 7	7	49	35 2 6
19 8 6	1	52	32 8 7
19 2 0	2	56	30 7 2
18 8 9	3	60	28 7 7
18 0 0	8	64	27 0 0
17 4 6	1	68	25 3 0
16 9 4	2	72	23 9 1
16 4 6	3	76	22 5 7
16 0 0	9	81	21 3 3
15 5 6	1	85	20 1 8
15 1 5	2	90	18 9 7
14 7 9	3	95	18 1 8
14 4 0	10	100	17 2 8
14 0 4	1	105	17 4 6
13 7 1	2	110	15 2 2
13 3 9	3	115	14 9 5
13 0 9	11	121	14 2 9
12 8 0	1	126	13 6 3
12 5 2	2	132	13 0 0
12 2 7	3	138	12 5 2
12 0 0	12	144	12 0 0
11 7 5	1	150	11 5 1
11 5 2	2	156	11 0 5
11 2 9	3	162	10 6 3
11 0 7	13	169	10 2 2
10 8 7	1	175	9 8 3
10 6 7	2	182	9 5 8
10 4 7	3	189	9 1 4

Boarde measure.			Units and parts.	Square rootes.	Timber measure			Boarde measure			Units and parts.	Square rootes.	Timber measure		
10	2	8	14	196	8	8	1	6	8	6	21	441	3	9	2
10	1	1	1	203	8	5	0	6	7	7	1	451	3	8	2
9	9	3	2	210	8	2	0	6	6	9	2	462	3	7	3
9	7	6	3	217	7	9	4	6	6	1	3	473	3	6	7
9	6	0	15	225	7	6	9	6	5	4	22	484	3	5	7
9	4	5	1	232	7	4	3	6	4	7	1	495	3	4	9
9	2	9	2	240	7	2	6	6	4	0	2	506	3	4	1
9	1	4	3	248	6	9	7	6	3	3	3	517	3	3	3
9	0	0	16	256	6	7	5	6	2	6	23	529	3	2	7
8	8	6	1	264	6	5	4	6	1	9	1	540	3	1	9
8	7	3	2	272	6	3	5	6	1	2	2	552	3	1	1
8	6	0	3	280	6	1	6	6	0	6	3	564	3	0	6
8	5	8	17	289	5	9	8	6	0	0	24	576	3	0	0
8	3	4	1	297	5	8	1	5	9	4	1	588	2	9	8
8	2	2	2	306	5	6	4	5	8	3	2	600	2	8	7
8	1	1	3	315	5	4	5	5	8	9	3	612	2	8	2
8	0	0	18	324	5	3	3	5	7	6	25	625	2	7	6
7	8	9	1	331	5	2	2	5	7	0	1	637	2	7	1
7	7	8	2	342	5	0	4	5	6	5	2	650	2	6	5
7	6	8	3	351	4	9	2	5	5	8	3	662	2	6	0
7	5	8	19	361	4	7	8	5	5	4	26	676	2	5	4
7	4	8	1	370	4	6	6	5	4	8	1	689	2	5	0
7	3	9	2	380	4	5	4	5	4	3	2	702	2	4	7
7	2	9	3	390	4	4	0	5	3	8	3	715	2	4	1
7	2	0	20	400	4	3	2	5	3	3	27	729	2	3	7
7	1	1	1	410	4	2	1	5	2	8	1	742	2	3	2
7	0	2	2	420	4	1	1	5	2	3	2	756	2	2	7
6	9	3	3	430	4	0	1	5	1	8	3	767	2	2	4

Boarde measure	Vnits and parts	Square rootes.	Timber measure.	Boarde measure	Vnits and parts	Square rootes.	Timber measure.
5 1 4	28	784	2 2 0	4 3 6	33	1089	1 5 9
5 0 9	1	798	2 1 6	4 3 3	1	1104	1 5 6
5 0 5	2	812	2 1 2	4 3 0	2	1120	1 5 4
5 0 0	3	826	2 0 9	4 2 7	3	1136	1 5 2
4 9 6	29	841	2 0 5	4 2 3	34	1156	1 4 9
4 9 2	1	856	2 0 2	4 2 0	1	1174	1 4 7
4 8 8	2	871	1 9 8	4 1 7	2	1190	1 4 5
4 8 4	3	885	1 9 5	4 1 4	3	1210	1 4 3
4 8 0	30	900	1 9 2	4 1 1	35	1225	1 4 1
4 7 6	1	915	1 8 9	4 0 8	1	1237	1 4 0
4 7 2	2	930	1 8 6	4 0 5	2	1247	1 3 7
4 6 8	3	945	1 8 3	4 0 3	3	1280	1 3 5
4 6 4	31	961	1 7 9	4 0 0	36	1296	1 3 3
4 6 1	1	974	1 7 6	3 9 7	1	1313	1 3 1
4 5 7	2	987	1 7 4	3 9 4	2	1331	1 2 9
4 5 3	3	1000	1 7 1	3 9 1	3	1350	1 2 7
4 5 0	32	1024	1 6 9	3 8 9	37	1369	1 2 6
4 4 6	1	1040	1 6 6	3 8 7	1	1388	1 2 4
4 4 3	2	1056	1 6 3	3 8 4	2	1416	1 2 2
4 4 0	3	1072	1 6 2	3 8 2	3	1425	1 2 0

An Appendix concerning the Resolution of the *Square* and *Cube* in numbers, to the finding of *their side. by* Ro: Hartwell.

A Figurate number is a number made by the multiplication of one number or more by another. A figurate number, what.

The sides of a Figurate number, are the numbers by whose multiplication it is made. The sides of a figurate number, what.

A Figurate number is twofold, a { Plaine. / Solide.

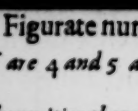
And it is { Of one multiplication, / Or consequently of many. } as a { Plaine. / Solide.

And in each { Both Equaliter. / And Inequaliter.

A figurate number made of one multiplicatiō by two sides or numbers muliplied together, is called a plain figurate number. A plaine figurate number.

For euery number made by the mutuall multiplication of two numbers, may be called a Plaine, because it bringeth foorth a right-angled parallelogramme according to his vnities disposed in length and breadth, the sides whereof are the two multiplying numbers. As the number 20, *made by the mutuall multiplication of* 4 *and* 5 *is called a plaine, and the*

sides thereof are 4 and 5 as heere.



Because the vnities thereof disposed in length & bredth, as the sides doe expresse, doe bring forth an inaqualiter Parallelogramme, for that the numbers or sides are inequall.

By like reason 36 made by multiplication of 6 by 6. is called an aqualiter plaine, for the sides thereof 6 and 6 are equall.

Moreouer one and the same plaine number may haue many sides, as the plaine number 24, hath sides 4 and 6: 3 and 8; 2 and 12. For it is produced from the mutuall multiplication of these numbers: whereupon for the inuention of the sides, to wit in inaqualiter plaines, it is needfull to giue one of the sides, by which the plaine it selfe diuided, the other side is made knowne. As the plaine 48 being diuided by the side 8, the quotient 6 is the remaining side. Notwithstanding another resolution and inquisition doth happen in the sides of the aquilater plaines.

An equilater plaine or quadrat what.

An æquilater plain is a number made by two equal sides or by any number multiplied by it selfe. It is vulgarly called a square or quadrat, by the Arabians *Zensu*, it is cõmonly expressed by this note ʒ, by vs q.

A quadrat or square in Geometrie is called a right lined plaine figure, made by foure equall right lines, and so many right angles,

and

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sides thereof are 4 and 5 as here.

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* * * * *
* * * * *

Because the vnities therof dispo-sed in length and breadth, as the sides doe expresse doe bring forth an inæquilater Parallelogramme, for that the nvmbers or sides are inequall.

By like reason 36 made by multiplication of 6 by 6. is called an æquilater plaine, for the sides thereof 6 and 6 are equall.

Moreover one and the same plaine number may haue many sides, as the plaine number 24, hath sides 4 and sixe: 3 and eight: 2 and 12. For it is produced from the mutuall multiplication of these numbers: wherevpon for the invention of the sides, to wit in inæquilater plaines it is needfull to giue one of the sides, by which the plaine it selfe diuided, the other side is made knowne. As the plaine 48 being diuided by the side 8, the quotient 6 is the remaining side. Notwithstanding another resolution and inquisition doth happen in the sides of the æquilater plaines.

An equaliter plaine or quadrat what.

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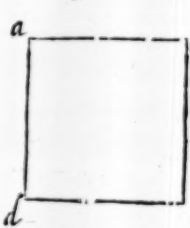
A quadrat or square in Geomerie is called a right lined plaine figure, made by foure equall right lines, and so many right angles, and

and euery one of the lines is called the side of the quadrat, as this figure a b c d, *whose side is* a b, *or* b d, *as also* c d, *and* a c.



To the similitude heereof that number is called a Quadrat, which is made, by the multiplication of two equall numbers, or of one in it selfe, of which maner 36 is made, by 6 multiplied in it selfe, or by the mutuall multiplication of 6 and 6. For if 36 vnites be placed in plaine forme, it bringeth forth a perfect Geometricall quadrat hauing in euery side sixe vnities as heere.

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* * * * * *
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The number whereof the quadrat is produced by multiplication in it selfe, is called the side or roote of the quadrat.

The side or roote of a number what.

Therefore to finde the quadrat roote, or the side of any quadrat number, is to search a number which brought or multiplied in it selfe maketh the number propounded, concerning the finding whereof as it is requisite that the sides (being lesser then 10) of the squares vnder an hundred should be gathered by the Ta-

ble of multiplication: so the sides of greater squares are to bee sought out by Art. First the squares whose sides are simple numbers, are heere set downe as you see.

The rootes.	1	2	3	4	5	6	7	8	9
The squares.	1	4	9	16	25	36	49	64	81

The knowledge of a square is by finding out his side expressed by a whole number.

Although the finding out of the side of a square, be applied to ech number giuen, as to a square, yet square numbers onely haue a side to be expressed by a certaine number of vnities, or by rationall numbers, the other are to be expressed but onely in power. The sides are commonly called rootes by a metaphoricall phrase.

The roote or side of a square is to bee found by the theorem following.

1 *If the odde degrees of a square number being marked from the right toward the left hand with points, you subduct from the number giuen, the particular square of the last period, setting the side thereof alone by it selfe.*

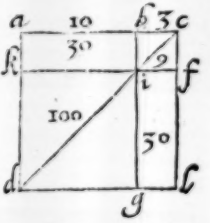
2 *Then going on if you deuide the remainder (if there be any) with the figure going before it by the double of the side set alone by it selfe.*

3 *And multiply the quotient found out (being placed by the side which was first set alone by it selfe, and also before the doubled number*

on

on the right hand, by both the numbers (namely by the double number, and the figure ſet by it ſelfe) beeing counted as one diuiſor. ſubducting the products from the giuen number, and then renew this laſt worke of diuiſion ſo many times as there are pricks remaining, the ſide of the ſquare ſhall be found out.

This artificiall deuiſe is taken out of the 4 P. 2 of Euclide. *Whereby demonſtration it is proued, that if a right line be cut into two ſegments, howſoeuer, the ſquare of the whole line is equall to the ſquares of the ſegments, and to the two rightangled figures made of the ſegments: as in the figure annexed, the two diagonals* k g, & b f, *are the ſquares of the ſegments* a b, *and* a c. *Alſo the complements* b k, *and* f g, *are the rightangled figures made by multiplying the line* a b, *by* b c.



To extract the ſquare roote.

The ſelfe ſame parts are to be found in any ſquare number. As for example, let the number be 169, *whoſe ſide is* 13. *This ſide being diuided into two peeces* 10 *and* 3. *multiplying each peece by it ſelfe once, namely* 10 *by* 10, *and* 3 *by* 3, *then multiply one by another, as* 10

The firſt example.

by 2, and 3 by 10, so shall you haue foure plain numbers, whereof two are squares as heere you see.

Therefore as the square 169 is made by adding together of these foure plaine numbers. so by subducting them seuerally it is resolued.

10		3
10		3
	100	
	30	
	30	
	9	
	169	

First therefore I marke each odde place with points, because the particular squares are to be found in the odde places. Then for so much as the vnitie standing vnder the first point next the left hand, & representing the last period is both a square and the side of a square: that figure therefore being set alone in the quotient, and being subducted from the vnitie standing ouer the point there remaineth nothing.

This vnitie set alone by it selfe in the quotient shall signifie 10, when another figure is set by it, representing the side of some other particular square. Whereupon I say that the greater Diagonall k g, is now subducted from the whole square, and the side of it k i, or a b, (for they are equall one to another) and also the side of one of the complements is found out. This is the first steppe to this resolution.

Moreouer I double the figure found out, because being doubled, it is the side of both the com-

complements taken ioyntly together, namely k i, *and* g i. *Then setting* 2 *the doubled number vnder* 6, *I diuide* 6 (*which in this place is as much as* 60, *and representeth both the complements*) *by* 2, *the quotient is* 3, *representing the other side remaining of the complement, namely* i f, *or* b c, *which number I set in the quotient, and count it for the segment remaining of the right line giuen. Wherefore because this number* 3 *is the side of the remaining Diagonall, that is to say, of the lesser square* b f: *therefore being set by the diuisor on the right hand and multiplied by it selfe, and also by the diuisor, it bringeth forth three plaine numbers, namely the square* b f, *and the two complements* a i, *and* i l, *which being subducted from the numbers standing ouer them, there remaineth nothing. The example is thus*

169(13	Which is all	169	
23	one as if you	100	The greater diagonal.
3	had put down	60	The complet twofold.
69	the numbers	9	The lesser Diagonall.
	found out in	169	
	this manner		

The subtilty of this inuention is illustrated by many examples Let the square giuen bee 1764. *This number beeing marked with two points telleth vs that the side thereof is to be written with two figures.* The second example.

First therefore beginning at the point on the left hand, I seeke the side of the last period, namely of 17. But for so much as it is no square number, I take 4 the side of the next lesser square, which I set alone by it selfe in the quotient, and then multiply it by it selfe, the product is 16, which being subducted from 17. there resteth 1. Moreouer I double the side found out, the product is 8, I place this doubled number vnder 6, and by it I diuide 16 standing aboue it, the quotient is 2, which must be set by 4. This quotient 2 must be set before the the diuisor 8 on the right hand vnder the point, and then must it be multiplied both by it selfe and into 8, the product is 164, which being subducted from the figures standing ouer them, there remaineth nothing, whereby I gather that the number giuen is a iust square. The example standeth thus.

$$
\begin{array}{cccc}
 & \cancel{2} & & \\
 & \cdot & & \cdot \\
\cancel{2} & \cancel{7} & \cancel{6} & \cancel{4} \;(42 \\
\hline
1 & 6 & 8 & \cancel{2} \\
 & & & \cancel{2} \\
\hline
 & 1 & 6 & 4 \\
\hline
1 & 7 & 6 & 4
\end{array}
$$

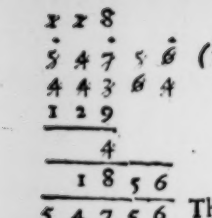
the collection

The same manner of working is to bee followed in greater square numbers giuen, sauing that the former part of the worke is to be vsed but

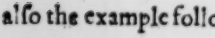
but once, but the latter part is to be followed ſo many times as there are points remaining excepting the laſt.

As in 54756, I ſay, that the ſide of the ſquare next vnto 5 is 2, therefore 2 being ſet in the quotient, and multiplyed by it ſelfe make 4, & taken from 5, the remainder is 1. Moreouer I double the quotient, the product is 4, which I ſet vnder the next figure toward the right hand, and thereby diuide 14. the quotient is 3, which 3 being ſet both in the quotient, and alſo before the diuiſor toward the right hand, I multiplie both the numbers by it, the product is 129, this being ſubducted from 142 ſtanding aboue it, the remainder is 18. But becauſe there is yet one point remaining with which I haue not medled, therefore I againe double all the whole quotient, for in this caſe I muſt take 23 for the one ſide of one former ſquare, and generally in great numbers when I light vpon more particular ſquares then two, I muſt eſteeme them but as two, and take the ſides which are firſt found out but as the ſides of one onely ſquare. Therefore twice 23 is 46, by this I diuide 185, the number to be ſet in the quotient is 4, which number alſo muſt bee ſet before the diuiſor on the right hand, then muſt 464 be multiplied by 4, the product is 1856, this product being ſubducted from the numbers ſtanding ouer it, there remaineth nothing. The example ſtandeth thus. The third example.

The



See also the example following.



¶ *Therefore out of this invention is this consectarie.*

4 Example of a surd number.

The number whose side cannot be expressed by whole numbers, is not a square number.

Such are all prime numbers, and (the squares themselues excepted) all other compound numbers. For if in them you desire to find out the square side you shall labour in vaine, because they are not squares, for to the whole numbers arising in the quotient, there will be some fraction adioyned, whereby it commeth to passe, that the number of the side is not to be expressed by a true number, and it is commonly called a surd number.

Not-

Notwithſtanding if you adioyne to the ſide found out, the number remaining, taking his denomination from the double of the ſide augmented by an vnity, you ſhall find the next ſide that may be like to the ſide of a ſquare.

As if from 40 you take the neareſt ſquare, to wit 36, the remainer is 4. Here therefore the ſide ſought for of the ſquare, exceedeth not the ſide found out by an vnity, but either by one, or more parts of ſome whole number: wherefore I double 6, the ſide found out, and adde an vnity to it being doubled, the totall is 13, this number I ſet vnder 4 the remainer, and ſay that the ſide of 40 demanded as neare as may be, is $6\frac{4}{13}$, the Denominator of the Faction being added to the greateſt ſquare in the number giuen, namely vnto 36, maketh the next greateſt ſquare aboue it, namely 49, whoſe ſide is 7. But this ſurd ſide, to wit $6\frac{4}{13}$, multiplied by it ſelfe maketh $39\frac{12}{169}$, which are not iuſt equall to 40 the giuen number.

Iudge the like concerning the reſt which are not ſquares.

Thus much concerning plaine figurate numbers, but eſpecially ſuch as are ſquare numbers.

Solid figurate numbers follweth.

A ſolide figuratiue number is made of two multiplications by three numbers, or ſides, multiplied together, admitting length, broadth, and thickneſſe.

There-

A solid figurate number.

Therefore euery number made by the mutuall multiplication of three numbers, may be called a solid, because it bringeth forth a right angled Parallelipipedon, disposed according to his vnites in length, breadth, and thicknesse, the sides whereof are the three multiplying numbers. As the number 30 made by the mutuall multiplication of 2, 3, and 5. is called an inequilater solid number, and the sides thereof are 2, 3, and 5, because the vnities thereof disposed by a certaine distance one from another, in length, breadth and depth, as the sides doe expresse, doe bring forth in resemblance an inequilater parallelipipedon, for that the numbers or sides are inequall.

By like reason 216 made by multiplication of 6 by 6, and the product thereof by 6, is called an Equilater solid for the sides thereof 6, 6 and 6 are equall.

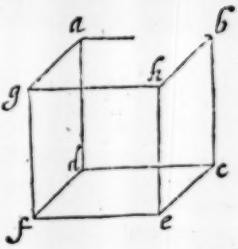
An Equilater Solide or Cube,

An Equilater solid, is a number made by three equall sides, or by any number multiplied by it selfe, and that product againe by the foresaid number. And it is called an Equilater and Equiangled Parallelipipedon or Cube, and is commonly represented by this note ꝛc, but by vs thus *C*.

A Cube in Geometry is a right angled Parallelipipedon, hauing six equall surfaces, & 8

solid

ſolid angles, & 12 ſides, as this figure a,b,c,d,e,f,g,h, *whoſe ſide is* a b, *or* a d, *alſo* b c, *or* c d, *either* c e, *or* e f, *likewiſe* e h, *or* h g, *alſo* g f, *or* d f, *or* d a, *and* g a.



The number whereof the Cube is produced by multiplication in it ſelfe twice, is called the ſide or the roote of the Cube, which being found out in whole numbers, the Cube is knowne. The ſide or roote of a Cube.

Therefore euery Cube in numbers hath ſuch a ſide as may be expreſſed in whole numbers, but in magnitudes it is not alwayes ſo, as indeed in magnitudes there are many things not to be expreſſed in whole number. Now for as much as the ſide of any Cube vnder 1000, *is a ſimple figure, it is neceſſary before we vndertake to find out the ſide of any great number, to know what Cube is made of each ſimple figure, and what is the ſide of any Cube leſſer then* 1000, *as I haue here ſet them downe.*

Rootes.	1	2	3	4	5	6	7	8	9.
Squares.	1	4	9	16	25	36	49	64	81.
Cubes.	1	8	27	64	125	216	343	512	729.

But in ſearching out the ſides of greater Cubes, we are to proceed as the theorem following teacheth vs.

If you diſtinguiſh with points as it were into periods, the giuen Cube beginning at the firſt figure on the right hand, and omitting each two figures continually, and firſt of all ſubduct the particular Cube of the laſt period from the number giuen, ſetting the ſide thereof in the quotient: and then ſet the triple of the quotient vnder the figure next following the former point on the right hand, and the ſquare of the quotient being tripled beneath it one degree more toward the left hand: and afterward diuide the number aboue written by the triple of the ſquare, ſetting the quotient by it ſelfe, and then multiply the diuiſor by the quotient found out, and the tripled ſquare by the ſquare of the quotient, and the quotient cubically, ſubducting the products (ſo orderly added together, that each figure may anſwere the numbers whereof it was multiplied) from the number giuen, and renew this laſt manner of diuiſion ſo many times as there are points remaining, the

ſide

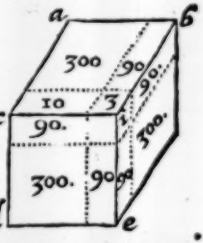
ſide of the Cube ſhall be found out.

This artificiall deuiſe is drawne out of that theorem which Ramus *made, imitating that of* Euclide *concerning ſquare numbers in this manner.*

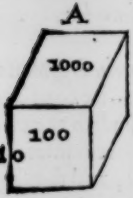
The extraction of the cubick ſide or roote.

It a right line be cut into two ſegments, the Cube of the whole line ſhal be equall to the Cubes of the ſegments, and to the two ſolid figures comprehended three times vnder the ſquare of his ſegment, and the ſegment remaining.

As the line c,i, *which is* 13, *is cut into two ſegments* 10 *and* 3, *therefore the Cube of the whole line, namely* 2197, *is equall vnto the Cubes of the Segments, namely vnto* 1000, *and* 27, *and alſo to the two-fold ſolids or Parallelipipedons thrice taken, whereof three haue like ſoliditie, the ſoliditie of ech of the three leſſer is* 90, *being made of the ſquare of the ſegment* 3, *that is to ſay of* 9, *multiplied by the other ſegment* 10. *Theſe three Parallelipipedons iointly taken together, make* 270 *But of the three greater Parallelipipedons each containeth* 300, *being made of* 100, *the ſquare of the greater ſegment* 10, *multiplied by the leſſer ſegment* 3, *and*

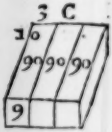


and they being taken joyntly together, make 900.

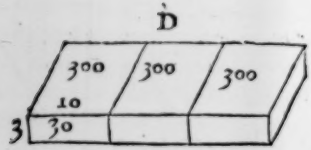


The Cube of the greater segmnt 10.

The Cube of the lesser segment 3.



The 3 lesser Parallelipipedons.



The 3 greater Parallelipipedons.

The Cube therefore hath eight particular solids in number which are made of the parts of the number given, namely of 10 *and* 3, *in this manner. First let there be foure plaine num-*

numbers made, each part being multiplied by it selfe; and one by another.

10	3
10	3
	9
	30
	30
	100

If againe I multiply the plaines by the same parts, there will arise 8 solids, as you see here.

9	9	
30	30	
30	30	
100	100	
3	10	
27	90	*All these being added together, are equall to the Cube of the whole, to wit,*
90	300	
90	300	
300	1000	2197.

¶ *Therefore the same way that is kept in making the Cube, is also to be followed in resolving the Cube.* The first example to extract the cubick Roote.

As for example, I marke the Cube giuen with points in this manner, 2197.

Then I subduct the particular Cube of the number set vnderneath the last point: but for so much as that number is no Cube, I take the neerest to it, namely an vnity, which also I set

in the quotient. This vnity in the number giuen, is 1000, *but in the quotient it is but* 10. *The vnity ſubducted from* 2, *the remainer is* 1, *which muſt be written ouer the number giuen. So that the greater Cube* A, *is to be ſuppoſed to be ſubducted from the number giuen. This is the firſt ſtep of this worke.*

1
2 1 9 7 (1
x

I triple the quotient found out (that is to ſay, I multiply it by 3) *this triple repreſenteth the three ſides (iointly taken together) of the three leſſer ſolids marked with* C, *I place the tripled number vnder* 9. *Againe I multiply the quotient ſquarewiſe, and triple the product which maketh likewiſe* 3. *This product reſembleth the three ſquare ſides, (taken iointly together) of the three greater ſolids, marked with* D, *I place the product one degree lower toward the left hand vnderneath* 1. *With it I diuide* 11, *which is written aboue it, the quotient is* 3. *This ſegment or quotient* 3, *being multiplied by* 3 *the Diuiſor, maketh* 9, *which in reſpect of the place wherein it ſtandeth is* 900, *and repreſenteth the three greater ſolids, marked with* D, *taken iointly together. Furthermore the ſame quotient being multiplied ſquarewiſe, maketh* 9, *and multiplied afterward by the tri-*

pled

pled number standing vnder 9, it maketh 27, which in respect of the place wherein it standeth is 270, and representeth the three lesser Solids marked with C. *Last of all, the same quotient multiplied cubically, breedeth the lesser Cube* B. *These three products therefore being added together, and the totall subducted from the numbers standing ouer it, there remaineth nothing, which importeth that the giuen number is a Cube. The example is as you see.*

```
1                    2̇ 1 9 7̇ (1 3
2̇ 1 9 7̇ (1 3        1 0 0 0   The greater Cube.
1   3                    3
  3                    3                (pedons.
  9          Or thus.  9 0 0   The 3 greater Parallelipi-
  2 7                  2 7 0   The 3 lesser Parallelipipe.
    2 7                  2 7   The lesser Cube.
1 1 9 7                2 1 9 7
```

The matter may bee explained by many examples.

The second example of the cubick Roote.

Let the side of the given Cube 16387064, *be sought out, continue it therefore (as it were) into certain periods with points. Then first of all, search out the side of the cube next to the left hand: But for as much as* 16 *is no Cube, take* 2 *the side of the next cube under it, that is to say, of* 8, *and set it in the quotient, and subduct* 8 *the Cube thereof from* 16, *there remaineth* 8. *This first worke is not to be renewed throughout the whole number, but the rules following must be repeated as often as there are points remaining. The first step to finde out the roote is in this manner:*

8

1 6 . 3 8 . 7 0 6 4 . (2

8

Moreouer triple the quotient now found out, & the product is 6, *which is to be placed vnder* 8, *namely vnder the figure following the next pricke toward the right hand. Then multiply the quotient by this tripled number (or which is all to one purpose square the quotient, and then triple the product) it maketh* 12, *set that number in a lower place one degree nearer the left hand: and make it the diuisor, diuide* 83 *by* 12 *obseruing this rule in chousing your quotient, that it be no greater then that the numbers afterward produced by multiplication may not exceed the numbers standing ouer it. So that here*

here you ſhal take 1 in 8, but fiue times. Afterward by this number 5, multiply the diuiſor 12, and by the ſquare of 5 multiply the tripled number 6 and laſt of al, multipl. 5 cubically ſo ſhall you produce three numbers, namely 60, 150, 125, to be deſcribed in ſuch ſort as you ſee. Theſe numbers added together, and ſubducted from 8387 the remainer is 762. The ſecond ſteppe to find out the roote, is in this manner.

```
   8 7 6 2
 1 6 3 8 7 0 6 4 (25
       6
   1 2
   6 0
   1 5 0
     1 2 5
   7 6 2 5
```

And becauſe there is yet one point remaining, this laſt manner of Diuiſion muſt bee wrought againe.

Firſt therefore I triple the quotient, the product is 75, which muſt be ſo placed, that the firſt figure thereof, namely 5, may ſtand vnder 6, the ſecond vnder the 0. Againe, multiply the quotient by this tripled number (or which is all one, ſquare the quotient, and triple the product) it maketh 1875, which muſt be the

Diuisor, whose first figure namely 5, must be placed vnder 7, the last figure of the tripled number. Then you see that 1 may be contained in 7 many times, but I can take it but 4 times, I set 4 in the quotient, and multiply the Diuisor by 4, the product is 7500, afterward I square 4, it maketh 16, which I multiply by the tripled number 75, the product is 1200. Last of all I multiply 4 cubically, it maketh 64, these products added all together, make 762064, which number being subducted from the Cube giuen, there remaineth nothing, whereby I gather that the number giuen is exactly cubicall. The third steppe to find out the side is in this manner.

```
      7 6 2
  1 6̇ 2 8 7̇ 0 6 4̇ (2 5 4
  ---------------
            7 5
      1 8 7 5
    -----------
      7 5 0 0
        1 2 0 0
              6 4
    -----------
      7 6 2 0 6 4
```

The third example of the Cubick root.

Behold also the example following.

```
6 1 4̇ 1 2 5̈ 0 0 0̇ (8 5 0
```

¶ *Another manner of working.*

¶ *Hither-*

¶ Hitherto the Princely high way to find out the ſide of the Cube hath beene declared.

But there are moreouer certaine other waies alſo bending thereto, and leaning to the ſame principles, whereof this is one. The ſecõd forme.

Hauing found out in the Table of ſimple Cubes, the firſt figure repreſenting the ſide of the Cube contained in the number ſtanding vnder the firſt point on the left hand, ſet it in the quotient, and ſubduct the particular Cube of that figure as you did before: then ſquare that figure, and triple that ſquare, the product ſhall be the Diuiſor, the firſt figure whereof ſhall bee ſet vnder that figure which is on the right hand, next of all to the point (now examined) before going. 1

See how many times the diuiſor is contained in the number written ouer it, and multiply the diuiſor in the quotient, and ſubduct the product from the diuidend: yet here you muſt take heed, that you take not a greater quotient, then that the products made afterward thereby may be ſubducted from the number giuen. 2

The ſubduction being done, triple the firſt figure which was ſet in the quotient, and adde to the triple the laſt number which was ſet in the quotient on the right hand of the product. 3

This totall multiplied by the ſquare of the figure laſt found out, and ſet downe the product ſo, that the firſt figure thereof toward the 4

right hand may stand vnder the point next before going on the same hand, and finally subduct the same from the number giuen.

The fourth example of the Cubick roote.

As in 804357. *The particular Cube, namely,* 729, *beeing taken from the number standing vnder the last period vpon the left hand, there remaineth* 75357, *the side of that particular Cube being* 9, *I set in the quotient. Then I square that side, it maketh* 81, *& triple the square, the product* 243 *is my diuisor, which I set vnder the giuen number, so that* 3 *may stand vnder* 3, *with this diuisor diuide the number standing ouer it, you shall find* 2 *to be conteined in* 7 *three times. Therefore I set* 3 *in the quotient, and multiply the diuisor by it, the product is* 729, *which being subducted from* 753, *the remainder* 24. *The induction is thus:*

```
      2
    7 5 4
  8 0 4 3 5 7  (93
    2 4 3
    -------
    7 2 9
```

Moreouer I triple 9, *the product is* 27, *by which on the right hand I set* 3, *the quotient last found out, the totall is* 273.

This number I multiply by 9 *the square of* 3 *the quotient last found out, the product shall be*

be 2 4 5 7, *which beeing subducted from the superiour number, there remaineth nothing.*

The induction is thus:

```
      2 4
      .   .
8 0   4 3 5 7 (93
        2 7 3
            9
---------------
      2 4 5 7
```

¶ *The selfe same worke may be dispatched another way a little differing from the former in this manner.*

The figure in the quotient being found out by subducting the particular Cube, and also the second figure in the quotient being found by diuision, let the totall quotient bee tripled, and let the tripled number be multiplied by the former figure in the quotient. Then let the product be multiplied againe by the latter figure found out, and let a cipher be set on the right hand of that product. Last of all, let the Cube of the latter figure found out, be added to this product, and let the totall summe be subducted from the number giuen. As in 3 7 3 2 4 8. The third forme.

The first induction is in this manner.

More-

The fift example.

$$\begin{array}{l} \quad 3\ \dot{0} \\ \underline{\cancel{3}\ 7\ \cancel{3}\ 2\ 4\ \dot{8}}\ (7 \\ 3\ 4\ 3 \end{array}$$

Moreouer I square the side found out, it maketh 49, and triple the square, the product is 147, which shall be the diuisor, by this I diuide 302, the number written ouer it, the quotient is 2. Now I triple the totall quotient 72, it maketh 216, and multiply this triple by 7, the former figure in the quotient, the product is 1512. I multiply this product also by 2 the later figure of the quotient, and set a cipher on the right hand of it, so as it maketh 30240, vnto this number last of all I adde 8 the Cube of the latter figure found out, the totall is 30248: which beeing subducted from the figures aboue it, there remaineth nothing.

The induction is thus:

$$\begin{array}{l} \quad 3\ \dot{0} \\ \underline{\cancel{3}\ 7\ \cancel{3}\ 2\ 4\ \dot{8}}\ (72 \\ \quad 1\ 4\ 7 \\ \hline \quad 3\ 0\ 2\ 4\ 8 \end{array}$$

¶ All the points of the number giuen being examined, if any thing remaine, it signifieth the number giuen is no cube: wherefore the true side of it cannot be exactly giuen in numbers,

bers. Yet if it pleaſe you to ſift out the neareſt ſide that may be, by the firſt kinde of reduction of mixt numbers, you ſhall reduce the number giuen vnto a cubicall fraction of a greater denomination, and afterward ſeeke out the cubicall ſide of that fraction.

To finde the neareſt Cubicke roote in a ſurd number.

For example ſake, becauſe 120 *is no Cube, therefore let it be reduced into ſixtie cubicall parts, after this manner. Multiply* 60 *cubically in it ſelfe, it maketh* 216000, *by this being taken for the denominator of the fraction, multiply* 120 *the number giuen, the product is* 25920000, *whoſe cubicall ſide is* $295\frac{}{60}$ *that is* $4\frac{11}{71}$ *the neareſt to the true ſide that can be.*

Deo Soli laus omnis honor & gloria tribuatur. *Amen.*

FINIS.

I. D. *To the earneſt Arithmetician.*

MY louing friend to Science bent,
Something thou haſt by this book won:
But if thou wilt be excellent,
Another race thou muſt yet run.

Supplies thereto but (few doe) neede,
And none but ſuch as in our phraſe,
(By *Records* pen) thou maiſt well reade:
Proceed therefore: Be not ſtunt dwaſe.

The ground moſt ſure, whereon this race
With ſpeedfull courage muſt be paſt,
Of late hath turn'd his Greekiſh face,
By Engliſh tilth, which aye will laſt.

The famous Greeke of *Platos* lore,
EVCLIDE I meane Geometer:
So true, ſo plaine, ſo fraught with ſtore,
(As in our ſpeech) is yet no where.

A treaſure ſtrange, that booke will proue,
With numbers skill, matcht in due ſort,
This I thee warne of ſincere loue,
And to proceed doe thee exhort.

Plus oultre.

Neare Fleetstreet within Fetter-lane, next the Hand and Pen is taught by *Robert Hartwell*, Practitioner in the Mathematickes, these *Arts*, *Sciences*, and *Faculties*, *heere vnder expressed.*

Viz.

- **Arithmeticke.**
  - I N whole Numbers and Fractions.
  - The extraction of Rootes, viz. { Square, Cube, Squared Square, Surfolid, &c. } At one operation.
  - Of Astronomicall or Physicall Fractions.
  - Of Proportions.
  - The Rules of Æquation, with Cosse, or Algebra.
- **Geometry**
  - Principles with Practise & Demonstration.
  - Measuring of Land, &c. & plotting of Countries with diuers and sundry instruments.
  - Reducing of Plots or Maps to any proportiō.
  - Mesuring of any superficiall or solid content. { Board, Glasse, &c. Timber, Stone, &c.
- **The doctrine of Triangles.** { Plaine & Spherical } With the vse of the Tables of { Sines, Tangents, Secants, & Logarithmes.
- **Description, Demonstration, and vse of Instruments, *viz.***
  - Quadrant, Theodolite,
  - Quadrat.
  - Crosse Staffe, and Jacobs Staffe.
  - Sector, Circular Scale,
  - Proportionall Ruler.
  - Circumferenter.
  - Pantometer.
  - Plaine Table, &c.
- **Nauigation.**
  - Principles thereof, with the making and vse of sundry Instruments fitting that Art.
  - The vse & proiection of Maps & Charts,
- **Vse and demonstration of**
  - The Sphere. { Cœlestiall. Both Globes { Terestiall.
  - Clestiall Globe in plaine by Mr. D. *Hood.*
  - All sorts of Astrolabes, & Mr. *Blagraues* Iewell.
- ***Dialling* of all sorts, *viz.*** { Fixed, and Instrumentall.
- Performing any proposition of the Sphere, in a plaine Circle, onely with Scale and Compasse.
- **Accompts for** { Merchants by order of Debitor and Creditors. Retaylers of all sorts.
- Perfecting of Accompts in Controuersie.

Fide —— *sed* —— *Vide.*

To the curteous iudicious Reader, and all the well-willers of the Mathematicks, The Corrector S. D.

FRiendly Reader, my selfe hauing receiued my first nutriment in the pleasant studie of the Arts Mathematicke, and also great benefit from this ground, so named by the learned Author Master Doctor RECORD, which since was illustrated by Master Dee, And further enlarged by my first Tutor therein Master Mellis. Who notwithstanding in a former Impression thereof about thirty yeares since, suppressed therefrom many excellent rules (the fault whereof be imputed to the Printer.) Therefore for thy sake, and my dutie to the common good, I haue annexed the chiefest of them thereunto; with an Appendix concerning the resolution of the Square and Cube, therein following the Method vsed by Christianus Vrstitius, with further illustrations to the booke. True it is that into the former Editions destitute many times of a Corrector, or else those very vnskilfull or careleſſe, almost infintie errors haue crept, which errors I haue endeauoured in this Edition to amoue, and so to rectifie the Authors worthie Treatise: passing by with easie neglect the vndeserued aspersion of that carping *Momus*, whose singular weakenes is more able to taxe in other, then himselfe to amend. Thus requesting thee friendly to reade, and with thy pen first to correct these faults annexed, (or any other ouer-slip) I humbly commend thee to the protection of the Almighty.

Thine Ro: Hartwell.

Philomathematicus.

Faults escaped.

R. P*Age* 172. *line* 32. *read* 9. *Pag*. 290 *l*. 3. *r*. $\frac{15}{60}$. *p*. 292. *l*. 19. *r*. $\frac{5}{8}$ *by* $\frac{2}{6}$. T. *p*. 304. *l*. 8. *r*. $\frac{5}{12}$. V. *p*. 323. *l*. 20. *r*. $\frac{20}{90}$ *and* $\frac{20}{23}$ *by* $\frac{4}{1}$ *in the margent* $\frac{20}{23}$ *by* $\frac{4}{1}$. X. *p*. 333. *l*. 7. *r*. 1. $\frac{56}{88}$. *p*. 335. *l*. . *r*. $\frac{68}{20}$ 31. A a *p*. 388 *l*. 16. *r*. 660. B b. *p*. 400. *l*. 30. *r*. 3325000. *In the examples of falshood, the signes of more and lesse are omitted, which place to the numbers at the foot of the crosse, thus. p*. 393. 208 — 112 +. *p*. 395. 10 + 18 + *p*. 397. *at* 4 + 2 +. *p*. 399 *at* 1775 – 1750 — *p*. 400 *at* 6825 – 6650 –. *p*. 404. 9 – 18. – *p*. 406. *at* 2 – 33 +. C c. *p*. 367. *l*. 17. *r*. $\frac{1}{4}$. *l*. 31. *r*. $\frac{7}{16}$. *p*. 368. *l*. 1. *r*. $\frac{3}{26}$. *in the example* $\frac{7}{12}$. *p*. 370. *l*. 2. & 3. *put out siluer and quicke siluer vnto water and read as* $\frac{48}{25}$. D d. *pag*. 379. *lin*. 22. *r*. 532. *yards*. *pag*. 380. *lin*. 28. 125 | 3. *pag*. 381. 1359 | 2 *groats*. *pag* 382. *l*.

Pag. 322. line 1. margent, set.

$$\begin{array}{r} 13\tfrac{3}{5}.\ 340 \\ \hline 68 \\ \hline 5 \end{array} \quad by \quad \frac{5}{8}$$

40

Page. 164. *line*. 14. *reade*.

But as these bee vnlike things, so the measures of their Vessels doe differ, for the measures of Ale are as followeth.

R. 268. *l*. 29. *read of the diuidend, and seeke how often I may haue the diuisor therein, and that.*

6. *r*.

Errata.

6.r.113. 385.r. *for* 3 D. 386. *l.* 3.r. 18. s̄. -- 8 D. E e. *P.*404.*l.*29. *and* 30.r. 7536. *p.*405. *l.* 1. *and* 2, r.8792. *l.*4.r. 9537. *l.*14.r, 3705. *p* 406.*l.*17. *r.maketh.* 12. Ff. *Pag.* 394. *l.*25. *r.*3 ℓ. 7 s̄. 4 d̄. *p.*395 *l.*14.r.6 ℓ. 3 s̄. *l.*20.r. 2 d̄. *l.*21.r.$\frac{3}{4}$. *l.*28.r.$\frac{1}{2}$. 401. *l.* 15.r. $\frac{2}{3}$. *l.*18.r.106 $\frac{2}{3}$. *l.* 20.r. *reductions.* Gg. *Pag.*414. *l.* 18.r. *by*.8. Hh. *P.*430.*l.*1.r. 2$\frac{2}{3}$ s̄. *l.*10.r.$\frac{1}{6}$ s̄. *l.*28.r.*by* 15. I i *P.*443.*l.*1.r.$\frac{3}{5}$. 448. *l.*3.r 174.$\frac{77}{96}$. *p.*449.*l.*19.r.246.$\frac{6}{25}$. *p.*453. *l.*3.r.8 s̄. Kk. *P.*457. *l.*11.r 8$\frac{3}{5}$ ℓ, *l.*12.r.34.$\frac{2}{5}$ ℓ *l.*23 r.8$\frac{3}{5}$.*P.*458.*l.*15 800. *p.*460 *l.*31.r.$\frac{2}{5}$.*p.*461. *l.*2.r.$\frac{3}{5}$. *l.*21.r.$\frac{1}{3}$.*l.*26.r.$\frac{1}{3}$. *p.*469.*l.*5.r.8 ℓ.3 s̄.4 d̄. Ll.*P.*480.*l.* 16.r.6$\frac{6}{7}$ d̄. *p* 482.*l.*13.3622 $\frac{2}{3}$ s̄. *p.* 483.*l.*2.r.1$\frac{7}{50}$ D. *p.*485.*l. last.* r. 200 – 5 – 8$\frac{4}{7}$. *starling, and* 266 – 11 – 5$\frac{1}{7}$. *pag.* 487.*l.*8.r.2$\frac{2}{3}$ D. Mm. *P.*489.*l.*6.r. 231 – 13 – 4 D. *p.*492.*l.*28.r. *exchange.* *p.*504 *l.*15 r.266 $\frac{2}{3}$ ℓ Nn. *P.*517.*l.*14. *read* 10000000. *p.*519.*l.*10.r. facit 814 *pounds* $\frac{1620000}{10000000}$ *parts &c.*

THE GROVND OF ARTS:

TEACHING THE PERfect worke and practise of Arithmeticke, both in whole Numbers and Fractions, after a more easie and exact forme then in former time hath beene set foorth: Made by

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Scientia non habet inimicum nisi ignorantem.

Fide ——— sed ——— Vide.

At London Printed by *I. B.* for *Roger Iackson*

1618.

THat which my friend hath well begun,
For very loue to Common-weale,
Neede not all whole to be new done,
But new encreaſe I doe reueale.

Some thing herein I once redreſt,
And now againe for thy behoofe,
Of zeale I doe, and at requeſt,
Both mend and adde, fitte for all proofe.

Of Numbers vſe, the endleſſe might,
No witte nor language can expreſſe,
Apply and try both day and night,
And then this truth thou wilt confeſſe.

I. D.

The Bookes Verdict.

TO pleaſe or diſpleaſe ſure I am,
But not of one ſort to euery man:
To pleaſe the beſt ſort would I faine,
The froward diſpleaſe ſhall I certaine.
Yet wiſh I will, though not with hope,
All eares or mouthes to pleaſe or ſtoppe.

TO THE MOST mighty Prince *Edward* the sixt, by the grace of God, King of *England*, *France*, and *Ireland*, &c.

He excellencie of mans nature being such, as it is by Gods diuine fauor (most mighty Prince) not onely created in highnes of degree far aboue all other corporal things, but by perfection, reason & search of wit, much approaching toward the Image of God, as not only the holy Scriptures doe testifie, but also those naturall Phylosophers, which exactly did consider the nature of man, and namely the far reach & infinit compasse of the works of the mind, were inforced to confes, that man scarcely was able to know himselfe. And if he would duely ponder the nature of himselfe, hee would finde it so strange, that it might seeme vnto him a very myracle. And thereof sprang that saying: *Magnum miraculum est homo: maximum miraculum sapiens homo.* For vndoubtedly, as man is one of the greatest myracles that euer God wrought, so a wise man is plainely the greatest.

And therfore was it that some did account the herd of a man the greatest miracle in the worlde, because not onely of the strange workmanship that is in it, but much more of the efficacie of reason, witte, memorie, imagination, and such other powers, & works of the mind, which can more easily conceyue any thing in a manner, then vnderstand it selfe. And amongst all the creatures of God it findeth none more difficult to be perceiued then these same powers of it selfe, whereby it doth conceiue & iudge as it may be well coniectured

by the diuersitie of opinions. th[illegible] he wiseſt Phyloſophers did vtter, touching t[illegible] man & the ſubſtance of it: wherof at thi[illegible] intend to make no rehearſall, but who ſo liſteth [illegible] reade thereof, may find it largely ſet forth, not only in *Ariſtotle* his bookes *De anima*, but alſo in *Galene*, his booke called *Hyſtoria philoſophica*, & againe, in *Plutarch* his worke, *De phyloſophorum placitis*, whoſe wordes are alſo repeated of *Euſebius* in the xv. booke τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue deſired to vnderſtand the intricate difficultie of knowing our owne ſelues, as touching our beſt parte, and that part whereby we deſerue to beare the name of men.

This matter ſeemed ſo obſcure & difficult in knowledge, that *Galen*, who for his excellent wiſdome and iudgement in naturall works, is called of many men a Miracle in nature, yet in ſearching the nature & ſubſtance of the ſpirite of man, hee not onely confeſſeth himſelfe ignorant, but counteth it plaine temerity to attempt to finde it. ſo farre aboue the hope of mans knowledge is that part, whereby man doth know and iudge of things. And althogh the ignorãt ſort (which hate all things that they know not) doe little eſteeme the profoundnes of mans ſpirit & of reaſon, the chiefe power & faculti[illegible] of it: yet as there is a kind of feare and obedience of vnreaſonable Beaſts vnto man, by the working power of God, ſo is there in theſe ſmall reaſoned perſons a certain kinde of reuerence toward wiſedome & reaſons, which they do ſhew oftentimes, and by power of perſwaſion, are inforced to obey reaſon, will they, nill they.

And hereby came it to paſſe, that the rudeneſſe of the firſt age of Man was brought vnto ſome more ciuile trade, as it is well declared by *Cicero*, in the beginning of his firſt booke *De inuentione Rhetorica*, where he ſaith thus: *Nam fuit quoddam tempus quum in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vi-*

tam

tam propagabant, nec ratione animi quicquam, sed pleraq; viribus corporis administrabat. Nondum diuine religionis, non humani ratio colebatur. Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos, non ius equabile quid vtilitatis haberet, acceperat: ita propter errorem atq; insciam cæca ac temeraria dominatrix animi cupiditas, ad se explendum viribus corporis abutebatur, perniciosissimis satellitibus. Quo tempore quidam, magnus videlicet vir & sapiens, cognouit quæ materia esset, & quanta ad maximas res opportunitas in animis in esset hominũ, si quis eam posset elicere, & præcipiendo meliorem reddere. Qui dispersos homines in Agris, & in tectis syluestribus abditos, ratione quadam compulit in vnum locum, & congregauit: & eos in vnamquamq; rem inducens vtilem atq; honestam, primo propter insolentiam reclamantes, deinde propter rationem atq; orationem studiosius audientes, ex feris & immanibus mites reddidit, & mansuetos.

This long repetition of *Tullyes* words will seeme tedious to them that loue but little, and care much lesse for the knowledge of reason, but vnto your Maiesty (I dare say) it is a delectable remembrance, and vnto me it seemed so pleasant, that I could scarce stay my pen from writing all that mine eyes did so greedily read.

This sentence of *Cicero* am I loath to translate into English, partly for that vnto your Maiesty it needeth no translation', but especially knowing how farre the grace of *Tullyes* Eloquence doth excell any Englishmans tõgue, & much more exceedeth the basenes of my barbarous style, yet for the fruite of the sentence, I had rather vnto my meere English countreymen vtter the rudenesse of my translation, then to defraude them the benefit of so good a lesson: trusting they wil so learne to loue reason, that they will also gladly and greedily embrace all good Sciences, that may help to the iust furniture of the same, whẽ they consider that informed reason was the onely instrument, or at least the chiefest meanes to bring men vnto Ciuile regi-

ment, from barbarous manners, & beastly conditions.

For the time was (saith *Tully*) that men wandred a-broade in the fields vp & downe like Beasts, & vsed no better order in feeding then they, so that by reasons rule they wrought nothing, but moste of theyr doings did they atchieue by force of strength. At this time there was no iust regard of Religion toward God, nor of dutie toward man. No man had seene right vse of mariage, neither did any man know their owne children from other, nor no man had felt the commodity of iust Lawes: so that through errour and ignorance, wilfull lust, like a blind and heady ruler, abused bodily strength as a most mortall minister for the satisfying of his desire. At that time was there one, which not onely in power, but also in wisedome was great, And he considered how that in the mindes of men was both apt instruments, and great occasion to the due accomplishment of most waighty affaires, if a man could apply them to vse, and by teaching of rules, frame them to better trade. This man with perswasion of reason gathered into one place the people that were wandring about the fields, and lay lurking in wilde cottages and woods: And bringing them into one common society, did trade them to al such things as eyther were profitable or honest, although not without repining at the first, by reason that they had not bin so accustomed before. Yet at length through reason and perswasion of words they obeyed him more diligently, and so of a wilde and cruell people, he made them curteous and gentle.

Thus hath *Tullie* set forth the efficacie of reason and perswasion, how it was able to conuert wild people to a mildnesse, and to change their furious cruelnesse into gentle curtesie: were it not now a great reproach in this our time (when knowledge raigneth so large) that men should shew themselues lesse obsequious to reason? Vnlesse it may be thought, that

now

now euery man hauing ſufficient knowledge of himſelfe, needeth not to hearken to the perſwaſion of others.

Indeed he that thinketh himſelfe wiſe, will not eſteeme the reaſon of any other, be he neuer ſo wiſe, ſo that of ſuch a one it may well be ſaid: Hee that thinketh himſelfe wiſer then he is, may iuſtly be counted a double foole: wherefore ſuch men are not to be permitted in open audience to talke, but muſt be put to ſilence, & made to giue care to reaſon, which reaſon conſiſteth not in a multitude of words heaped raſhly together, and applied for one purpoſe, but reaſon is the expreſſing of a iuſt matter with witty perſwaſions, furniſhed with learned knowledge: ſuch knowledge had *Moſes*, beeing expert in all learning of the Egiptians, as the Scriptures declare, and therefore was able to perſwade the ſtubborne people of the Iewes, although not without great paine. Such knowledge and ſuch reaſons did *Druys* ſhew, which was the firſt Law-maker of all the Weſt part of Europe. Like reaſon and wiſedome did *Xamolxis* vſe amongſt the Gothes: *Lycurgus* vnto the Lacedomonians, *Zeleucus* to the Locrians, *Solon* to the Athenienſes, and *Dunwallo Molmutius* two thouſand yeares paſt amongſt the olde Britains of this Realme. And thereby came it to paſſe, that their Lawes continued long, till more perfect reaſon altered many of them, and wilfull power oppreſſed moſt of them.

Druis was ſonto King Sarron, & ſucceeded him in his kingdome.

At the beginning when theſe wiſe men perceiued how hard it was to bring the rude people to vnderſtand reaſon, they iudged the beſt meanes to attaine this honeſt purpoſe to depend of learning in euery kinde, for by learning (as *Ouid* ſaith): *Pectora moleſcunt, aſperitaſque fugit*: Stout ſtomackes doe waxe milde, and ſharpe fierceneſſe is exilde. Therefore as *Beroſus* doth teſtifie, *Saron* that was the third King ouer all this Weſt part of Europe, for to bring the people

the 8 d. I set downe under the place of pence, and the one shilling I kepe in mind to carry to the nert denominations of shillings. Then come I to the shillings, and say, one that I bring or haue in minde, and 6. is 7. and 3. is 10. and 6. makes 16. which, because it containeth not one pound, I set directly vnder the place of shillings. Then come I to the pounds, whose parcels are 2, 3, 4. that is in all 9. that 9. doe I set downe directly vnder the pounds: And so the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

l	s	d
14	12	8
4	6	8
3	—	—
2	3	4
	6	8
9	16	8
4	16	0 *Rest.*

Now, for the worke of Subtraction, I must rebate that totall summe of Addition out of the highest number, that is to say, from the 14 l. 12 s. 8 d.

Therefore to performe the worke, I say, 8 d. out of 8 d. remaineth or resteth nothing, therefore, in the place of the rest or remaine, right vnder the denomination, I set downe 0. Then comming to the shillings where I find 16. which should be taken out of 12. but I cannot: therefore I imagine to borrow 1. of the nert Denomination, that is,

is, of the 9 l. and put that one pound so borrowed vnto 12 s. that maketh 32 s.

Now 16 s out of 32 s resteth 16 s. which 16 s. I set downe directly vnder the place of the rest.

Lastly, counning to the pounds, saying one pound in mind that I borrowed, and 9. make 10. then 10. out of 14. there resteth 4. So doth my whole rest or remaine appeare to be 4 l. 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as they are set in order, beginning with the pence, saying, 8. 4. 8. make 20 d. which 20 d. you should take out of the 8 d. aboue the line, but you cannot, therefore shal you borrow 1. of the next denominati-

l	s	d
14.	12	—8
4	—6	—8
3	—0	—0
2	—3	—4
0	—6	—8
9	—16	—8

on, that is to say 1. of the shillings, & put it to the 8 d. that maketh 20 d. now 20 d. out of 20 d. resteth 0. which Cipher I set downe directly vnder them.

Then one shilling that I borrowed or had in

in mind, & 6. make 7. and 3. make 10. and 6. make 16. then 16. out of 12. I can not take, therefore of the next Denomination I doe borrow one l. and put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly, I come to the pounds, saying: 1 l. in mind, of that I borrowed, and 2. make 3. and 3. is 6, & 4 is 10. then 10. out of 14. there resteth 4.

So doth my remainer or rest appeare as before to the 4 l. 16 s. 0 d.

Scholar. This doe I perceiue very well, and if there bee none other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckoned to be next in order.

A proofe of Subtraction in numbers of one denomination.

Master. We haue done indeed with the art of Subtraction, as touching the working. But yet before we goe to Multiplication, I will instruct ȳ how to examine your worke, whether it be well done or not. For the performance whereof, if you marke what I said right now in the last manner of proofe, you may easily perceiue what is to be done for this proofe, which is onely made (as before was taught you) by the aid of Additiō thus

Draw vnder the lowest number (which is your Remainer) a line, and then adde the summe paid in all, and the Remainer together. And if these twaine added together in one summe make the contents of the vpper number

number aboue the line, which in this example is 14 l. 12 s. 8 d. then is the subtraction well wrought, or else not.

As for example, in our first summe, which stood thus.

	l	s	d
	14	12	8
	4	6	8
	5	0	0
	2	3	0
		6	8
Paid in all.	9	16	8
Rest.	4	16	0
	14	12	8

Where, in the title of pence, I find 8. and 0. the 8. I set downe directly vnder him in that place of pence. Then in the place of shillings I find 16. & 16. which make 32. shillings, wherein is contained 1 l. and 12 s. the 12 s. I set down directly vnder them in their due place of shillings and one pound I kepe.

Then comming to the pounds, saying, 1. that I kepe, & 4. is 5. and 9. is 14. which 14. in due order I set downe directly vnder the as this figure sheweth. Which is also 14 l. 12 s. 8 d. agreeing with the vpper number aboue. So I find the worke is good, and the Subtraction well wrought.

Now for the proofe of the latter Subtraction, w we reckoned for the shorter worke: Draw vnder the Remainer also a line, then adde that remainer & all the other numbers that ye did subtract before together, & write that that amounteth vnder the lowest line, and

Proofe in Subtraction of diuers denominations.

and if the ſumme that commeth thereof be equall to the higheſt number aboue, then is the Subtraction well wrought, or elſe not.

As for example alſo in the laſt ſummes which ſtood thus.

	£	s̃	d̃
	14	12	8
	4	6	8
	3	0	0
	2	3	4
	0	6	8
	4	16	0
	14	12	8

Firſt, in the title of pence I adde 8. 4. 8. that maketh 20 d̃. which containeth one ſhilling and 8. pence.

The 8. I ſet downe vnder the loweſt line in the row or title of pence, and that one ſhilling I kæp to carry to the nert Denomination or place of ſhillings.

Then returning to the ſhillings, ſaying: one in minde, or that I kæpe, and 16. make 17. and 6. make 23. and 3. make 26. and 6. make 32 ſhillings, which amounteth to one pound, 12 s̃. the 12 s̃. I ſet downe vnder the title of ſhillings, and 1 pound I kæp or haue in mind to carry to the nert Denomination or place of pounds. Then come I to the pounds, ſaying, 1. that I bring and 4. make 5. and 2, make 7. & 3. is 10. and 4. make 14. then doe I write 14 vnder the pounds, & ſo haue I ended the Addition, and I ſee that the lowelt line is like vnto the vppermoſt line in number, wherefore I know that I haue well done.

And thus now haue I taught you the Art of

of Subiraction, and the meanes to proue whether it be well wrought or not.

Many other workes and proofes might be shewed of Subtraction, but one of these, whether of them you please, are most aptest, and best allowed of any other worke or proofe, whether it be of l, s, d, or any other grosse summe whatsoeuer.

Scholar. Sir, I thanke you most hartily, for now I vnderstand well both the feate of Subtraction and his proofe.

Master. Therfore now will I make an end of Subtraction, and wil instruct you in Multiplication.

Multiplication.

Vltiplication is an operation that two summes produce the third, which third summe so many times shall containe the first, as there are Vnities in the second. And it serueth in steade of many Additions. As for example. When I would know many are 30: times 48: if I should adde 48. thirty times, it would be a long worke. Therefore was this worke of Multiplication deuised, which shall doe that at once that Addition should doe at many times.

Muliplication what it is.

Scholar,

Scholar. I perceiue the commodity of it partly, but I shall not see the full profit of it till I know the whole vse of it. Therefore Sir, I beseech you, teach mee the working of it.

Multiplycation of Digits.

Master. So I iudge it best, pnt because that great sums cannot be multiplied, but by the Multiplication of Digits, therefore I thinke it best to shew you first the way of multiplying then: As when I say, 8. times 8 or 8. times 9. &c. And as for the small digits vnder 5. it were but folly to teach any rule, seeing they are so easy that euery childe can doe it. but for the Multipication of ehe greater Digits, thus shall you doe,

First, set your Digits one ouer the other right then from the vppermost downeward, & from the nethermost vpward, draw straight lides, so that they make a crosse, cōmonly caled Saint Andrewes crosse, as you see here. Then looke how many each of them laketh of 10. and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7. times 8. I must write those Digits thus.

The difference.

Digit difference

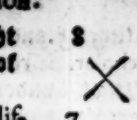
8

7

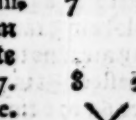
Then doe I looke how much 8 doth differ from 10 and I find it to be 2, that 2

doe

doe I write at the right hand of 8, at the end of the line thus.



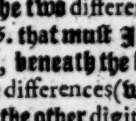
After that I take the difference of 7 likewise from 10. that is 3, and I write that at the right side of 7. as you see in this example.



Then do I draw a line vnder them, as in Addition thus.

Last of al, I multiply the two differences, saying 2, times 3, make 6. that must I euer set vnder the differences, beneath the line: then must I take 1. of the differences (which I wil, for al is like) from the other digit (not from his owne) as the lines of the Crosse warne me, and that that is left, must I write vnder the digits. As in this example if I take 2, from 7, or 3, from 8, there remaineth 5. that 5, must I write vnder the digits, & then there appeareth the multiplication of 7. times 8. to be 56. And so likewise of any other Digits, if they be aboue 5. for if they be vnder 5 then will their differences be greater then themselues, so that they cannot be taken out of them. And againe, such little summes euery child can multiply, as to say: 2, times 3,

Digit differece.



8 4 times 5. and such like.

Scholar. Truth it is. And seeing me seemeth that I understand the multiplying of the greater digits, I will proue by an example how I can do it. I would know how many are nine times 6.

Master. It is all one in value to say nine times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and so of all other summes.

Scholar. Then would I know how many are 6 times 9, therefore I set the digits thus, and make the crosse thus.

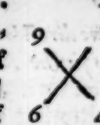
Then do I set their differences from 10. at the right side, the difference of 9, which is 1, against it, & the difference of 6, which is 4, against it also, as in this example.

And vnder them I draw a line. Then doe I multiply the digits together, saying: one time 4 maketh 4, that 4 doe I write vnder the differences thus.

Then take I one of the differences from the other digit, as one from 6, or else 4 from 9, & each waies there resteth 5, which I do write vn-

der

der y digits thus. And so appeareth the multiplication of 6 times 9 to be 54. Thus I sée the feat of this manner of multiplication of digits.



Maſter. Now might you goe ſtraight to the multiplication of great numbers, ſaue that both for your eaſe and ſuretie in working I will draw you here a Table, whereby ſhall appeare the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, ſeeke your firſt or laſt Digit in the greater figures, and from it go right forth toward the right hand, till you come vnder the number of your ſecond Digit, which is in the higheſt rowe, and then the number that is in the meeting of the rowes of little ſquares (which

come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this table the multiplication of 7. times 9. seeke first 7. in the greater figures, & then goe right forth toward the right hand, till you come vnder 9. of the highest row, in which place where you so come vnder the other digit (as here for example you come vnder 9.) is alwaies contained the ofcome or product, which you seeke: and that place we terme to bee in the common angle, in respect of the two numbers so taken on the outsides, as here in that common angle, where the rowes of little squares (directly proceeding from 7 & 9. doe meet, you haue 63, which 63. is the summe of the multiplication of 9 by 7.

To multiply greater sums.

Scholar. This is very good & ready. And so may I find the multiplicatiõ of any digits: but now how shal I do in greater summes?

Master. *When you would multiply any sum by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number vnder it, for that is the place of the Multiplier or Multiplicator, that is to say, the number by which Multiplication is made, and it in English alwaies put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times, is*

Multiplier.

Times.

that

that which must be multiplied.

Therfore when I would multiply one number by another, I must write the greatest highest, & the lesser vnder it, as in Addition. And vnder them must I draw a line. As for example: If I would multipy 264. by 29, I must set them thus.

264
29

¶ Of which numbers thus set downe to be multiplied, may be formed a question, as thus. There are 29 men, and each man hath 264 Lambes. The question is, how many Lambs they haue in all.

To the performance wherof, I must multiply euery figure of the higher row, by euery figure of the nether row: and that that amounteth, I must set vnder the line. as thus

First I doe multiply 4 by 9, saying: 9 times 4 (or 4 times 9 which is al one) that maketh 36, as the Table before of digits doth declare, of that 36 I must write the 6. that is the Digit, vnder the 9, and the Article 3. I kepe in mind to carry to the nert place.

264
29
6

Then come I to the second figure of the higher row, which is 6, and say: 9. times 6. make 54, and with the 3. in me mind make 57 the 7 I set downe vnder the 2, and 5 I kepe in minde.

264
29
76

after that I come to the next figure, which is 2, and multiply it by 4, and that maketh 8, and with 5 that I haue in mind maketh 13: wherfore because it is the last worke of the Multiplier, I set downe in order as you see:

264
29
2376

And so haue I ended the first figure of the Multiplier. Wherfore I giue it now a fine dash with my pen.

Then begin I with the next figure, and multiply it into all the higher figures as thus:

264
29
2376
8

First, 2 times 4 make 8, that 8 doe I write vnder the second place: for euermore the Digit or first figure of the Multiplication that amounteth of the first figure of the higher number, must be set vnder the Multiplier of it, and the other in their order toward the left hand

Scholar. I vnderstand you thus, that the Digit of the sum amounting of the Multiplication of the first figure of the higher row, by the first figure of the lower row, or Multiplier, must be set vnder the first place: and that that amounteth of the same first figure by the second Multiplier must be set vnder the second place, and so of the other, if there be more Multipliers.

Master. So meane I indeed: and if there amount but a Digit, then must it be set vnder the Multipler.

And

And now to goe forth: I multiply by the same 2, the second figure of the higher row, which is 6, saying: two times 6, make 12: whereof I write the digit 2 vnder the third place, and the Article 1, I kæpe in minde.

Then doe I multiply the last figure of the higher summe by that same 2, saying: two times 2 is 4, and with the 1 that I haue in mind maketh 5, which 5 I write vnder the fourth place. And so haue I ended the whole Multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as sone as I haue dispatched any Digit, by which I multiply: and the summes stand thus.

Then must I drawe a line vnder all those summes that mount of the multiplication, and must adde all them into one summe, as in the example you may see.

```
264
 29
2376
 28

264
 29
2376
 528

264
 29
2376
 528
7656
```

Where in the first place I find but 6, and therefore write I it vnder the line. Then in the second place 8 and 7 make 15, whereof I write 5, and kæpe one in my mind, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amounteth of the Multiplication of 264.

by 29. & that is the iuſt number of Lambes the 29. men had.

Scholar. If there be no more to be obſerued in it, then can I doe it, I ſuppoſe, as by this example I ſhall proue.

¶ There is a peece of ground which containeth 1365. yards in length: and 236 yards in breadth, I would know how many yards ſquare there is in all this peece of ground, which numbers I ſet downe with the greater aboue, and the leſſer vnder, as you ſée.

	1365
	236

Then doe I multiply 5 by 6. ſaying: 6. times 5, make 30. of which I write the cipher in the firſt place and the Article 3, I doe kéepe in minde to carry to the next place.

	1365
	236
	0

Then doe I by the ſame 6, multiply the ſecond figure of the higher ſum, which is 6. ſaying, 6 times 6, make 36, and 3, in my mind make 39, of which I write the 9, vnder the ſecond place, & the Article 3, I kéepe in minde.

	1365
	236
	90

Then doe I multiply the third figure, which is 3, by the ſame 6, and that maketh 18, and with 3, in my minde make 21. Then 1. I ſet downe, and kéepe 2, in mind.

	1365
	236
	190

Then come I to the laſt figure of the higher ſumme,

summe, & multiply it by 6, saying, 6 times 1, make 6, and 2, in my mind make 8, that 8, doe I write vnder the fourth place. And so haue I ended the first Multiplier, and dash him sleightly with my pen.

Then beginne I with the second multiplier, and say first 3, times 5, that maketh 15, of which I set the 5, vnder the second place, because that the Multiplier is there, and the Article 1, I keepe in minde.

Then come I to the second Figure, that is 6, and multiply it by 3, which maketh 18, and with one in minde maketh 19, the 9, I set down vnder the third place, and 1 I keepe in minde.

Then come I to the third Figure, which is 3, and multiply it by 3, saying: three times three make 9, and with 1, in my minde make 10, the Cypher I set vnder the fourth place, and the Article I keepe in minde.

And then comming to the last figure 1, I multiply it by 3: and it maketh 3, and with the one in mind it maketh 4, which 4, I set in the fift place, and then haue I ended

1365
236
8190

1365
236
8190
5

1365
236
8190
95

1365
236
8190
095

1365
236
8190
4095

ended two of the Multipliers, & the summes stand as you may see in the latter end of the page going before: & then I give 3 his rath.

1365
236
8190
4095
0

Then come I to the third Multiplier, and multiply it into euery figure of the higher sum, and first I say: 2 times 5 make 10, of which I set the Cypher vnder the Multiplier in the third place, and the Article 1 I kepe in minde.

And so multiplying the second figure 6, by that same 2, there amounteth 12: & with one in my mind make 13. whereof I write the Digit 3 vnder the fourth place, and the Article 1 I kepe in mind.

1365
246
8190
4095
30

Then doe I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6. and with the one in minde make 7. which 7. I set downe vnder the fift place, as appeareth by the example.

1365
263
8190
4095
730

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the first place, and then doth the summe stand thus.

1665
236
8190
4095
2730

And so haue I ended the whole

whole Multiplication.

But now (as you taught me) to know what this whole sum is, I must adde all those parcels together, and then vnder the line will appeare, as you may see, the grosse or totall summe is, 322140. Whereby I know there is so many yards square in that peece of ground.

	1365
	236
	8190
	4095
	2730
	322140

Master. This is well done.

Scholar. Then, me thinketh I could call it well done, when I know, whether I had well done or no.

Master. It is to be proued by 9, as Addition was, but the surest proofe is by diuision, and therefore I will reserue that proofe by diuision, till you haue learned the Art of Diuision. And anone I will shew you how it is commonly proued.

¶ *But first, for your further instruction in this exercise of Multiplication, I will with one example more try your cunning and so make an end: And the question is this. I would know how many dayes it is since the Natiuitie of our Lord and Sauiour Iesus Christ vnto the yeere 1590. Which to performe, you must multiply by the dayes in one whole yeere, which are 365.*

Scholar. Now, for that you haue giuen me so

so much light into the question, you shall see I will handsomely finish the worke, for according to your former instructions I set them downe with a line vnder them, thus.

1590
365

Then say I, 5 times 0. is 0. the 0. I set downe vnder the first place, as here appeareth. Then say I 5, times 9, make 45, the digit 5, I set downe in the second place vnder 9, & the article 4. I keepe in mind to be added to the next multiplication. Then saying fiue times 5, make 25, & 4, in mind makes 29. the 9, I set downe in the third place, & 2, I keepe in minde. Then comming to the last figure, I say once fiue is 5, or fiue times 1, is 5, & 2, in mind make 7, that 7, doe I set downe vnder the fourth place: and thus haue I ended my first Multiplier, and therefore I giue it a dash with my Pen.

1590
365
7950

Then come I to the second Multiplier, which is 6, and doe likewise multiply it into the vpper number, saying, 6, times 0, is 0. the 0. I set downe in the second place right vnder his Multiplier: then say I, 6, times 9, make 54, the 4 I set downe vnder the third place, and 5 I keepe in mind. Then say I 6, times 5, make 30. and 5. I keepe in mind make 35, the digit 5, I set downe in the fourth place, & 3. I keepe

1590
365
7950
9540

kéepe in minde. Then say I, 6 times 1 is 6: or once 6 is 6, and 3 in mind make 9, which I set downe next, and so haue I ended two Multipliers: wherfore I dash the 6 with my penne.

Then I begin to multiply the third Multiplier into the ouer number, saying, 3 times 0. is 0.: the 0. I set downe in the 3 place right vnder his Multiplier. Then say I, 3 times 9 make 27, the digit 7 I set downe in order next, & 2 I kéepe in minde, then say, I 3 times 5 is 15, & 2 in mind is 17. the 7 I set downe, & 1 I kéepe. Lastly I say, once 3 is 3. & 1 I kéepe is 4, which I set downe orderly next: And so haue I ended ỹ Multiplication, and my figures stand thus.

1590	
365	
7950	
9540	
4770	

Master. I commend you for yont diligence, the worke is very perfectly done, which parcels if you now adde together into one sum. it will be 580350, which is the grosse or totall summe of that Multiplication, and declareth the number of daies since our Lord and Sauiour his incarnation. vnto the end of 1590 yéres, beside 397 daies and twelue houres for leape yéres.

Scholar. This is maruelousme thinketh, that such great matters may so easily be atchieued by this Art, which heretofore I euer thought had béne impossible, as infinite

nite sorts of people are of that minde.

Maſter. Truth it is, that knowledge hath no greater enemy then ignorance, for this is one of the leaſt of tenne thouſand things that may be done by this Art, as hereafter you ſhall be able to iuſtifie.

Scholar. This manner of Multiplication I perceiue, if there be no more in it.

Maſter. *Yes, there are other formes and helpes for eaſe and ſhorter labour of the worke of Multiplication: but I will remit them till you haue a little taſted Diuiſion, where alſo the like helpe into Diuiſion may be vſed: and ſo therefore vnder one example for both, will I ſhew you both eaſe in Multiplication, and alſo in Diuiſion.*

But ſith the other formes and workings do nothing differ from theſe works in effect, but onely in ſetting of the numbers, I will ouerpaſſe them till a more meet place and time. And now will I inſtruct you in Diuiſion, ſo that you thinke your ſelfe ſufficiently to perceiue what I haue taught you.

Scholar. Yes, Sir, I thanke you, but I do not perceiue how to examine my worke, to trie whether I haue well done, or no: therefore as you promiſed me ere-while, I pray you firſt ſhew me how I ſhall proue it.

Maſter. *That is commonly vſed by the proofe of 9. as you learned before in Addition, ſauing that it differeth from that forme in*

diuers

diuers respects: as for example.

First, you must make a crosse after this manner.

Then must you examine your sum that should be multiplied, & loke what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoeuer remaineth in it after casting away 9 so often as you can, write that at ỹ other side of the crosse: thẽ must you multiply those two numbers together, & loke what amounteth thereof, if it be vnder 9, write at the higher part of the crosse: but, if it be aboue 9, then take then 9 as often as ye can, and write the rest at the head of the crosse: as first for example, we will proue the example you put forth of the pece of ground that contained 1365 yards in length, and 236 yards in breadth. Proofe of multiplication.

Therefore first, I cast away all the nines from the summe to be multiplied, saying: 5 and 6 make 11, cast away 9, rest 2: then 3 and 2 makes 5, and 1 is 6: that 6 I write at the one side of the crosse thus.

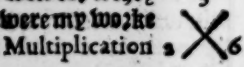
6

Then do I examine the Multiplier which is 236, wherein, when the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the crosse.

2 6

Then doe I multiply 6 by 2, and it maketh

maketh 12, from which 12, I withdraw 9. then resteth 3, that 3, doe I set at the head of the crosse. Then doe I examine the grosse summe amounting of the multiplication, which is 322140, where I find 9, once, & 3, remaining; that 3. I set at the foote of the crosse, & then I see it to agree with the other 3, at the top of the crosse, & so know I that I haue done well: for if they two did differ, then were my worke vaine, and the Multiplication false.



This is the common proofe, but the most certaine proofe is by Diuision, of which I will anone instruct you.

A sure proofe of Multiplication.

Scholar. Sir, what is the chiefe vse of Multiplication?

Master. The vse of it is greater then you can yet vnderstand: howbeit, these plaine commodities it hath, that if you would resolue any great and whole value into many smal & lesse portions, as if you would chang pounds into shillings, pence, or any other greater or smaller parcels by Multiplication, you shall doe it speedily & easily. Also if you should [illegible] to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more speedily, readily easily, and surely, then by often and sundry Additions. Take you these commodities grossely shewed for an answer at the [illegible], and

and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Scholar.

WEll Sir, then in Diuision I pray you to instruct me. But me thinketh by the name of it, that it should be all one with Multiplication: for I call that Diuision, when any thing is parted into diuers and manie parts.

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceiue that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrary wayes, it bringeth many parcels into few, but yet so, that these few taken together, are equall in value to the other many: for by Diuision, pence are turned into shillings, and shillings into pounds: As for example, of 120 shillings it maketh 6 pounds, so are 120 turned into 6. which is a smaller number: but then if you consider the Denominations, you shall see that they are such that

70.) you shall haue for your quotient 426 Duckets, your desire.

Thus much haue I said of money. Now will I shew you in like sore, the distinction of weights.

Weights.

Troy weight.

A peny weight.

An ounce.

A pound Troy.

After a Statute made, Anno 11, H 7. there ought to be but one sort of weights. As 24 Barly cornes drie, and taken out of the middest of the eare, doe make a peny waight, 20 of those peny waights make an ounce, & 12 ounces a pound of troy waight, by which is weighed Bread, Gold, Siluer, Pearle, Silke & such like.

But cõmonly there is vsed another weight called Haberdepoise, in which 16 ounces make a pound. Therefore when you would reduce ounces into pounds, you must consider whether your weight be Troy weight or Haberdepoise: and if it be Troy weight, you must diuide your onnces by 12, to bring them to pounds, but if it be Haberdepoise, you must diuide them by 16. Now againe, there be greater weights, which are called an hundred, halfe a hundred, a quarterne, and also a halfe quarterne, &c.

Haberdepoise.

A hundred weight.

Scholar. Why so there may be reckoned 20 pound, 40 ℔, 200 ℔ & such innumerable.

Master. All these are numbers of weight; but they haue not common weights made to their rate, as the other haue. And againe these that I did name are not iust in number as they seeme by their name, for an hundred

dred is not but 100, but is 112 pound. And to the halfe hundred is 56: the quarter 28, & the halfe quarter 14. And these be the common weights used in most things that are sold by weight.

Howbeit there are in some things other names, as in Wooll, 28 pound is not called a quarterne but a Todde: and 14 pound is not named halfe a quarterne but a Stone, & the 7 pound halfe a stone. Other names because they differ in many places, and agree in few, I let them passe. Woolle. Weights. Todde. Stone.

But a sacke of Woll by the Statutes, is limitted to be 26 stone: Sacke of Woolle.

¶ Now in Cheese, though it be sold by the hundreth, and by the stone in some places, yet the very weights of it are Cloues and weyes. So that a cloue containeth 8 pound and a wey 32 Cloues, which is 256 pound, that is 12 score & 16 l. & so much weigheth a wey of Suffolke Cheese, & the like weight is or should be the Barrel of Suffolke butter. Cheese weights.

The Wey of Essex Cheese containeth fiue score and fiftene pound: and so much is also the Barrell of Essex Butter.

Moreouer this weight is vsed by the Appothicaries in their Phisical compositions & mixture of medicine wherin the least is a graine. G

The Apothecaries waits	20 Graines, 3 Scruples, 8 Dragmes, 16 Ounces.	make	A Scruple, A Drachme, or Dragme, An Ounce, A Pound,	thus characterized. ℈ ʒ ℥ lb

The Apothecaries weights.

Measures for liquors. Now of weights are made other measures both for graine and liquor. For 8 pound in Troy weight, maketh a pint in measure, so that 8 pound of 8 pints do make a gallon: halfe a gallon is named a pottle, a halfe pottle is called a quart, which containeth two pints. Now aboue a Gallon the next measure is a Firken: then the Tercian, a Kilderkin, or halfe Barrell, and a Barrell. And by those measures are sold commonly Ale, Beere, VVine, and Oyle, Butter and Soape, Salmon, Herrings and Eeles.

A Pint. Gallon. Pottle. Quart. Firken. Tercian. Kilderkin. Barrell.

But as these bee vnlike things, so the measures of their Vessels doe differ, for the measures of Ale are as followeth.

Ale measures. Of Ale { *the Firkin.* / *the kilderk.* / *the Barrel.* } *containeth.* { 8 / 16 / 32 } *gallons,*

Beer measure. Of Beer { *the Firkin.* / *the kilderk.* / *the Barrel.* } *containeth.* { 9 / 18 / 36 } *gallons*

Sope measures. Sope measures, both Frking, Kilderkin, and Barrell should be equall to Ale measure *Moreouer the Statutes doe limite the weight of euery of those three vessels being empty.*

{ *A Barrell* / *Halfe a Barrell* / *A Firken* } *to weigh empty* { 26 / 13 / $6\frac{1}{2}$ } *pounds.*

Herrings. *Herrings also be sold by the same measures that Ale and Soape be sold by.*

Her-

Herrings also are sold by the tale, 120 to the hundred, ten thousand to the Last.

Salmon and Eeles haue a greater measure. Salmon & Eeles.

Salmon & Eeles	the Butte	holdeth	84	gallons.
	the Barrell		42	
	halfe Bar.		21	
	the Firkin		10½	

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels.

Now as for wine vessels they are seldome smaller then Hogsheds, which are of 63 Gallons: Euery Hogshed, is two Barrels: yet there are many other wine vessels, but of them all sée this Table; and marke the measures one by another. Wine measures.

Of wine and oyle	the Ronlet	holdeth	18½	Gallons.
	the Barrel		32½	
	the Hogsh.		63	
	the tercian		84	
	the Pipe.		126	
	the Tunne.		252	

But you shall marke that there be other kinds of Tertians: for there be Tertians, (that is to say) Thirds of Pypes, of Hogsheds, and of Barrels, as wel of other things as of wine. Tercians.

Also

But. Also Malmeseyes, and Sacke, &c. the halfe Tun is not called a Pipe, but rather a Butte, And thus much haue I thought méet to tell you at this time.

Scholar. And is that alwaies true?

Master. I haue told you how it should be, but how it is, I may not say: how they do differ daily from their iust measure, that Gaugiers can tell you better then I. But I will let this passe now, and speake briefely of the other measure.

Drie measures. And as of weights there did spring the liquid measures, (whereof I spake last) so of the same springeth drie measures, as Pecks, Bushels, Quarters, and such like, whereby are measured corne and like graines, also Salt, Lime, Coales and other like. And this is the order and quantitie of them.

A Pecke: A Pecke in the measure of two Gallons.

A Bushell Quarter Wey:

A Bushell.	containeth	*foure Peckes.*
A Quarter.		*eight Bushels.*
A Wey.		*sixe quarters.*

These are the common names & measures, but in diuers places there be diuers sorts.

The Bushell in many places is two Bushels: but then is that Bushell there called a Strike, and in some places halfe a quarter is called a Cornocke. But these diuersities are too many to tell you briefly them al, and againe, sith they are against the law and statutes, I count them vnméete to be vsed.

Strikes: Cornock.

But

But now remaineth yet another kind of measures, whereby men meate length and breadth, and thickenesse, and those are, an Inch, a Foote, and such other: whose names and quantities this Table sheweth. Measure to meate length, breadth, & thickenesse.

3 *Graines of Barly in length make an inch.* An Inch.
12 *Inches* } *make* { *a Foote.*
3 *Foote* } { *a Yard.* Foote. Yard.
3 *Foote and* 9 *Inches* } *make* { *an Elle.* Elle.
5 *Yards and a halfe* } { *a Pearch.* Perch.

1 *Perch in breadth and* 40 *in length doe make a rodde of Land, which some call a rood, some a yard land, and some a Farthendele.*

2 *Farthendels* } *make* { *half an acre of ground.*
4 *Farthendels* } { *an Acre.* Acre.

¶ *More,* 40 *rods in length doe make a furlong,* 8 *furlongs make an English mile, which containeth* 320 *Perches.*

So that an English mile, grounded upon the Statute, is in length 1760 yards, 5280 foote, and 63360 Inches.

Somewhat greater then the Italian mile of 1000 paces, and 5 foote to a pace.

Here might I tell you many things else touching measure, and also how to reduce strange measures to our measures, but because it cannot bee well done without the knowledge of Fractions, which as yet you have not learned, I will let them passe til another time that I have taught you the knowledge of broken numbers.

I Scholar.

The parts of time.
A day.
An houre.
Weeke.
Moneth.
Yeere.

Scholar. But yet sir of the parts of time, I pray you tell me somewhat.

Master. You know that a naturall day hath 24 houres, and euery houre hath 60 minutes. It nedeth not to tell you, that 7 daies make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeere, lacking one day, and certaine houres, and minutes: but of that I shal instruct you hereafter.

Here will I make an end of Reduction for this time, which though it be counted no kinde seuerall of Arithmeticke, you see it is no lesse needfull to be knowne, or easier to be done, then any of the other.

Scholar. Marry sir, it semeth vnto me much harder the any other sort, for it requireth the knowledge of so many things: but now sir when you see time, I am ready to learne forth: for as much of Reduction as you haue taught me, I remember; but if I doe at any time forget, I shall haue recourse to the tables which you set forth for me.

Master. So doe you, for it will not be remembred without exercise. But in as much as you vnderstand so much as wee haue intreated of, I will now instruct you in Progression.

Progres-

Progreſsion.

ALthough vntill this day the moſt part of Writers haue defined Progreſſion as a compendious kind of Addition, yet truly it is not ſo: for Progreſſion (as the very nature of the word doth informe any man) is a going forward and proceeding in numbers, and that regularly and orderly, whoſe place is aptly choſen to be very neere, or rather next after the expoſition of the foure principall parts of Arithmeticke, for in it after a moſt eaſie manner, are all the foure former parts exerciſed and practiſed: and not onely Addition, as cuſtomably is done. Which cuſtome hath beene the cauſe, why it hath ſo ſpecially beene named a kind of Addition, and defined to be a quicke and briefe Addition of diuers ſummes, proceeding by ſome certaine and reaſonable order.

You ſhall alſo vnderſtand that there are infinite kinds of Progreſſions, but for you (as yet) two are ſufficient to be exerciſed in, of which the one I call Arithmetical, and the other Geometricall.

Arithmetical Progreſſion.

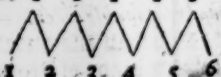
Arithmeticall Progreſſion is a rehearſing or placing downe of many numbers, number after number, in ſuch ſort that betweene euery

two next numbers rehearsed or placed downe, the difference, diuersity, or excesse, be equall and alike.

Scholar. Sir, I thanke you for that you haue both opened vnto me what Progression is truly, and also why it is here placed. But I pray you with an example make plaine your definition.

Master. Examples cannot want, seeing all reasonable creatures naturally vse the order of one kind of Arithmetical Progression (which therefore is also named natural) whensoeuer they distinctly doe count or number any multitude by one, saying, 1, 2, 3, 4, 5, 6, whereby the proceeding from the number to number, and euerie one surmounting and exceeding his fellow next before by a like quantitie (which here is 1) declareth the same to be Arithmetical Progression. And for the more plainenesse, I set it downe in this manner.

The common excesse. 1 1 1 1 1



1 2 3 4 5 6

The Progression.

Scholar. This is most euident. And I thinke that I am able to tell you now of any Progression Arithmeticall propounded, what is that common excesse or difference whereby it proceedeth, if this order be kept in it.

Master.

Master. What say you of 3, 6, 9, 12, 15?

Scholar. They excæde each other by 3. And that may I set downe in such evident order, as you did your Example of naturall Progression in this wise.

The common excesse. 3 3 3 3

The Progression. 3 6 9 12 15

Master. And doe you not also now perceiue, that the whole Table of Multiplication may be made by the order of Progression Arithmeticall? either if you will begin at the first number of any of them on the left hand, and so procæd right ouerthwart: or at any of the first numbers of the vpper row, and goe directly downeward.

Scholar. I pray you let me consider the thing a little, and I will answere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now very wel, for the commō excesse or difference betwēn any two next, is continually as much as the first number of euery row, either frō the left hand ouerthwart taken, or from any of the vppermost ouerthwart rowes downeward

To know the total summe of an Arithmeticall Progression.

Master. Now then of any such Progression, you would spēdily know the total sum much quicklier then by common rules of Additiō: first tel how many numbers there are (which numbers here wee call places or parcels) and if they be odde, write their sum downe by it selfe, as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7 as you may see, therefore set down 7 in a place alone, then adde together the first number and the last, as in this example: adde 2 to 14, and that maketh 16, take halfe of it, and multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together, as in this example, 8 multiplied by 7 maketh 56, and that is the summe of all those figures.

Scholar. What will I worke by another example. I would know how much this summe is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places and there are 9, that I note. Then I put the first number 5, and the last 29, together, and they make 34, I take the halfe of it, that is, 17, and multiply by 9, and it

it maketh 153. That you say is the summe of all the numbers.

Master. So shall you find it if you trie it.

Scholar. How shall I trie it?

Master. By your common Addition, for if you adde all the parcels together, you shal see the same sum amount, if you did worke well. And that manner of Addition trieth all kinds of summing any Progression.

Scholar. Then can I summe a Progression, if the numbers of the parcels be odde. But what if they be euen: as in this example, 1,2,3,4,5,6,7,8?

Master. When the number of the parcels is euen, then note that also as you did before, and likewise adde the first sum to the last, and by the halfe of the number of the places doe you multiply it: as in our example, the parcels are 8, that I note: then adding the first sum to the last, there amounteth 9, that doe I multiply by halfe of the parcels, that is, by 4. and maketh 36, which is the sum of the parcels.

But if you will take one rule for these both, doe thus; multiply the halfe of the one, by the other whole, and the sum wil amount al one. For sometime it chanceth that the number of the parcels be odde, so that their halfe cannot be taken: and sometime it chanceth the Addition of the first number & the last, doe bring forth an odde number, so that the

A generall rule.

halte of it cannot be taken: but they will neuer be both odde.

Scholar. Then I perceiue this, if there beno maze belonging to it.

Master. As accustomably it hath béne taught, this hath béene the chiefe & only exercise in Progression vsed. But that you may perceiue how diuers waies, & to how great profit so simple a thing (as this Arithmeticall Progression is) may be considered and vsed, I will here propound you six propositions: of which, foure of them were inuented by a friend of mine, & neuer before this published: and the two first, were neuer to my knowledge written of, but by 3 men.

Scholar. This doth greatly encourage me to be attentiue vnto your words, séeing I shall not onely be instructed at your hands in the common knowne rules of this excellent art, but besides that, so abundantly in other new rules enformed, as my very enterance shal seeme to passe a great many mens further study, and longer continuance. Therefore sir I beseech you, let me know your sixe propositions.

Master. These they are.

1 *To know the last number without proceeding by continuall Addition, till you come vnto it, so that the common excesse, the first number, & the number of the places be known.*

2 *The first number of the Progression and the last*

last being knowne, with the common excesse to finde the number of the places.

The excesse being giuen, and the first or last, to know the quantitie of any middle number, whose place is giuen from the first or last. 3

The totall sum being giuen, and the first and last to find out the number of the places. 4

The totall summe of any Arithmaticall progression being giuen, and the first and last, to finde out the common excesse. 5

The totall sum being giuen, and the mutuall excesse, with the number of the places, to giue the first or last number of the same progressiõ. 6

Many moe considerations could I propound you in these *Arithmeticall progressions*, but these are sufficient, to giue you occasiõ to think, that rulers of knowledge & *Arts* are infinitely capable of enlargement.

Sch. Happy were I, if I did but well vnderstand that which is already inuented and written, And yet in my simple fantasie these things offer themselues (in manner) to be studied for about progression, therefore I pray you to proceede to the rules answering to these propositions.

Master. I will orderly for euery of these sixe propositions giue you rules, and with euery one an example, vnlesse the plainnesse and easinesse neede no farther exemplifying.

For the Solution of the first, Multiply the excesse by a number lesse by 1 than the num- 1 Propositiõ

ber of the places, and the off-come adde to the first number, so you shall have the last number, which is sought for.

As for example: If there were seuen places in a progression Arithmeticall, whose continuall increase or mutuall excesse were 4, and the first number were 5, and I would know what the last and seuenth number is: I multiply 6, which is one lesse than 7, (the number of the places) by 4, thereof commeth 24. which I adde to 5, that maketh 29: and that is the last number which I desire to know. And this you may streight way proue, by continuall proceeding from 5 tyll the seuenth place, encreasing euery one by 4, as thus.

5 9 13 17 21 25 29.

Lo here, the last, being also the seuenth, is 29.

Scholar. I perceiue already one good property in this rule, which in all workes is to be desired: that is, it wil ease one from great labor, if a progression were propounded of a hundred or two hundred places, or more. And also it is very easie to worke, and most necessarie for the totall summe finding, in a very long progression.

2 Proposition. Master. *The second rule is this. From the last subtract the first, the remainder diuide by the common exces, to the* quotient *adde* 1, *and you haue the number of the places, which*

you would know: As in this progreſſion,

6 11 16 21 26 31

If I know onely 6 and 31, and that they encreaſe by 5, then according to the rule, from 31 I ſubtract 6, there remaineth 25: which 25 I diuide by 5 (the common exces) the quotient commeth forth 5, to which I adde 1, that maketh 6: and ſo many are the places, as you ſée.

Scholar. This rule is ſo eaſie, that I were much to blame, if I could not remember it.

Maſter. *The third propoſition may alwaies* 3 Propoſi-
thus be ſolued. Multiply the exces *by a numb-* tion.
ber leſſe by one, than the diſtance of the place *is from the* firſt, *or the* laſt uumber giuen: *the off come adde to the* firſt, *if the* diſtance *be reckned from the* firſt, *and the* firſt *alſo known, or ſubtract from the* laſt. *if the diſtance be from the* laſt counted *& the* laſt giuen *alſo, and that which commeth forth, either in that* Addition to the firſt, *or* ſubtraction from the laſt, *is the number* ſought. As for example, *I propound you this* Progreſſion.

8 15 22 29 36 43 50 57

And for the apt conſidering the manner of this queſtion, I will note ouer euery place his diſtance from the firſt, and vnder euery place his diſtance inclуſiuely frō ỳ laſt, thus.

1 2 3 4 5 6 7 8
8 15 22 29 36 43 50 57
8 7 6 5 4 3 2 1

Now

Now if the exceſſe whereby this progreſſion ſtandeth, be knowne to be 7, & firſt number giuen, being 8, I would know what number ſtandeth vnder 4, that is to ſay in the fourth place. I multiply 7 by 3 (which is leſſe by 1 than the number of the place propounded) that yeeldeth 21 to which I add 8 ỳ firſt number ſo commeth 29: which I ſay to belong to the fourth place, as ye ſee in the example it alſo doth: or if in the third place from the laſt, you would know what number in this example ſhould ſtand the laſt number being knowen to be 57, and the common exceſſe 7, then by 2 which is leſſe by 1 than the place propounded, I multiply 7, that giueth 14: which I ſubtract from 57, ſo remaineth 43: which appertaineth to the third place incluſiuely reckned from the laſt, and ſo my example giueth you.

Scholar. I perceiue right good vſe of this rule: for if I had forgotten what the firſt number were, and remember ſtill but the laſt, the common exceſſe, and the number of the places, then might I come by the knowledge of my firſt number againe.

And me thinketh, ỳ it differeth not much frõ the firſt propoſition, ſauing that which you make hẽre a middle number, there was made the laſt: and alſo in this point it differeth, that in it the laſt was only ſought, and no conſideration had in numbring the places

ces

es from the laſt, as here I marke in your numbers noted vnder your progreſſion.

Maſter. And thinke you not, the middle numbers of Progreſſion ſtanding of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continual encreaſſing from the firſt by the common exceſſe, or abating from the laſt continually the common exceſſe as the very finall numbers in a ſhorter progreſſion would doe?

Sch. Yes ſir, that I thinke right wel and therfore I am glad of this new framed propoſition, & the manner of the working of it.

Maſter. *The rule of the fourth is this. Adde the firſt and the laſt together, and by the ſame diuide the totall ſumme. Double the quotient, and that will be the number of the places.* 4 Propoſition.

Scholar. Then if in a Progreſſion, whoſe ſum were 207, and the firſt number 12, and the laſt 57, if I adde 57 & 12 together, that maketh 69, & by it I diuide 207, the Quotient will be 3, which I double, & ſo I haue 6, & ſo many muſt bee the number of the places that this Progreſſion ſtandeth on.

Maſter. Whether it be ſo or no how wil you trie?

Scholar. Halfe 6, which is 3, being multiplied by 69, muſt make 207, the toral ſum; if 6, bee the number of the places. For

ſo

so the whole work of your rule in summing any Arithmeticall Progression did enforme me. I will then multiply 69 69, by 3, thus, 3

It commeth forth iustly. 207

Master. I must much herein commend your promptnesse, both in memorie and in well applying your rule: although in manifest words it did containe no such matter.

Scholar. Sir I pray you heare me frame one example more.

Master. I am well pleased, so that ye be short, for you make me more longer heere than willingly I would haue beene: but I cannot perceiue how I could haue omitted any thing as yet, without your great lacke thereof.

Scholar. If I had received 85 pounds of certaine men, but of how many I haue forgotten, yet I remember that the first gaue me 7 l. and the last 27 l. & euery payment after other did rise by a like sum. And the man for whom I receiued this money, conditioned with me, that of euery payment I should haue twelue pence for my labour: now vnlesse I can by Art finde the truth of this case, I am like to lose the most part of my reward.

Master. I perceiue you can handsomely frame an example, which should concerne your owne gain: I pray you let me see how you

you would do iustice in this point.

Scholar. I adde the first and the last together, that maketh 34: by which I diuide 85, thus.

1	
27	17
85	(2
34	34

Why how now Sir, here is a remnant of 17, in which 34 cannot be had: so that now I am in the bypars for doubling of my quotient, & farewell then both my Iustice, and a good lump of my gains.

Master. Yée are neuer the farther from the matter, though it fall into a Fraction. For you shall vnderstand, that the Fraction which of any such worke proceedeth, is euer halfe of one such, as the vnits of the quotient before are. And that you may trie, if you double that which so remaineth, for then it will be equall to your Diuisor, as if ye double 17 (the remnant) it maketh 34, and your Diuisor also was 34, this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard example. For with it I haue a general rule for the Fraction that may happin in this worke. So that the qnotient being two and a halfe, I double that, and it maketh 5, therefore should my gaine be 5 shillings. And to bee sure, (by your leaue) I will trie it, for I will multiply halfe of 34. (which is the first and last number ioined together) by 5, thus.

17	
5	
85	

It

It is most true (I see) that I should leese thing by the former working.

5 Proposition.

Master. *The fifth proposition hath this rule appertaining vnto it: By the fourth rule find the number of the places that being done, from the last subtract the first, and the residue diuide by a number lesse by 1. then the number of the places, and the quotient will shew the excesse which is sought for.*

An example hereof shall be this: If one had distributed 685 pounds to a certaine number of men, you neither can tell how many they were, or how much the one mans money exceeded his next before, but you are sure that the excesse was equall betweene euery two next: and also you remember that the first had 19. & the last 118 pounds, how would you find both the number of the men and the excesse, continually obserued in the successions of their payments?

Scholar. Your rule doth plainly bid, first to find the number of the places.

which I will do according to the fourth rule. I adde 19, and 118 together thus.

	118
	19
	137

By this 137, I diuide 685, thus.

Seeing there is no fraction but a whole number, being 5. I double that, and then must the number of the pla-

	x3
685	(5
	x37

res bee 10. Now from the last I subtract the first, as 19 from 118, thus: And so remaineth 99

118	
19	
99	

This 99 I diuide by a number lesse by 1 then the number of the places, and seeing the places were 10, I diuide 99. by 9, thus the quotient is 11, and so was the excesse, if I haue followed your rule right.

99	
99	(11

Master. You haue wrought euery part of this question both well in order, and truely in the practise of your rules.

Scholar. I will then set it downe also formably, so that the number of the places, the excesse and the totall sum may streight appeare, as your first example stood.

11	11	11	11	11	11	11	11	11	11	11
19	30	41	52	63	74	85	96	107	118	

The common excesse.

That the places be 10, and that from the first to the last the common excesse is 11. I perceiue most euidently, but whether the totall sum be 685, I haue not yet proued, which I will now doe: I adde 19 and 118 together, that maketh 137. I multiply that by halfe the number of the places, thus.

137	
5	
685	

The Progression.

All things agree most exactly, so that I am perfect enough in these rules, if I forget them not againe.

Master. Use maketh all things perfect.

6 Proposition.

Your sixth rule is this. By the number of the places diuide the totall summe, double the quotient, and that will be the first and the last ioined in one summe. Then by a number lesse by 1 then the number of the places, multiply the excesse, that ofcome subtract from the first doubled quotient, and the halfe of the residue is the first number. The last number you may diuersly find out, as by the first of our sixe rules, or by subtracting this first number from the sum which here contained both the first and the last iointly, (or thirdly) by continuall adding the excesse.

Scholar. I pray you make this somewhat more plainer with an example.

Master. If euery moneth in the yeare (counting them now as 13) you gained clearely 40 shillings more then you did the moneth next going before, and at the yeeres end you finde the whole gaine 5720 shillings, but ye remember not how much either the gaine of the first moneth or the last was, by this rule it may be tried out.

Scholar. So that here ye same to apply the 13 moneths to thirteene places, the 40 shillings euery one more then the other next before it, to be the common excesse, and 5720 s. to be the totall summe.

Master. It is true: by 13 then I diuide 5720

5720. in this manner.

I double this Quotient, ſo haue I 880 for the first and the laſt ſumme ioined together, by 12, which is leſſe by one then the number of the places; I multiply 40 (the common exceſſe) ſo commeth 480.

x	
x5	
x720	(440
x333	
xx	
	40
	12
	80
	40
	480

This 480 I ſubtract from 880, ſo remaineth 400: halfe whereof is the first number which we deſired to know: that is 200.

And as for the laſt number I can giue you it three waies: As by the first of my fixe rules, I multiply the exceſſe by a number leſſe by 1, then the number of the places, as 40 by 12, that giueth 480, which I adde to the first, being 200, ſo ſhal the laſt be 680.

The ſame ſumme commeth forth, if yee ſubtract 200 from 880:

And thirdly, if I beginne at 200, and ſo proceed, encreaſing by 40, I ſhall at the thirteenth place haue 680, as thus:

200, 240, 280, 320, 360, 400, 440, 480, 520, 560, 600, 640, 680

Scholar. I thanke you moſt heartily for theſe fixe rules. Now if it be your pleaſure I would heare & learne ſomewhat of Progreſſion Geometricall.

Master. There are yet very many rules and propositions, which fall into this Arithmeticall Progression: but these shall suffice for this time.

And in Geometricall Progression I will be more briefe, both because I haue béene so long in this part of Arithmeticall Progression, and also for that it would require the knowledge of Rootes & numbers surd, (whereof ye haue yet learned nothing) if I should frame the like propositions in them as I haue done in these. Therefore I will onely teach you two practises, about it, & so end the considerations and workes of these progressions.

Progression Geometricall.

Progression Geometricall is when the numbers increase by a like proportion, that is, if the second number containe the first, 2, 3, or 4 times, and so forth: then the third containeth the second so many times also: and so the fourth the third, and the fifth the fourth, wherefore I set these 3 examples.

1,	2,	4,	8,	16,	32,
1,	3,	9,	27,	81,	
2,	10,	50,	250,		

Here in the first example you see, that euery number containeth the other (that goeth next before him) two times: and in the second example three times. & in the third example fiue times. Now if you will know how to find easily the sum of any such numbers, doe thus: Consider by what number they be multiplied, whether by 2, 3, 4, 5, or any

any other, and by the same number doe you tiply the last summe in the Progression.

Scholar. I pray you worke it by this example, 2, 8, 32, 128, 512. 2048, which I haue framed by proceeding from 2, and continuall multiply by 4.

Master. Then must I multiply the last summe (which is 2048) by 4 also, & it will be 8192. Now must I abate from this sum the first number of the Progression, w here is 2, then resteth 8190. which sum I must diuide by 1 lesse then was the number that I multiplied by. Seeing then I multiplied by 4, I must diuide by 3, so diuiding 8190, by 3, the Quotient will be 2730, which is the summe of all the Progression. And now to proue whether you can doe the same, I giue you these numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number this Progression doth increase.

Master. In any such doubt doe thus: Diuide the second number by the first, and the Quotient will shew you the number that engendreth the Progression.

Scholar. Then is that number in this example 5, for so many times is 3 in 15.

Master. So is it. Now worke as I taught.

Scholar. The last number is 46875, which I multiply by 5, & it yeldeth 234375, from which I abate the first number of the progression, that is 3. & there resteth 234372,

which I diuide by 4, for that is one lesse then 5, and the quotient is 58592, which is the whole summe of the Progression.

Master. If you remember well this, you haue learned the Art of Progression both Arithmetical, and also Geometricall, which you may proue either by subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding together of all the parcels, for so will the same summe amount.

¶ And now for the vse and better vnderstanding of both these rules, I will propone vnto you certaine pleasant and necessary questions Arithmeticall and Geometrical, and to the performance of their workings, such necessary rules and documents, as are requisite for the vnderstanding of them, or any such like.

X A question of Velvet. A certaine Mercer sold 20 yards of Velvet to be paid in 12 weekes, by Arithmeticall proportion: that is to wit to receiue the first weeke 6 shillings, the second week 12, shillings, the third weeke 18 shillings, and so forth, encreasing the number of weekes by 6 shillings, till the twelfth and last weeke were expired. The question is how many pounds he had for 20 yards of velvet.

To the performance of this question and such other the like, I set forth the 12 payments in such sort, as for example, here appeareth in the margent. Then

Then touching the adding together of theſe ſummes, without the aid of Addition, according to the rules I taught you in Progreſſion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the Progreſſion, which is 72, and the firſt number together make 78, and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23 l' 8 s. And ſo much hath the Mercer for his 20 yards of veluet, which is nye about 23 s, 5 d, a yard:

6
12
18
24
30
36
42
48
54
60
66
72

Scholar. I underſtand this worke very well: but is there any proofe for the iuſtifying hereof as you haue of other workes?

Maſter, The worke of it ſelfe (being ſo perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for iuſtifying of the ſame: yet notwithſtanding becauſe your requeſt is reaſonable, I will propone an example for the proofe hereof.

A certaine man is bound to pay for 20 yards of veluet, the ſum of 23 l' 8 s: and it is to be paid weekly in 12 weekes or termes by Arithmetical Progreſſion. The queſtion is therefore to know with what number the ſame Progreſſion is to be begun & continued in ſuch equall proportion Arithmeticall, The proof of the laſt queſtion.

that in 12 weekes the same may iustly be accomplished.

A general rule.

For the resolution whereof, and of all such other like, reduce 231 8 s. all into shillings, which maketh 468 s.

Then adde 1 vnto 12, the number of the termes, it maketh 13, which 13 you shall multiply by halfe the number of the termes, which is 6, it maketh 78, then diuide 468 by 78, and you shall find 6 in the quotient, which is the true number that shall begin & continue the said Progression. That is to say, the first weeke 6 s. the second 12 s. the third weeke 6 s. more, which is 18 s. and so euery weeke as they rise, 6 s. more then the weeke before, as is manifest in the question aforesaid.

A question of a Farme.

A Farme is to be solde to be paid by the weeks in a yeere, the first weeke to pay 4 s. the second weeke 8 s. the third weeke 12 s. & so forth, increasing each number by 4, till the number of 52 (which are the number of weeks in a yeer) be expired. The question is what the price of the Farme commeth to.

Scholar. I doubt not, but by that you haue already taught me, to end this question very wel, wherefore I set forth the Progression with his excesse 52 times.

Master. May stay a while. And here for your further ease, (to abridge you of great labor that appeareth to fall out in this question,

sion, & so may doe in any other the like, if a question were proponed of 100 or 200 places or moe, and that this question nor any other y^like can be ended, vnlesse you know absolutely what the last number of the Progression at the 52 place is or ought to be) I will giue you a generall rule how to know the last number of any Progression Arithmeticall, aswell as if you had ordinarily proceeded by continuall Addition, till you had come to the last worke, which is this.

Multiply the excesse by a number lesse by one than the number of the places, and thereto put the first number of the Progression, and you shall haue your desire. A generall rule.

Sch. This rule is wel worth the noting: for if I vnderstand you aright I consider that my excesse is 4, which I multiply by 51, which is one lesse than y^number of the places, & it maketh 204: whereunto I adde the first number of the Progression, which is 4, and then it is 208: which you say is, or should be the last number of the Progressiō.

Master. This is a most approued truth if there were neuer so many places.

Scholar. This rule is so easie, that I were much to blame, if I do not remember it. For by the benefit hereof I haue such an ease & light into this excellent Art, that my first entrance doth seeme to passe a great many mens further studie & longer continuance.

Master. Many moe considerations could I proponnd you in these Arithmetical Progressions: but these are sufficient for a taste, to giue you occasion to thinke that rules of knowledge and Artes are infinite capable of enlargement.

Scholar. Happie were I, if I did but well vnderstand that which is already inuented and written. But these things, in my simple fantaste, offer themselues to be greatly beneficiall vnto the ayde of Progression. Therefore now I will goe forward with your question.

Now considering that the 52 and last place is 208, I add thereunto the first number of the Progression, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 shillings. And so much is the totall summe or addition of this Progression: which maketh 275 l, 12 s, as appeareth here by my tables.

Master. I like well your labour and commend you for your diligence, I will here propone one example more, & therewithall for this time wil end progression Arithmatical.

A question of Holland.

A certaine man bought 20 els of Holland to be paid in 17 weekes or termes by progression Arithmeticall. And the first weeke to pay 1 s. 8 d, the second weeke 3 s. 4. d, the third week 5 s. the fourth week 6 s. 8 d.

&

& so forth each weeke succeeding 20 pence more than the weeke before. The question is, what the sum of his 20 els commeth to.

Scholar. Because here is mention made both of shillings and pence, I feare there is some harder matter contained herein, than in the other before: therefore I pray you worke it your selfe, and I will diligently marke your labour.

Master. There is no more to be done in this then in the other before, but because your request is so reasonable, be attentive vnto me.

First by the generall rules I seeke to find out the last number of the 17 place, what this Progression ought to be. Therefore here in my tables multiplying the excesse 20 by 16, which is one lesse then the number of the termes or places, and it commeth to be 320, and thereunto adding the first number of ỹ progression which is 20 pence, all is 340 pence, or 28 s, 4 d: for so much ought the last number of the paiments to be.

Then finally to know what the whole 17 places amount vnto, I adde the first number of the Progression and the last together, which make 360. Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360, which is 180: and multiplying 180 by 17, commeth to 3060 pence, which maketh as you see by Diuision

12 l. 15 s. And so much is the buyer to pay for his 20 elles of Holland. Which 3060 pence if you diuide by 20, the number of elles that was bought, you shall find 12 s. 9 d. and so much paid he for an elle one with another.

The Proofe.

Aquestion of debt

A certaine man doth owe 12 li. 15 s. to be paid in 17 weekes or termes by Arithmeticall Progression. The question is, to know with what number he shal begin and continue the Progression in such equal proportion, as the same may be truely paid and satisfied in 17 weekes.

The Answere.

First I reduce 12 l. 15 s. all into pence, which as you se here in my Tables make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I should multiply by halfe the number of the weekes or termes, which is $8\frac{1}{2}$, which $8\frac{1}{2}$ multiplied by 18 cannot well be done vnlesse you were acquainted with Fractions or broken numbers, therefore you shall let that passe & multiply 17 by the halfe of 18:

which

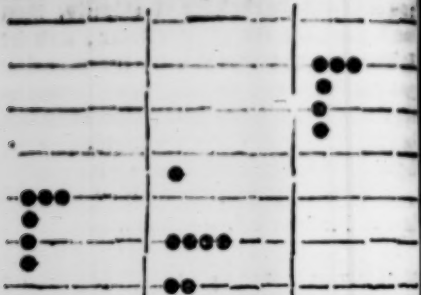
which is 9, (for that is all one with the multiplication of $8\frac{1}{2}$: and the multiplication of 9 into 17, maketh as you ſee 153, with which number you ſhall diuide the 3060 pence before ſaid: and the quotient bringeth forth 20 pence, which is the firſt number or payment to begin the progreſſion with all: and ſo each weeke ſucceeding to riſe 20 ſ more than the weeke before, and thereby in 17 weekes ſhall 12 l 15 ſ, be paid: as before was ſufficiently declared. Thus much for Progreſſion Arithmeticall.

Scholar. Certainly ſir, I know not how to render you condigne thanks for theſe benefits ſhewed me, which me thinketh are ſo eaſie, delightfull and pleaſant, that I count my ſelfe happy to be in your company.

Maſter. I am glad you delight ſo well herein, which is an art of wonderfull dexterity to al ſorts of men of what decree or profeſſion ſoeuer they be. And now will I propone a queſtion or two of progreſſion Geometricall.

A Mercer hath 12 yards of Satten which he valueth at 16 ſ. the yard, and ſelleth the ſame 12 yards to another man to be paid as followeth. That is to wit, for the firſt yard to haue one ſhilling, for the ſecond yard 2 ſ: for the third yard 4 ſ, for the fourth yard 8 ſ, &c. doubling each number following, til the twelfth & laſt yard. The queſtion is, who

A queſtion of Satten.

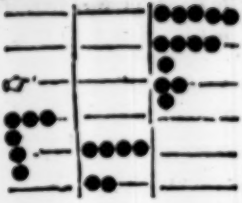


Scholar. I perceiue well, for indeede the ſumme that you haue ſet downe is 365000: for ſo much doth amount of 1000: multiplied by 365.

Maſter. Well then goe forth, in the next ſpace I find one Counter, which I remoue forward, but take it not vp, but (as in ſuch caſe I muſt) ſet downe the greater halfe of my multiplier (ſeeing it is an odde number) which is 182, and here I doe ſtill let the fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſée.

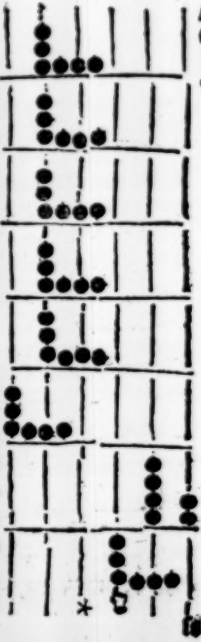
Where I haue ſet the Multiplication with other

ether; but for the ease of your vnderstanding, I haue set a little line betweene them. Now shuld they both in one summe stande thus:



Another forme of Multiplication.

Howbeit another forme to multiply such counters in space is this: first to remoue the finger to the next line beneath the space, and then to take vp the counter, and to set downe the multiplier fiue times, as here you see.

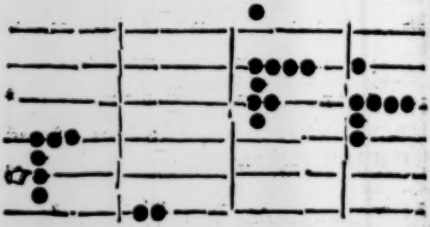


Which sums if you do adde together into one sum, you shall perceiue that it will be the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter

so this is plainer to reason, for such as have had small exercise in this Art.

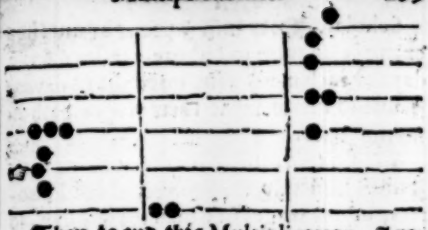
Notwithstanding you may adde them in your mind before you set them downe: as in this example you might haue said, five times 300 is 1500, and 5 times 60 is 300, also 5 times 5 is 25, which all put together doe make 1825, which you may at one time set downe if you list.

But now to goe forth, I must remoue the hand to the next Counters which are in the second line, and there must I take vp those foure Counters, setting downe for them my multiplier foure times severally, or else I may gather the whole sum in my mind first and then set it downe: as to say, 4 times 300 is 1200: 4 times 60 are 240: and foure time 5 make 20, that is in all 1460: that shall I set downe also, as here you see.

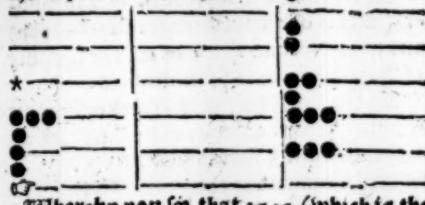


Which if I ioyne in one summe with the former numbers, it will appeare thus.

Then



Then to end this Multiplicatton, I remoue the finger to the loweſt line, where are only 2, them doe I take vp and in their ſtead doe I ſet downe twice 365, that is 730, for which I ſet one in the ſpace aboue the third line for 500 and 2 more in the third line with that one that is there already, and the reſt in their order, and ſo haue I well ended the whole ſumme thus:



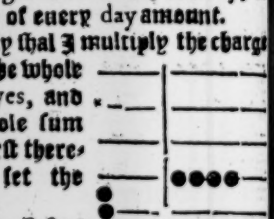
Whereby you ſee, that 1542 (which is the number of yeeres ſince Chriſt his incarnation) being multiplied by 365 (which is the number of the daies in one yeere) doth amount to 562830, which declareth the number of daies ſince Chriſts incarnation vnto the end of 1542 yeeres, beſide 385 daies and twelue howres for leape yeeres.

Scholar. Now will I proue by another example, as this: 40 Laborers (after 6 d the day for each man) haue wrought 28 dayes, I would know what their wages doth amount vnto.

A question of wages.

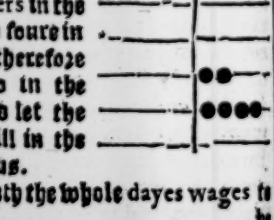
In this case must I worke doubly: first I must multiply the number of the Laborers, by the wages of a man for one day, so will the charge of euery day amount.

Then secondly shal I multiply the charge of one day by the whole number of dayes, and so will the whole sum appeare: First therefore I shall set the summes thus.



Where in the first place is the Multiplier (that is one dayes wages for one man) and in the second place is set the number of the workemen to be multiplied.

Then say: 6 times 4 (reckoning that second line as the line of vnites) maketh 24, for which sum I should set two Counters in the third line, and foure in the second, therefore doe I set two in the third line, and let the foure stand still in the second line thus.

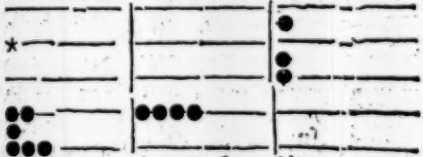


So appeareth the whole dayes wages to

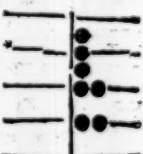
bi

be 240 ß, that is 20 ß.

Then doe I multiply againe the same sum by the number of daies, and first I set the numbers thus: then because there are Counters in diuers lines, I shall beginne with the highest, and take them vp, setting for them the Multiplier so many times as I take vp Counters, that is twice, then will the summe stand thus.



Then come I to the second line, and take vp those 4 Counters, setting for them the multiplier 4 times, so will the whole sum appear thus.



So is the whole wages of 40 workemen for 28 daies after 6 ß each day for a man 6720 ß, that is, 560 ℔ or 28 pound.

Master. Now if you would proue Multiplication, the surest way is by Diuision: therfore wil I ouerpasse it til I haue taught you ẏ art of Diuision, which you shal worke thus.

Diuision

Diuision in Fractions.

First set downe the Diuisor for feare of forgetting and then set that number that shall be diuided on the right siae so far from the Diuisor that the quotient may be set between them as for example.

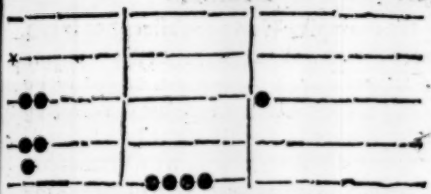
Example of sheepe.

If 220 sheepe cost 45 l, what did euery sheepe cost? To know this I would diuide the whole sum, that is 45 l, by 225, but that cannot be, therefore must I first reduce the 45 l, into a lesser denomination, as into shillings, then I multiply 45 by 20, and there 900: that summe shall I diuide by the number of sheepe which is 225, these two numbers therefore I set thus:

Then begun I at the highest line of the dividend [illegible], and that I may doe foure times: then say I foure times 2 are 8, which if I take from 9, there resteth but 1, thus:

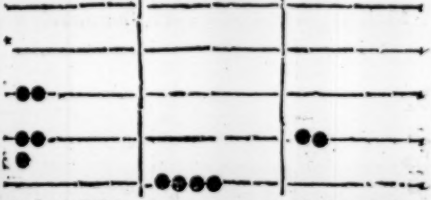
And

R. 268. l. 29. *read of the dividend, and seeke how often I may haue the diuisor therein, and that.*



And becauſe I found the Diuiſor 4 times in the diuidend, I haue ſet, as you ſee, 4 in the middle roome, which is the place of the quotient: but now muſt I take the reſt of the Diuiſor as often out of the remainer therefore come I to the ſecond line of the Diuiſor, ſaying: two times 4 make 8, take 8 from 10, and there remaineth 2 thus;

Then come I to the loweſt number which is 5, and multiply it 4 times, ſo is it 20, that take I from 20, and there remaineth nothing, ſo that I ſee my quotient to be 4, which are in value ſhillings, for ſo was the diuidend: and thereby I know that if 225 ſheepe coſt 45 l. euery ſheepe coſt 4 s.



Schol. This can I do as you ſhal perceiue by

by the example. If 160 souldiers doe spend euery month 68 ℔, what spendeth each man?

Example of wages

First because I cannot diuide the 68 by 160, therfore I will turne the ℔, into ß by multiplication, so shal there be 16320 ß, now must I diuide the sum by the number of souldiers, therfore I set them in order thus

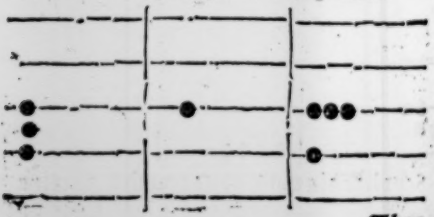
Then begin I at the highest place of the diuidend, seeking my Diuisor there, which I find once, therfore I set 1 in the nether line.

☞

Master. Not in the neather line of the whole summe, but in the neather line of that worke, which is the third line.

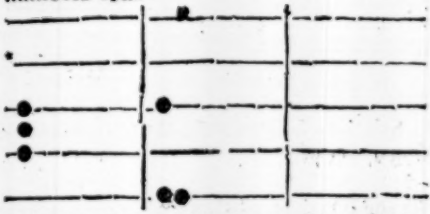
Scholar. So standeth it with reason.

Master. Then thus doe they stand.



Then

Then séeke I againe the rest, how often I may find my diuisor: and I sée that in 300 I might find 100 thrée times: but then the 60 will not be so often found in 20, therfore I take 2 from my quotient: thē take I 100 twice from 300, and there resteth 100, out of which with the 20 that maketh 120, I may take 60 also twice, and then stand the numbers thus:



Where I haue set the quotient 2 in the lowest line: So is euery souldiers portion 102 d, that is 8s, 6d.

Master. But yet because you may iustly perceiue the reason of Diuision, it shall be good that you set your Diuisor still against those numbers from which you doe take it, as by this example I will declare:

If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost? Example of purchase.

First will I turne the pounds into pence, so will there be 69600 pence. Then in setting downe these numbers I shall doe thus.

First set the diuidend on the right hand as ought

ſi ought, e then the Diuiſor on the left hand againſt thoſe numbers from which I intend to take him firſt, as here you ſée where I haue ſet the diuiſor two lines higher then his owne place.

Scholar. This is like the order of Diuiſion by the pen.

Maſter. Truth you ſay, and now muſt I ſet the quotient of this worke in the third line, for that is the line of vnites in reſpect of the Diuiſor in this worke.

Then I ſée how often the Diuiſor may be found in the diuidend, and that I find 3 times, then ſet I 3 in the third line for the quotient: and take away that 60000 from the diuidend, and further I ſet the Diuiſor one line lower, as you ſée here.

And

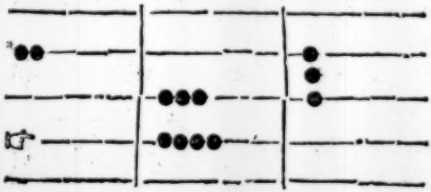
And then ſeke I how often the Diuiſor will be taken from the number againſt it, which will be foure times, and 1 remaining.

Scholar. But what if it chance that when the Diuiſor is ſo remoued it cannot be once taken out of the diuided againſt it?

Maſter. Then muſt the Diuiſor be ſet in another line lower.

Scholar. So was it in Diuiſion by the pen, & therfore was there a Cipher ſet in the Quotient: but how ſhal that be noted here?

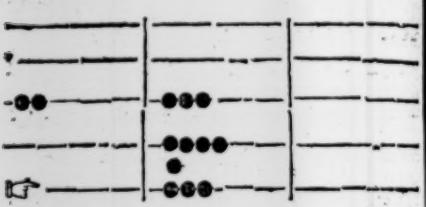
Maſter. Here needeth no token, for the lines doe repreſent the places, only looke that you ſet your Quotient in that place which ſtandeth for vnites in reſpect of the Diuiſor. But now to returne to the example: I find the Diuiſor foure times in the diuidend, & 1 remaining: for 4 times 2 make 8, which I take from 9, & there reſteth 1 as this figure following ſheweth: and in the middle ſpace for the Quotient, I ſet 4 in the ſecond line, which is in this worke the place of vnites.



Then remoue I the Diuiſor to the ner-

lower

lower line, and ſéke how often I may haue it in the Diuidend, which I may doe here3 times iuſt, and nothing remaine, as in this forme.



Where you may ſée that the whole quotient is 3480, that is 29s, whereby I know that ſo much coſt the purchaſe of one Acre.

Scholar. Now reſteth the p3oues of Multiplication, and alſo Diuiſion,

Maſter. Their beſt p3oues are each one by the other, fo3 Multiplication is p3oued by Diuiſion, and Diuiſion by Multiplication, as in the wo3ke by the pen you learned.

Scholar. If that be all, you ſhall not néd to repeate againe that which was ſufficiently taught already: and except you will teach mée any other feate, héere may you make an end of this Art, I ſuppoſe.

Maſter. So will I doe as touching whole number, and as fo3 broken number, I will not trouble your wit with it, till you haue p3actiſed this ſo wel that you be full perfect,

ſo

so that you néed not to doubt in any point ỹ I haue taught you, and then may I boldly instruct you in the Art of Fractions or broken numbers: wherein I will also shew you the reasons of all that you haue now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings & pounds, proceeding by no grounded reason, but only by a receiued forme and that diuersly, of diuers men: for the Merchants vse one forme, and Auditors another. The reason of all the former rules.

Merchants vſe.

Merchats Accompt.

But first for Merchants forme marke this example here, in which I haue expressed this summe 198l, 19s 11d. *So that you may see that the lowest line serueth for pence, the next aboue for shillings, the third for pounds, and the fourth for scores of pounds.*

And further you may see that the space betweene pence and shillings may receiue but one counter (as al other spaces likewise do) and that one standeth in that place for 6d.

Likewise betweene the shillings and the pounds, one counter stands for 10 shillings.

And

And betweene the pounds and 20 l. one Counter ſtandeth for 10 l.

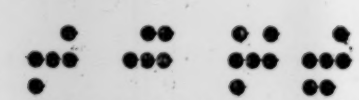
But beſide thoſe, you may ſée at the left ſide of ſhillings that one number ſtandeth alone, and betokeneth 5 s.

So againſt the pounds, that one Counter ſtandeth for 5 l. And againſt the 20 pounds, the one Counter ſtandeth for fiue ſcore ponnds, that is, 100 pounds, ſo that euery ſide Counter is fiue times ſo much as one of them againſt which he ſtanceth.

Auditors account.

Auditors Accompt.

NOw for the account of Auditors, take this example.



Where I haue expreſſed the ſame ſumme 198 l.—19.—11 d.

But here you ſée the pence ſtand towards the right hand, and the other increaſing ouerly toward the left hand.

Againe you may ſée, that Auditors will make two lines (yea and more) for pence, ſhillings, and all other values, if their ſums extend thereto. Alſo you ſée that they ſet one Counter at the right end of each row, which ſo ſet there, ſtandeth for fiue of that roome, and

and on the left corner of the row it standeth for 10 of the same row.

But now, if you would adde, or subtract after any of both these sorts, if you marke the order of the other feate which I taught you, you may easily doe the same here without much teaching: for in Addition, you must first set downe one summe, and to the same set the other orderly, and in like manner if you haue many: but in Subtraction, you must set downe first the greatest sum, and from it must you abate the other, euery Denomination from his due place.

Scholar. I doe not doubt but with a little practise I shall attaine these both: but how shall I multiply & diuide after these formes?

Master. You cannot only doe any of both by these sorts, therefore in such case you must resort to your other Arts.

Scholar. They that vse such accounts that it excéed 200 in the summe they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand therof, and the thousands they set in farther row yet, and at the left side thereof they set the 5000, and in the space ouer they set the 10000, & in a higher row 200 0, which all I haue expressed in this example, which is 97869 l 12 s, 9 d, ob, q. Ninety seauen thousand, eight hundred threescore and nine pounds, twelue

twelue ſhillings and 9 pence halfe peny farthing, for I had not told you before where neither how you ſhould ſet downe farthings, which (as you ſée héere) muſt be ſet in a void place ſteeling beneath the pence, for a farthing, one Counter, for ob. 2 Counters, for ob. farthing, 3 Counters, and more there cannot be: for 4 farthings make 1ð, which muſt be ſet in his due place.

☞

And if you deſire the ſame ſumme after Auditors manner, loe here it is.

But in this thing you ſhall take this for ſufficient, and the reſt you ſhall obſerue as you may ſée by the working of each ſort, for the diuers wits of men haue inuented diuers & ſundry waies, almoſt innumerable.

The

The second part of Arithmeticke, touching Fractions, briefely set forth.

Scholar.

Lbeit I perceine your manifold busines doth so occupy or rather oppresse you, that you cannot as yet completely end the Treatise of Fractious Arithmeticall, which you haue prepared, wherein not only sundry workes of Geometry, Musicke, and Astronomy be largely set forth, but also diuers conclusions and naturall workes, touching mixtures of Mettals and compositions of medicines, with other strange examples: Yet in the meane season, I cannot stay my most earnest desire, but importunely craue of you some briefe preparation toward the vse of Fractions, whereby at the least I may bee able perfectly to vnderstand the common workes of them, and the vulgar vse of those rules, which without them cannot well bee wrought. Arithmeticall fractions.

Master. If my leasure were as great as my will is good, you should not neede to vse any importunate craning, for the attaining of that thing, whereby I may bee perswaded that I shall any waies profit the cõmon

common wealth, or helpe the honest studies of any good members in the same: whereof while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

What a fraction is.

Therefore first to begin with the explication of this name, Fraction, what take you it to be?

Scholar. Mary Sir, I thinke a Fraction (as I haue heard it often named) to be a broken number, that is to say, to be no whole number, but a part of a number.

Master. A Fraction indeed is a broken number, and so consequently the part of another number: but that must be vnderstood of such another number as cannot be diuided into any other parts then Fractions, for although I may take the third part of 60, or the fourth part of it, and so of other parts diuerslie, yet these parts be not properly, nor ought not to be called Fractions, because they may be expressed by whole numbers, for the third part of it is 20, the fourth part is 15, the twelfth part is 5, and so forth of other parts, which all bee whole numbers.

What a fraction is properly.

Wherefore properly a Fraction expresseth the parts or part onely of a vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not be greater then one: and therefore it followeth that

that no one Fraction alone can be so great, that it shall make 1, as by examples I will declare as soone as I haue taught you to know the soome how a Fraction is expressed or represented in writing:

Numeration.

Numeration of Fractions.

BVt first to begin with expressing of a Fraction, *which is the numeration of it: you must vnderstand that a* Fraction *is represented by two numbers, set one ouer the other, and a line drawn betweene them, as thus,* $\frac{1}{3}\ \frac{3}{4}\ \frac{2}{7}\ \frac{10}{17}$, *which 4 Fractions you must pronounce thus:* $\frac{1}{3}$ *one third part,* $\frac{3}{4}$ *three quarters,* $\frac{2}{7}$ *two fift parts,* $\frac{10}{17}$ *ten 17 parts.*

Scholar. I vnderstand the soome of their expression & pronunciation: but their meaning or valuation seemeth more obscure. Yet I thinke that by the two first fractions I vnderstand the valuation of the two latter fractions, and consequently of other.

Master. Value them then, that I may perceiue your taking of them.

Scholar. $\frac{2}{7}$ Betokeneth two fift parts, that is to say, if one be diuided into 5 parts, that fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be diuided into 17 parts, I must take tenne of

them. And this I gather of the two first examples: for $\frac{1}{3}$ that is one third part, doth easily declare, that if one thing be divided into three parts, I must take out one of them: so $\frac{1}{4}$, that is three quarters, doth declare that one being divided into four quarters, I must take (for this Fraction) three of those quarters.

☞ If there be no more difficultie in their Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kinds of workes. For I understand that the same kindes of workes be in fractions that be in whole numbers.

Master. There are the same kindes of workes in both, albeit the order of them is diuers, as I will anone declare: but yet more in Numeration before we leaue it. You must vnderstand, that those two numbers which expresse a Fraction, haue seuerall names, The ouermost, which is aboue the line, is called the Numerator, & the other beneath the line, is called the Denominator.

Numerator. Denominator.

Scholar. And what is the reason of their diuers names? (For, in mine opinion, both be Numerators, seeing both they do expresse the numeration of the Fraction.

Master. You are deceiued: for one onely (which is the ouermost) doth expresse the Numeration: and the Denominator doth declare the number of parts, into which the vnit

vnite is diuided, as in this example, when I say: diuide a pound weight of Gold betweene foure men, so that the first man shall haue $\frac{3}{17}$, the second $\frac{5}{17}$, the third $\frac{7}{17}$, and the fourth $\frac{2}{17}$.

Now doe you perceiue that by the Denominator (which is one in all foure fractions) it is intended that the pound weight should be diuided into so many parts, I meane 15, and by the foure seuerall Numerators is limited the diuers portion that each man should haue, that is, that when the whole is parted into 15, the first man shall haue two of those 15 parts: the second man three of them: the third man foure: and the fourth man fiue. And so may you see the seuerall offices (as it were) of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceiue that a man can haue no more parts of any thing then it was diuided into, neither yet aptly so many: so that it were vnaptly said: You shall haue $\frac{15}{15}$ that is 15, 15 parts of any thing, sée it were better said you shall haue the whole thing.

Scholar. So doth it appeare reasonably, for the labour is vaine to diuide any thing, and then to apply the Diuision to no vse. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be diuided into 15 parts

onely, it is not possibly to take 16 of them, that is to say, more then altogether.

☞ Master. This is true touching the proper and apt vse of the name of a Fraction: yet improperly, and after a vulgar acceptation (for easinesse in worke) both those formes be called fractions, because they be written like Fractions, although they be none indeede: for $\frac{15}{15}$, and generally all such other, where the Numerator and Denominator be equal, are not Fractions, but the whole thing with all his parts. And so $\frac{16}{15}$ is not to be called a Fraction but a mixt number, of a whole number and a Fraction, for it is as $1\frac{1}{15}$ that is one whole 1 and $\frac{1}{15}$ parts, as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions where the Numerator is eyther equall or greater then the Denominator.

Mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in work write some numbers after that sort like Fractions: but they need not to call them Fractions, but (as they be) whole numbers or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions.

Now must you understand, that as no Fraction properly can be greater then 1, so in smalnesse vnder one the nature of Fracti-

ons

ons doth extend infinitely, as the nature of whole numbers is to increase above one infinitely, so that not onely one may be diuided into infinite Fractions or parts, but also euery fraction may be diuided into infinite Fractions or parts, which commonly be called Fraction of Fractions, and they be expressed diuerslie: as for example $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{1}{3}$, that is, three quarters of two third parts of one halfe part. Whereby is signified, that if one be diuided into two halfes, and the one halfe into three parts, and two of these three parts be diuided iointly into foure quarters, this fraction of fractions doth represent three of those quarters. Fractions of Fractions.

Scholar. I pray you let me proue by an example in common money, whether I do rightly vnderstand you or no. One Crowne which I take for an vnite, doth contain 60 d therfore the halfe of it is 30 pence: $\frac{1}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fiftéen pence, so then 15 pence is $\frac{3}{4}$ of $\frac{1}{3}$ of $\frac{1}{2}$ of a Crowne: and so is 3 pence $\frac{3}{4}$ of $\frac{1}{3}$ of $\frac{1}{2}$ of a shilling.

Master: You perceiue this well enough: yet this note I giue you by the way, that the forme of expelling these fractions is voluntarie, and hath no other reason than the will of the Diuisor, which forme many follow: for some expresse them thus $\frac{3}{4}\,\frac{1}{3}\,\frac{1}{2}$ without any figure of distinction betwéen them, which forme many follow. Some other do

for $\frac{12}{15}$ set 45 ob, for $\frac{4}{5}$ take 108 ob, and for $\frac{1}{3}$ put 75 ob. Now doth 45 beare the same proportion to 108 that 75 doth beare to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also of 180.

But these reasons may be better reserued till another time, when the knowledge of proportions in due order shal be taught: in the meane season I will shew you how it commeth to passe that in fractions the three summe must needs be lesser then any of the other two.

Note: Consider this, that when a fraction is postponed, as in the former example: if it be multiplied by more then 1, it will make more then one entire number. As if I multiply $\frac{1}{3}$ by 5: that is to say, if I take it 5 times, it will make three entire vnites: example in a Crowne, $\frac{1}{3}$ of it maketh 3 s, which if I take five times it wil amount to 15 s, that is, three entires Crownes: so if I take the same $\frac{1}{3}$ but twice it will yeeld 6 s, that is, one entire Crowne and $\frac{1}{3}$. Now if I take it but once, it cannot be more then it was before, that is 5 s. And if I take it lesse then once, it cannot be so much as it was before. Then seeing that a fraction is lesse then one, if I multiply a fraction by another fraction; it followeth that I doe take the first fraction lesse then once, and therefore the summe

ſumme that amounteth, muſt needs be leſſe then the firſt fraction.

Scholar. Sir, I thanke you much for this reaſon. And I truſt I do perceiue the thing, as by example of this ſame fraction ½ I wil experiẽce. If I take ½ of a Crowne once, that is to ſay, if I multiply ½ by 1, it will be as it was before, but ½: ſo if I doe multiply it by ½, that is, if I take but halfe one time, then will it be but halfe ſo much: likewiſe if I multiply it by ⅓ that is, if I take but the third part of one, it will yeeld but 12 pence, that is, the third part of the firſt fraction.

And ſo to make an end: If I take it but the twelfth part of once, that is, If I doe multiply it by 1/12, it will yeeld but the twelfth part of the firſt fraction, which is but three pence. And it followeth, that if 1/12 make three pence, then 1/15 muſt needs make fiue times ſo much, that is 15 pence, which was the ſumme that hath giuen the occaſion of all this doubt.

Maſter. When I perceiue you haue ſufficient vnderſtanding in this ſort of Multiplication for this time, wherefore I will omit that I might ſay more of Multiplication, till we come to Reduction, and will paſſe to the other worke, and firſt to Diuiſion, whoſe place followeth Multiplication, both by naturall order, and alſo in eaſineſſe of worke.

Diuiſion.

Diuision.

Diuision of fractions.

When seuer two Fractions be proponed, that one should be diuided by the other, I must set downe first the fraction that shall be diuided, (which is called the Diuidend) and then after it the other, which is the Diuisor. Then shall I multiply the Numerater of the Diuidend by the Denominator of the Diuisor, & that which amounteth I must put for a new Numerator. Againe, I shall multiply the Denominator of the Diuidend by the Numerator of the Diuisor, and the number that amounteth thereof, I must put for the new Denominator. And this third Fraction is the Quotient of the said Diuision.

Scholar. This seemeth easie in forme, as by example thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{3}$, first I multiply 5, (being the Numerator of the Diuidend) by 8, which is the Denominator of the Diuisor, and thereof riseth 30: then I multiply 8 being the Denominator of the Diuidend) by 2, being the Numerator in the Diuisor: and so riseth 16, the which I must make in a third Fraction, thus $\frac{30}{16}$.

Master. Me seemeth you are quicker in vnderstanding now, then you were when I taught you the Art of whole number: but that

that is no marueil: for the more knowledge that any man getteth, the readier shal hee finde his wit, & quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued here for the ease of worke, and lightnesse of vnderstanding, the nature of the Quotient.

Whensoeuer you diuide one fraction by another, eyther they be both equal together, either else the one is greater then the other: if they be equall, their Quotient shall be such, that the Numerator and the Denominator of it shall be equall also. And if the two first fractions be equall, their Quotient shall declare the same by the vnequalitie of the Numerator & Denominator, as in these examples following shall appeare.

First of equal fractions $\frac{2}{3}$ and $\frac{12}{27}$ be equal together, and if the one be diuided by the other, the quotient will be $\frac{1}{1} : \frac{81}{81}$, as you may perceiue by that rule afore said.

Now in the vnequall fractions, as $\frac{2}{3}$ and $\frac{4}{9}$, the quotient will be $\frac{18}{27}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see the reason why it should be so.

Maister. The reason is this: When any Fraction is diuided by another, the quotient declareth what proportion the diuidend beareth to the Diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$ maketh 2, which must be founded, not 2, but

Note how to know the proportion betweene two numbers.

but twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the quotient representeth the Diuidend, and the Denominator representeth the Diuisor. And this is alwaies true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne, till you haue learned the Art of proportions: notwithstanding somewhat of it I will declare in the next Rule of Reduction. But now for the easie remembrance of the quotient in Diuision: as soone as you haue set downe your two fractions the one against the other, then make a straight line for the quotient: and as soone as you haue multiplied the Numerator of the Diuidend, by the Denominator of the Diuisor, set the number that amounteth ouer the said line, and then multiply the other two numbers, and set their totall vnder the same line.

Scholar. I perceiue you would not haue me trust to memory till I were better expert, lest oftentimes I happen by misse remembrance to be abused. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$ I must set the numbers one against the other, (as here both appeare) & then make another line for the Quotient

2	3
by	
3	4

in

in some good distance, where I may set the numbers of the quotient, as soone as any of them is multiplied. So then as soone as I haue multiplied 2 by 4, which maketh 8, I shal set that 8 ouer that line, thus: 8 ——— And then multiply 3 by 3, which yeldeth 9: & that 9 must be set vnder the same line, and then will the whole quotient appeare thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but how may I perceiue that? Note.

Master. Although you shall better perceiue it by the Rule of Reduction, yet this example may be declared in commō coines, as in a common shilling of 12 d., of which $\frac{2}{3}$ maketh 8 d., and $\frac{3}{4}$ doth make 9 d., and so you may easily see that their proportions doe agree. And if you had taken this example before when you toke the example of $\frac{4}{6}$ and $\frac{2}{3}$, your Quotient would appeare (as this doth) more easie to vnderstand; whereas that Quotient being $\frac{10}{12}$, is not an easie proportion for you to perceiue, being yet little acquainted with proportions: wherof to giue you some taste, I will enter to the rule of Reduction: In which also I will declare other workes, both of Multiplication, and also of Diuision, which now I must for a time omit, as things that doe neede the helpe of Reduction.

Reduction.

Reduction of Fractions.

Fiue kinds of Reductions.

Herefore will I now declare the diuerſities of Reduction of Fractions, which commonly haue fiue formes of Reduction.

1 *Firſt, when there be ſundrie Fractions of intire vnite, they muſt be reduced to one denomination, and alſo into one fraction.*

2 *Secondly, when there be proponed fractions of fractions, they muſt be reduced likewiſe into one fraction: for otherwiſe they cannot be brought into one denomination.*

3 *Thirdly, when an improper fraction is proponed, that is to ſay a fraction in forme, which indeed is greater then an vnite: it muſt be reduced into apt forme, expreſſing the vnite and vnites of it, and the proper fraction diſtinctly. And ſometimes alſo it ſhall be needfull to conuert ſuch a mixt number of vnites with fractions, into the forme of a fraction, that is, into an improper fraction: which two formes I eſteeme but as one, becauſe they worke on one kinde of number.*

4 *Fourthly, there happeneth ſometimes fractions to be written in great numbers, which might be written in leſſer nnmbers: therefore is there a meane to reduce ſuch great numbers into their ſmalleſt termes.*

Fifthly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certaine parts, but none of like denomination with that Fraction, then may you reduce the said fractions into another, whose denomination shall expresse the common parts of that whole thing. 5

Scholar. This distinction in doctrine delighteth me much, but more with hope then present fruit: for as yet I do not understand scarcely the varieties, & much lesse the practise and use of their workes.

Master. Reduction is an orderly alteration of numbers out of one forme into another which is neuer done orderly but for some needfull use, as in every of the said fiue severall formes I will distinctly declare.

First therefore, when two or more severall fractions of any vnite be proponed, as for example $\frac{1}{12}$ and $\frac{1}{8}$, because it is hard to tell what proportion of the intire number those two fractions do expresse, therfore was Reduction diuised, to be a mean whereby these severall fractions might be brought into one denomination and fraction. The first forme of Reduction

And in these fractions this is the Art for bringing them to one denomination.

Multiply first the denominators together, and the totall thereof you shall set twice downe vnder two seuerall lines for two new denominators, or rather for one common Denomi- How to reduce Fractions.

Of divers denominations into one denomination.

nator. Then multiply the Numerator of the first fraction, by the Denominator of the second and set the totall thereof for the Numerator ouer the first line. Likewise multiply the Numerator of the second fraction by the Denominator of the first, and set that totall ouer the second line for the Numerator of that fraction, and so are those two first fractions of seuerall denominations, brought to one denomination.

Scholar. If I vnderstand you, as I thinke I doe, my example shall declare the same. The fractions which you propoped were these, $\frac{3}{16}$, and $\frac{5}{6}$, whose denominators (being 16 and 6) I multiply together, and there amounteth 96 which I set vnder two lines thus: $\frac{\quad}{96}$, $\frac{\quad}{96}$.

Then I multiply the Numerator of the first fraction by the Denominator of the second saying, 3 into 6 maketh 18, that I set ouer the first line for a new Numerator, and it will be thus $\frac{18}{96}$;

Likewise I multiply the Numerator of the second fraction by the Denominator of the first, saying, 4 times 16 maketh 64, that I set for the second Numerator, and the fraction will appeare thus, $\frac{64}{96}$.

So that both fractions brought to one denomination must stand thus $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You haue done well.

Scholar. I beseech you let me examine it after

after my accuſtomed forme, by common parts of coyne or other meaſure.

Maſter. Goe to.

Scholar. I haue a peece of Gold which is accounted worth 8 ſhillings, and containeth 96 pence, whereof $\frac{1}{12}$, that is, the 12 part, is 6 pence, and $\frac{3}{12}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{1}{2}$ of the ſame peece of Gold is 16 d. ſo that $\frac{1}{2}$ part maketh 64 d. that is $\frac{64}{96}$. And ſo I finde the ſums to agree with the other before.

Maſter. *So haue you now the Art to bring two ſuch fractions into one denomination. And if there be more then two, then muſt you multiply all the denominators together, and ſet the totall thereof ſo many times downe as there be fractions, and then to get for each one a new Numerator, multiply the Numerator of the* firſt, *by the Denominator of the* ſecond, *and the totall thereof multiply by the Denominator of the* third, *and ſo forth, if there be more. Likewiſe multiply the Numerator of the* ſecond, *by the Denominator of the* firſt, *and the totall thereof by the Denominator of the* third. *And in the ſame ſort multiply the Numerator of the third into the Denominator of the* firſt, *and the totall thereof into the Denominator of the* ſecond, *and ſo forth, if there were moe. So theſe three fractions* $\frac{2}{3}, \frac{1}{4}, \frac{3}{5}$ *doe make by reduction theſe other 3 fractions of one denomination* $\frac{24}{60}, \frac{15}{60}, \frac{36}{60}$. *All which you*

Note the reduction of 3. Fractions (or more) to one.

may bring into one fraction by adding the numerators together, and putting that totall for the common Numerator, reserving still that same common denominator. And those 3 fractions make one improper fraction thus, $\frac{59}{20}$.

Scholar. All this I perceiue, and also that this last fraction is more then an vnite, and therfore yon did cal it an improper fraction.

Master. There be certaine other formes of working in this Reduction, which I will briefly touch also, to giue you an occasion to exercise your wit therein.

The first varietie of Reduction

The first varietie is this: When you haue made and written downe your common Denominator (as I haue taught before) then to get a Numerator for the first, do thus: Diuide the common denominator by the denominator of the first fraction, & the quotient multiplied by the Numerator of the same yeeldeth a new Numerator for the first new fraction. So likewise do with the second and the third, and with all the residue, if there be more.

Sch. That wil I proue in your last example of these 3 fractions $\frac{2}{5}\ \frac{3}{4}\ \frac{2}{3}$. When the denominators be multiplied, they make 60: for 5 into 4 maketh 20, & 20 by 3 yeldeth 60, that I set downe 3 times, thus: 60, 60, 60: then to haue a numerator for the first I must diuide 60 by 5, (ỹ denominator of the first) & the quotient is 12, which I must multiply by 2 (the numerator of the first) & that maketh

teth 24, & so haue I for the first fraction $\frac{24}{60}$.

Likewise for the second fraction: I diuide 60 by 4, & there commeth 15: which I multiply by 3, and so haue I 45, and the second fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{20}{60}$. The second varietie.

Master. *Another way is this: If it happen so, that the lesser denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numerator ouer that lesser denominator, and for the lesser denominator put the greater, as thus in these two fractions,* $\frac{3}{12}$ *and* $\frac{2}{3}$, *three being the lesser denominator multiplied by 4, will make 12, which is the greater denominator: therefore by the same 4 I do multiply 2, which is the Numerator ouer 3, and that maketh 8: vnder which I do put 12 being the greater denominator, which is also made by multiplication of 4 into 3, and so haue I these two fractions* $\frac{3}{12}$, $\frac{8}{12}$, *thus shortly reduced, without altering the one fraction.*

Scholar. This I vnderstand.

Master. *Then marke this third way: if the denominators do not happen so, that one by multiplication may make the other, then looke whether they both may be parts of any other one number, as in* $\frac{5}{12}$, *and* $\frac{7}{18}$, *although the lesser taken but twice, be too great to make 18, yet they both may be parts vnto 36: therefore looke how many times twelue is in 36, and that quotient being multiplied by the Nume-* The third varietie.

rator ouer 12, *the totall shall be put in stead of Numerator ouer* 12, *and for* 5 *put* 15, *thus*, $\frac{15}{36}$. *So likewise looke how often is* 18 *in* 36, *and because it is twice, therefore by* 2 *multiply* 7, *which is ouer* 18, *and it will be* 14, *set that for the Numerator, and in stead of* 18 *put* 36, *and then shall your fractions reduced stand thus* $\frac{15}{36}, \frac{14}{36}$, *in stead of* $\frac{5}{12}$ *and* $\frac{7}{18}$.

Proofe. And if you will proue whether you haue wrought well or no, that may be proued by Reduction of them againe to their former denominations, which Art shall be taught in the fourth kind of Reduction, where greater termes of Fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction of the same termes or numbers come againe that were before, then is the worke good, els not.

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Scholar. Sir, I heare your words, but I doe not vnderstand many of them: which if it may please you declare.

Master. With a good will, when conuenient place serueth, but that must be in the said fourth kinde of Reduction. In the meane season I will declare the second forme of Reduction, which teacheth how to reduce fractions of Fractions into one Fraction, and so to one Denomination.

The second sort of Reduction. *When fractions of Fractions bee proponed, you shall multiply the Numerators of each into other, and set the totall for the new Numerator,*

merator, and then multiply all the denominators likewise, and take their totall for the new denominator, and so are they speedily reduced.

Scholar. If that be all, then I vnderstand it already, as by this example I will declare. These be the fractions, ½, of ⅓, of ⅔ of ⅞, which I would reduce to one denomination. Reduction of fractions of fractions into one denomination.

Therefore begin I with the Numerators, and multiply them together, saying: 3 into 2 maketh 6, & 6 by 6 maketh 36, which multiplied by 7, yeldeth 252, that I set ouer a line for the Numerator thus. 252

Then I multiply the Denominator, 4 by 3 maketh 12, & that by 7 bringeth 84, which multiplied by 9, yeldeth 756, the new denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to vnderstand yet. 252 / 756

Master. You thinke so, and no maruel, but anone you shall learne to iudge it easily: for this fraction is no more indeed then ⅓, although it be in greater termes, and therefore more straunger, and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kind of fractions, and also the due vnderstanding of the Reduction.

The

The entire measure parted into 9:

Here you see the longest measure, (which standeth for the whole and entire quantity) first parted into 9 diuisions, whereof 7 are seuered by the second measure: and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being diuided into foure parts, the lowest measure doth containe $\frac{1}{4}$, so that the same $\frac{1}{4}$ must be named, not $\frac{1}{4}$ of the whole measure, but indeed is $\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar: This example is so sensible that I cannot chuse but see it. And furthermore I see also that the same fraction is equall to $\frac{1}{3}$ of the entire measure, as the lines which run vp and downe doe expresly set forth. Also I see here that $\frac{1}{3}$ of $\frac{6}{7}$ is equall to $\frac{6}{21}$. And further yet that $\frac{1}{3}$ of $\frac{7}{9}$ is equall to $\frac{7}{27}$.

Master. I am glad that you see it so well, not doubting but you will gather greater light of knowledge hereby.

But

But now it is time that we come to the third forme of Reduction which teacheth of improper fractions, that is to say, mixt numbers of vnites and Fractions, although they appeare like Fractions, as this $\frac{16}{7}$, which doth include 5 vnites wholly, and $\frac{2}{7}$ ouer. Wherefore first you shall know them: by that the Numerator is greater then the Denominator.

The third forme of Reduction of improper fractions.

Scholar. **Indeed sir, that appeareth reasonable, that if the Numerator doe expresse more parts to be taken of any vnite, then the Denominator doth signifie that vnite to be diuided into, it must needs follow that such a fraction importeth more then the whole, that is to say, the whole with certain parts ouer: but what reductiō is therein it?**

Master. There be two seuerall kinds of reduction, concerning such fractions. Sometimes it shall bee needfull to conuert these fractions into vnites, and the proper fractions that will remaine. And sometimes contrariwise, it shal be meet to reduce mixt numbers, that is vnites written with fractions, into the forme of one simple fraction, and so be there two wayes.

Two seuerall wayes in this Reduction.

Scholar. **What is the meane of the first way to turne improper fractions into vnites with their fractions?**

Master. *That is thus. Your Numerator being greater then the Denominator, must bee diuided by the same Denominator, and the quotient*

The first way.

Reduction of improper fractions into vnites, with their proper fractions.

quotient thereof expresseth the vnites: the Remainer shall be put for the Numerator of the fraction that resteth, and the Denominator must be the same that was before.

Scholar. For example, I take $\frac{17}{5}$. And dividing 17 by 5, the quotient will be 3, and there will remaine 2.

Master. That must you write thus, $3\frac{2}{5}$ where (you see) I haue written 3 without any line, as entire numbers ought to be written, and the 2 that remained I haue set ouer the former Denominator with a line, as a proper fraction. And this number doth signifie now 3 vnites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by vnites here vnderstand crownes, so it were 3 Crownes, and $\frac{2}{5}$ that is 2 s.

The second way.

Master. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this hapeneth sometime that when the reduction is so wrought, there remaineth nothing. And then it is not a mixt number, but a simple entire number, represented like a fraction.

Scholar. As $\frac{15}{5}$ will make 3 iust, and $\frac{18}{3}$ will make euen 6. This I will remember. But now, what is the second forme of Reduction, that you speake of, for these sorts of fractions?

Master. *Whensoeuer you haue any of these two sorts of numbers, that is to say, whole numbers without fractions, or whole numbers with fra-*

fractions, and you would turne them into the forme of a fraction, you must multiply the whole number by that denominator which you will haue to remaine still, and to the totall thereof adde the Numerator, which you haue already, and all that shall you set for the new Numerator, keeping still the former Denominator: as if you haue 6 3/4 which you would conuert into an improper fraction, you must multiply 6 by 4, wherof commeth 24 & therto adde the numerator which is 3 & so haue you 27 for the numerator, & 4 still for the denominator.

Reduction of whole numbers, either alone, or ioined with fractions into fractions.

Scholar. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Maister. Euen iust, and so backward, (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proose of the others worke.

Note.

Scholar. This I perceiue, but now if you would turne whole numbers without fractions into any fraction, I see not how that may be done, because there is no Denominator to make the multiplication by.

Maister. That was well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication must be made: for the proose whereof I set downe 7, which is a whole number. And if you will haue this number conuerted into any certaine fraction, will me to doe it.

Scholar.

Sho. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some fraction.

Scholar. No, I passe not for the sorte of the fraction.

Master. Then how can you thinke that you require mée to doe any thing certaine, when you leave mée to doe as I list? And seing you stand at that stay, whether thinke you that I must first intend in mind what fraction I will make of it, before I can doe it indéed?

Scholar. Else you should doe ignorantly.

Master. Then will I limit my selfe (seing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the Denomination of quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the Denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar. Indéed I perceiue this to be reasonable, for without much triall I vnderstand that $\frac{1}{4}$ of any thing doth make 7. And so then if I would turne 8 into 5 parts, it will make $\frac{5}{4}$, which is all one with 8: for 8 crownes turned into 5 parts (that is, into shillings) will make 40 shillings, that is $\frac{5}{4}$ of a crowne.

The fourth forme of Reduction.

Master. *Seeing you vnderstand now these three kinds of Reduction, I will declare vnto you the fourth kind, that is, when fractions be written*

written in greater termes then they need, how they may be brought to lesser termes.

Scholar. To write any thing in greater termes then néedeth, séemeth to be a fault, & so this rule séemeth to amend that fault.

Master. It were a fault to doe any thing without néed, which after must be redressed: but in this case it is not so; neither did I say absolutely (as you doe) that it néedeth not to expresse those fractions in so great termes, but that the fractions doe not need, I meane for their value to be understood: but yet it may be néedfull for the ease of those workes whereto they be applied, as for example: In the first kind of Reduction this was your owne example: $\frac{1}{12}$ and $\frac{4}{8}$, which when you would reduce you were faine to turne them first into one denomination, & so appeared they thus, $\frac{12}{96}$, and $\frac{48}{96}$, where the fractions (for their owne understanding) néded not to be turned out of smaller terms into greater, but yet the easinesse of working néeded it.

Scolar. Sir, I understand now, not onely the difference of this néed (for the fractiõs might better be understood as fractiõs seuerall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you meane by greater termes and lesser termes, wherof before I was in doubt: for I sée you call

Termes of Fracti ons.

call the Numerator and Denominator, the termes of the fraction.

Maſter. *I am glad you underſtand it ſo well. Now then when you would value any fractions (becauſe that they may beſt be done when the termes are ſmalleſt) you ſhall reduce them to the ſmalleſt that you can, which thing you may doe thus: Diuide the greateſt of any ſuch two termes by the leſſer, and if any thing remaine, by that remainer diuide the laſt diuiſor: and if any thing remaine now, by that diuide the firſt diuiſor (which was before the remainer of the laſt diuiſion) and ſo continue ſtill, till nothing doe remaine in the diuiſion: and then marke your laſt diuiſor, for it is the number that will eaſily reduce your fraction, if you diuide both the numerator and the denominator by the ſame number, and put for the numerator the quotient of his diuiſion, and for the denominator alſo his quotient, that riſeth by his diuiſion.*

Reduction of fractions into their ſmalleſt termes or leaſt value.

Scholar. I take for example $\frac{18}{96}$, and becauſe 96 is the greateſt number, I diuide it by 18, and the quotient is 5, and there reſteth 6: what ſhall I doe with this Quotient?

Maſter. Nothing in this worke, but now ſeeing there remaineth ſomewhat, by that remainer muſt you diuide the laſt Diuiſor.

Scholar. If I ſhall diuide 18, (which was the laſt diuiſor) by 6, that was the remainer, ſo is the quotient 3, and nothing reſteth.

Maſter.

Master. As for the Quotient I omit him yet: but because there doth remaine nothing, therefore is 6 (which was your last Diuisor) that number by which you may reduce the Fraction proponed.

Scholar. Then as you taught me, I must diuide the Numerator 18 by 6, and the quotient is 3, which I must put for the Numerator ouer a line, thus: $\frac{3}{\ }$. And then by the saide 6 must I diuide also the Denominator 96, and the Quotient will be 16, which I must take for the Denominator, and so is the Fraction $\frac{3}{16}$. And so mee thinketh this Rule doth proue the worke of the first Reduction.

Master. That is true if the first Reduction were made of fractions into their least termes, and else not, without some helpe, as the second number in that place will declare.

Scholar. The second number was $\frac{3}{4}$, which was turned into $\frac{48}{64}$ by that Rule. Now if I shall by this Rule reduce it againe into the least termes, I must diuide 96 by 64, and there remaineth 32, wherefore I must take that 32 for the Diuisor, to reduce the saide fraction. Then doe I diuide 64 by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96 by 32, and the quotient will be 3, and so I haue but $\frac{2}{3}$.

Master. Muse not at the matter, for you

haue done well enough: but you thinke you haue not the fraction that you looked for, that is, $\frac{2}{3}$, yet haue you one equall to it, as by the parts of a shilling you may proue.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$, and $\frac{2}{3}$ and $\frac{4}{6}$, be all three equal. And now I perceiue that because $\frac{4}{6}$ was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now vnderstand I this Rule well. But is there any other way to worke this Reduction?

Another way to worke this reduction.

Master. *Yes, but first note this, that if you find no such Diuisor to reduce the Fraction to you come to 1, because one doth make no Diuision: therefore that Fraction is already in his least termes, as by* $\frac{71}{100}$ *you may proue, and so* $\frac{85}{98}$, *and many other like.*

Note that mediate any number is to diuide by two.

But now for your better aide to find the due proportion in least termes, with more ease for a young learner, you shall mediate or take the halfe of the Numerator, and also of the Denominator as long as you may vpon a line, alwaies parting them with a right downe dash of your pen as you worke, which may easily be done, if the numbers be euen: as 2, 4, 6, 8, or 10, but if they bee odde (though it be but one of them) then must you abreuiate them by 3, 5, 7, or 9, &c.

And because examples doe most instruct,

I haue

I haue here set downe the manner of two or three, whose last number at the end of the line sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawne from it thus $\frac{288}{576}$. And because both the Numerator, & the Denominator end in euen numbers, I (as this may be abreuiated by 2, or 4, or 6, &c. Therefore on the other side of the right downe dash toward the right hand, I first take the halfe of the numerator: saying, the halfe of 2 is 1, the halfe of 8 is 4, and again, the halfe of 8 is 4: which 144 is now a new numerator: and therefore I part it with a right downe dash, as before:

Then doe I also take the halfe of 576, in saying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a new Denominator.

Then beginning againe: saying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2, vntill you come to the last worke, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$, which is equal to $\frac{288}{576}$.

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2
	28	14	7	1			
	112	56	28	4			

Abbreuiated first by 5, then by 293.

1465	293	1
4395	879	3

Scholar. Sir I thanke you much, this is very easie and good for a young learner.

Master. So it is, but yet notwithstanding if you can without that diuision by memory espie the greatest number that may diuide exactly both termes of your fraction proponed, then need you not to vse that diuision, as in this fraction $\frac{1465}{4395}$, I see that 293 is the greatest number that I can diuide them both: and therfore without any worke by memory onely, I turne that into $\frac{1}{3}$, but this ability in knowledge is got by exercise.

Yet one other way of easie reduction in this kind there is when your fraction hath any ciphers in the first places of both termes, then may you by casting away the ciphers, make a briefe reduction, as thus $\frac{1000}{4000}$ here take away the cyphers, and it wil be $\frac{1}{4}$, which is the same in value with $\frac{1000}{4000}$.

Sch. And so if I haue $\frac{1000}{7000}$, it will be $\frac{1}{7}$.

Master. You are deceiued, for you take away more cyphers from the Numerator then you doe take from the Denominator, which you may not doe.

Scholar. I confesse my fault, which came of

of to much hast, I was more gladder of the rule then wise in using it: but now I understand it I trust.

Master. Then may I goe in hand with the fift or last kind of Reduction, which teacheth how to turne any fraction proponed into any other Denomination that you list, or into any parts of common coynes, weights, or measures, or such like.

The fift kinde of Reduction

For declaration whereof, first you shal marke whither your fraction be a simple Fraction, either else a Fraction of sundry parts, I meane of more termes then two. And if your Fraction be a Fraction of Fractions, or otherwise compound, you must reduce it to one simple fraction. And then make well the Denomination of that other Fraction into which you would turne this: For by that Denominator you must multiply the Numerator of your first Fraction, and the totall product thereof shall you diuide by the Denominator of your first Fraction, and that quotient shall be the Numerator of the Denominator proponed: as for example, I haue this Fraction $\frac{3}{5}$, *which I would turne into ten parts: therefore I multiply this* 10 *by* 3, *that is the Numerator of my Fraction, and there ariseth* 30, *which I diuide by* 5, *and the quotient is* 6, *which must be the Numerator to* 10 *and so* $\frac{3}{5}$ *will be* $\frac{6}{10}$.

Scholar. This is easie enough to doe.

Master. Then shall you see another ex-

ample of the same fraction that is not so [illegible] fie: as if I would turne $\frac{3}{5}$ into 8 parts, prove you that worke.

Scholar. I must multiply 8 by 3, & then amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And sée you nothing doubtful in this worke?

Scholar. I sée that when 24 was diuided by 5, there remained 4, which I did not passe of, because ye spake nothing of any remainer, but onely of the quotient.

Master. By likelihood you remember what I said to you in diuision of whole numbers, that you should not passe of the remainer there, but onely note it as a sum that could not be diuided without knowledge of fractions. Wherefore now marke this, that in all diuisions of whole numbers, when there is any remainer, you shall set it ouer a line as a Numerator, and set the Diuisor for the Denominator, and that fraction doth make the Diuision compleat, & is part of the quotient: as if I would diuide 48 by 5, the quotient will be 9 $\frac{3}{5}$: so in your former worke when 24 was diuided by 5, the quotient should be 4 $\frac{4}{5}$, and so the new fraction should be thus: $\frac{4}{8}$ and $\frac{4}{5}$, of 8, that is, $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may proue by example of some coyne.

Scholar. Then I take a crowne, whose $\frac{1}{5}$ is

is 3 s. Now if I would proue whether the 3 s be ⅞ and ⅞ of ⅛, I shall haue a cumbrous worke to doe.

Master. Indeed for whole pence, your example is a little troublesome: yet turning ỳ crown into halfe pence, it is easie enough.

Scholar. That will I trie.

¶ First I see that ⅞ of a crowne is 3 s, which is 36 d, 0372 halfe pence. Now if I can find that this fraction ⅞ and ⅞ of ⅛ be equall vnto 3 s, then am I fully answered.

Because I cannot take ⅞ of a crowne, I turne the crowne into halfe pence, as you willed me, which makes 120, which I diuide by 8, my quotient is 15, which taken foure times, make 60 d. Now resteth me to haue ⅞ of the ⅛ part of a crowne, whereof ⅛ part is 15 d the 15 being parted in 5 parts, the quotient is 3, which taken 4 times maketh 12 d. which with my 60 before, amounteth to 72, which are then equal to ⅞ my desire.

Master. I commend you for your diligence, you might haue wrought it thus either ⅞ being abreuiated as before I taught, is ⅛. Now, halfe of a crowne is 2 s, 6 d. Then ⅞ of ⅛ is a fraction of fractions, which if you doe reduce into one entire fraction, as before you haue learned, in saying, fiue times 8 is 40, for a new Denominator, and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, and abreuiated also make $\frac{1}{10}$. Now the

tenth part of a crowne is 6 s, which put to 2 s, 6 s, make also 3 s, your desire.

But now one example more for this rule, and then we shall end it. If I haue $\frac{7}{15}$ of a Soueraigne (accounting the Soueraigne 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholar. I must multiply 7 by 20, & that maketh 140, which I shall diuide by 15, & the quotient will be $9\frac{5}{15}$, or else in lesser termes $9\frac{1}{3}$.

Master. That is 9 s, & one third part of a shilling, that is 4 d, as by the same Rule you may proue. And this for this time shall suffice for Reduction, saue that I must now repeat a little touching Multiplication and Diuision, and so goe forward.

Multiplication.

Multiplication of fractions.

IN Multiplication it happeneth sometime, that there bee whole numbers to be multiplied with Fractions: and may be in two sorts: for either the whole number is seuerall from the Fraction, and is the Multiplier, or else the whole number is ioined with one, or both of the Fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no Reduction, but onely multiply the Numerator of the Fraction by that whole number, and the totall thereof set for the new Numerator.

Reduction of whole numbers into fractions.

Scholar.

Scholar. I understand you thus. If I have $\frac{6}{13}$ to be multiplied by 16, then must I multiply that 16 with 6, which is the Numerator whereof commeth 96, & that must I set for the new Numerator, keeping still 13 for the Denominator, and so the fraction will be $\frac{96}{13}$, that is $4\frac{4}{13}$.

Master. And in this sort of work you may abridge the labour, thus. If it happen the Denominator to be such a number, as may cunnly be divided by the said whole number proposed, then divide it thereby, and set the quotient of that division for the former Denominator: but reserve still the Numerator, and so is the Multiplication ended.

Scholar. Then I take this example $\frac{7}{20}$ to be multiplied by 5: and because 5 will justly divide 20, therefore I take the quotient of that division which is 4, and set in stead of 20, and so the fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$ that would have followed of the other sort of work.

Scholar. I perceive it very well.

Master. *Now then for the other sort, where the number is mixt take this way: first to reduce the said whole number, and fraction into one fraction improper (as I shewed you in* Reduction) *and then multiply them together as if they were proper fractions.* How to multiply mixt numbers.

Scholar. $13\frac{3}{5}$ being set to be multiplied by $\frac{1}{4}$ first I must reduce the mixt number,

as

as appeareth in the margent, by multiplying 13 by 5, and that maketh 65, whereto I must adde the Numerator 3, and so the fraction will be $\frac{68}{5}$, which two fractions now I shal multiply after the accustomed forme, and it will be $\frac{340}{5}$ or $13\frac{3}{5}$.

Master. You haue done well: and so may you see, that although most part of the formes of Multiplication may be wrought without Reduction, yet some cannot, as namely mixed numbers.

And yet one note more will I tell you of Multiplication, before wee leaue it: That is, whensoeuer you would multiply any fraction by

Duplation. 2, *which commonly is called* Duplation, *you may do it not onely by doubling the Numerator, but also by parting the Denominator into halfe if it be euen.*

Scholar. Then if I would double $\frac{1}{12}$, I may chuse whether I will make it, $\frac{1}{6}$, or else $\frac{2}{12}$. And indeede I see that is all one, but that the diuiding of the Denominator seemeth the better way to make smaller termes of the fraction, and so they shall neede the lesse reduction.

Master. It is so: & now I shall not need to tell you that Multiplication is proued by Diuision, and Diuision likewise by Multiplication: but the like workes that I shewed you in Multiplication, will I shew you in Diuision also.

Diuision,

Diuision.

When any whole number shall be diuided by a fraction, you must multiply the said whole nnmber with the Denominator of the fraction, and set the totall thereof for the new Numerator, *and for the* Denominator *set the* Numerator *of the fraction.*

Diuision of fractions.

To diuide a whole nnmber by a fraction

So $20 \div \frac{1}{4}$ by $\frac{1}{4}$ 3

Scholar. Then 20 diuided by $\frac{1}{4}$ will make $\frac{80}{1}$, as heere appeareth $\frac{80}{1}$.

Master. *Euen so: but if you would diuide the fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for the Denominator, without changing the Numerator.*

To diuide the fraction by the whole number.

Scholar. Then to diuide $\frac{20}{31}$ by 4, it will be $\frac{4}{93}$, as heere appeareth $\frac{20}{31}$ by $\frac{1}{4}$ in this example. $\frac{20}{93}$.

Master. You say well. And by the same example you giue me occasion to remember another briefe way to do the same: for if you had diuided the said numerator by 4, and set the quotient for the numerator, keeping still the olde denominator, it would haue beene not only as well done, but also in a fraction of lesser termes.

Another briefe way.

Scholar. I guesse it to be euen so, by a like worke

worke that you taught me in multiplication: And for proofe thereof $\frac{20}{23}$ being the diuidend, and 4 the diuisor, I diuide the Numerator 20 by 4, & the quotient is 5, which I set for 20 ouer 23, thus, $\frac{5}{23}$: And I see that it is all one with $\frac{20}{23}$ as by diuiding or abbreuiating both these termes by 4, & so reducing them to their least denomination, I may easily proue: as appeareth by this example $\frac{20}{23}|\frac{4}{1}$.

Master. You conceiue it well. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction, and then worke as you haue learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you goe forward to some other thing.

Master. Then take this note yet for Diuision, if the Denominators be like, thē diuide the Numerators as if it were in whole numbers, & the quotient whether it be fraction, whole number, or mixt, is a good quotient for that diuision. And generally if one of the Numerators may iustly diuide the other, by that quotient multiply the Denominator of the lesser Numerator, & set it that both amount in the terme of the same denominator, & then for a numerator to it, set the Denominator of the other fraction.

Scholar. Then if I would diuide $\frac{1}{4}$ by $\frac{12}{19}$

I

I see that 3 will diuide 12, and the Quotient will be 4, by which I must multiply the other 4, that is the denominator vnder 3, & then it is 16, which I set for the denominator 4, & ouer it instead of 3, I must set 17 the other Denominator, & so is it thus, $\frac{17}{16}$.

Master. And so is $\frac{17}{16}$ instead of $\frac{51}{48}$, which would haue risen by the common worke as here appeareth. $\frac{3}{4}$ by $\frac{12}{17}$ $\frac{51}{48}$

And now for mediation (which is to diuide by 2) marke this, if the Numerator be euen, set the halfe of it in his place without the Diuisor, and so haue you done: and if the Numerator be not euen, then double the Denominator. Mediation.

Scholar. That is, if I would mediate $\frac{6}{11}$ I may make the quotient $\frac{3}{11}$, & if I would meditate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Master. Now trust I that you haue sufficient knowledge in Reduction, Multiplication, and Diuision: and therefore will I goe in hand with Addition & Subtractiõ, which now will appeare easie enough:

Addition.

WHensoeuer you haue any Fractions to be added, you must consider whether they be of one denomination or not: and if they be of one denomination, then adde the Numerators together, and set that that amounteth for the Numerator, ouer the com- Addition of fractions of one denomination.

mon

mon Denominator, and so haue you done: The reason is because that such differ little in Addition *or* Subtraction *from the worke of vulgar denominations, where the denominators be of odde numbers: as* 3 *d, and* 5 *d, make* 8 *d, where the* Denomination *is not altred. But if the fractions be not of one Denomination, or any of them be mixt of whole numbers and fractions, then must you first reduce them to one* Denomination, *and after adde them. And if they be many, then adde first two of them, and so the summe that doth amount of the Addition, and the third, & then the fourth &c. if you haue so many.*

To adde fractions of diuers denominations.

Scholar. This seemeth easie enough, now that I haue already learned to multiply & to reduce, without which two I could neuer haue wrought this. And therefore now I see good reason, why you did place Multiplication, and Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your vnderstanding of it by an example.

Scholar. Then would I adde first $\frac{1}{2}$ with $\frac{3}{4}$, and because the Denominators are like (and so needeth no reduction) I adde 7 to 5 which maketh 12, and then is my summe $\frac{12}{4}$, that is in smaller numbers, being abbreuiated, 3.

And if I haue many numbers to be added as here $\frac{1}{2}, \frac{3}{4}, \frac{2}{10}$, first I must reduce them (because

cause they haue diuers denominators) into one denomination, & then they will be thus $\frac{150}{400}$, $\frac{320}{400}$, $\frac{360}{400}$, or in lesser termes $\frac{15}{40}$, $\frac{32}{40}$, $\frac{36}{40}$, which by Addition doe make $\frac{83}{40}$ that is $2\frac{3}{40}$.

Master. Now may we go to Subtraction.

Subtraction.

SVbtraction hath the same precepts that Addition had: for if the Denominators be like then must you subtract the one Numerator from the other, and the rest is to be set ouer the common Denominator, and so your Subtraction is ended: but and if you haue many fractions to be subtracted out of many, then must you reduce them to one Denomination, and into two seuerall fractions, that is all that must be subtracted into one fraction, and the residue into another fraction, and then worke as I said before. Subtraction of fractions.

Scholar. For the first example I take $\frac{15}{12}$ to be subtracted out of $\frac{17}{12}$ & ẏ rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For another example, I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I subtract 24 out of 28, and there resteth 4, which I set ouer the common Denominator for a Remayner, thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Now

Now for the third example, I take $\frac{1}{3}$ and $\frac{2}{5}$ to be subtracted from $\frac{7}{8}$ & $\frac{2}{15}$: And because their Denominators be diuers, I doe reduce them into one denomination thus $\frac{1600}{1920}$ $\frac{1680}{1920}$ $\frac{1728}{1920}$.

Then doe I adde the two first, and they make $\frac{3040}{1920}$. Also I adde the two last, and they yeld $\frac{3408}{1920}$. Then doe I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$, that is in smaller termes $\frac{23}{120}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Master. Proue one example more of two fractions of diuers denominations.

Scholar. I take these two fractions, $\frac{7}{8}$ to be subtracted from $\frac{2}{24}$, which being reduced will stand thus $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I cannot.

Master. Then may you perceiue that you misstake the fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiply, or diuide, the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his termes lesser then $\frac{2}{24}$, yet is $\frac{2}{24}$ the lesser fraction: for generally if you multiply the Numerators and the Denominators of two fractions crosse waies, that Fraction is the greatest of whose Numerator commeth the greatest summe, as in this example: 7 multiplied by 24, maketh 168; and 9 being multiplied by 8, yeldeth but 72: there-

The greatest of two fractions.

therefore is the first fraction 7 the greatest of these two, so can you not subtract it out of a lesser fraction.

But if you should subtract a fraction out of a whole number, what would you doe?

Scholar. Marry I would reduce the whole number into a fraction of the same Denomination that my fraction is, and then worke by Subtraction.

Master. So may you doe, but it is easier much, if your fraction be a proper fraction, that is to say, lesse then an vnite, to take an vnite from the whole number, and then turne it into an improper fraction, and so worke your Subtraction. As if I would subtract $3\frac{1}{7}$ from 4, I may take 1 from 4, & turne it into $\frac{7}{7}$, frō which if I abate $3\frac{1}{7}$ there will remaine $\frac{1}{7}$. And if the first Fraction be an improper Fraction, then may I take so many vnites from the whole number, that they may make an improper Fraction greater then that first, and then worke by by Subtraction. As if there be proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more then 3, and not so much as 4, I must take 4 from 6, and turne them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$: so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$ which is $\frac{10}{3}$, from 6, whole: then set 1 vnder 6, as thus $\frac{6}{1}$: And then to reduce those two fractions into one Denomination, as here appeareth $\frac{10}{3}$ from $\frac{6}{1}$.

Then $\frac{12}{7}$ from $\frac{12}{7}$ resteth $\frac{8}{7}$ which maketh $2\frac{1}{7}$ your desire. $\frac{12}{7} \times \frac{4}{7}$

And thus will I make an end of the worke of cómon Fractions for this time, not doubting, but you can apply them both vnto the Rule of Progression, and also vnto the Golden Rule, without any other teaching then you haue learned before, which might séeme tedious to repeat, sauing that in some speciall diuersities, which be peculiar to fractions, I cannot ouerpasse, but instruct you somewhat by the way.

The Golden Rule.

The rule of proportion.

THerefore as touching the Golden Rule, for the placing of the 3 numbers proponed in the question whereby to find the third, and for the forme of their worke, with other like notes, I refer you to that which you haue already learned.

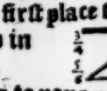
But this easie forme of working by fractions shall you note, that if your three numbers be fractions, for an apt worke and certaine, multiply the Numerator of the first number in the question, by the Denominator of the second. And all that againe multiply by the Denominator of the third number, and the totall thereof shall you keepe for to be the Diuisor. Then multiply the Denominator of the first number by the Numerator of the second, and the whole

thereof

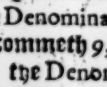
thereof by the Numerator of the third, and the totall thereof shall be your Diuidend.

Now diuide this Diuidend by the diuisor which you found out before, and that number shall be the fourth number of the question, which you seek for, as in this example. *If ¾ of a yard of velvet cost ⅞ of a Sovereigne, esteemed at 20 s., what shall ⅝ cost?* A question of velvet.

Scholar. If it please you to let me make the answere, I would first place these three numbers, as I learned in whole numbers, thus.



And then according to your new rule, I must multiply 3, being Numerator in the first number, by 3 the Denominator of the second, and thereof commeth 9, which I multiply againe by 6, the Denominator of the third number, and so haue I 54, which I keepe for the Diuisor: Then multiply 4, the Denominator of the first, by 2 the Numerator of the second, and there ariseth 8, which againe I multiply by 5, the Numerator of the third, and it maketh 40: Then must I diuide 40 by 54, and it will be $\frac{40}{54}$, that is, $\frac{20}{27}$ in lesser termes, and then the figure will stand thus.



But what that is in money I cannot tell, except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may resolue it when you list, but it were disorderly

done here to mingle diuers workes together, where we do not seeke the value of the thing in common mony, but in an apt number, which ye haue well done. And therefore will I yet shew you another like way of easmesse in worke, how you may change your three fractions into three whole numbers, by which you shall worke as if the question were propouned in whole numbers. The first number you shall find as I taught you: now to find the Diuisor of the second number, take the Numerator for the second fraction: and for the third number take that that ariseth of Multiplication of the Denominator of the first, by the Numerator of the third, and then worke your question.

A question of Siluer.

Scholar. *For example hereof, I put this question. If* $\frac{11}{12}$ *of* 1 *l' waight of siluer be worth* $\frac{12}{4}$ *of a Soueraigne, what is* $\frac{1}{2}$ *of* 1 *l' waight worth? For the answere, first I place the fractions in order thus.* $\frac{11}{12}$ Z $\frac{12}{4}$ $\frac{1}{2}$

Then to turne these fractions into whole numbers, I multiply 11, which is the Numerator of the first by 4 (the Denominator of the second) and there commeth 44, which I multiply by two the Denominator of the third, and so amounteth 88, which I set for the Diuisor in the first place. Then in the second place I set 12, which is the Numerator of the second fraction, and in the third place I set the summe that amounteth of 12,

being

being the Denominator in the first number, multiplied by one, being Numerator in the third number, add to the figure 88 Z 12 it will stand as here you see. 12

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be $\frac{16}{88}$, or in lesser termes, $1\frac{7}{11}$, and then the figure will stand thus: $\frac{11}{12}$ Z $\frac{11}{4}$ $\frac{1}{3}$ $1\frac{7}{11}$

Master. These two formes now you understand well enough, and as for any other at this time I will not repeat, onely this shall you mark for the proofe of this rule whether your worke be well wrought or no: Multiply the first number by the fourth, and note what amounteth: Then multiply the second by the third, and marke what amounteth also. Now if those two numbers so amounting be equal, then is your worke wel done, else you have erred. And this shall suffice for the former Rule. But in the backer Rule, this shall you note for ease of worke, that you multiply the numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the third, and that amounteth thereof, shall be the Diuidend. Then multiply the Denominator of the first by the Denominator of the second, and that whole by the Numerator of the third, and that that riseth thereof shall bee the Diuisor. Example of this:

The proofe of the golden Rule.

The backer Rule.

A question of Loane. *I did lend my friend $\frac{3}{4}$ of a Porteguise seauen moneths, vpon promise that he should doe as much for me againe, and when I should borrow of him, he could lend me but $\frac{5}{12}$ of a Porteguise: now I demand how long time must I keepe his mony in iust recompence of my loane, accounting 13 moneths in the yeere?*

Scholar. The first number must be the first mony borrowed, that is $\frac{3}{4}$ of the porteguise: the second number the 7 moneths, that is $\frac{7}{13}$ of a yeere: and the third number the money that was lent in recompence, that is, $\frac{5}{12}$ of a Porteguise: then I set the numbers thus:

$\frac{3}{4}$ — $\frac{7}{13}$

$\frac{5}{12}$

Then (as you taught me) I multiply 3 (being Numerator in the first number) by 7, the Numerator of the second number, and it maketh 21, which I multiply by 12, the Denominator of the third, and so haue I 252 for the diuidend: then I multiply 4 the Denominator of the first, by 13 the Denominator of the second, and it yeldeth 52, which I multiply againe by 5, the Numerator of the third, and it will make 260, that is the Diuisor. Then must I diuide 252 by 260, so it will be in the small Fraction $\frac{63}{65}$ of yeere.

Master. And thus doe you see some ease in working better then to multiply & diuide tediously so many fractions.

Statute of Assise of Bread and Ale. *Another question yet will I propose, to the intent you may see thereby the reason of the Statute of Assise of Bread & Ale, which in all*

Statute

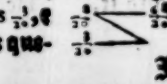
Statute Books, in Latine, French, and English is much corrupted for want of knowledge in this Art, for the right understanding whereof I propone this question.

When the price of a quarter of wheat is 2 ℔, the farthing white loafe shall waigh 68 ℔, then I demand, what shall such a loafe waigh when quarter of wheat is sold for 3 ℔? Question of Bread

Scholar. This question must be wrought as it is proponed in whole numbers, and not in Fractions.

Master. You saime to say reasonably, howbeit in the Statute of Assise, the rate is made by the proportion of parts in a pound waight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often, as all other monies doe: but this Statute being well understand, is a continuall rule for ever, as I will anon declare by a new Table of Assise, converting the shillings into ounces & parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound waight, and so by a penny $\frac{1}{240}$ of an ounce: wherefore although you might worke this question proponed by whole numbers well enough, for that time when the Statute was made, yet to apply it to our time, & to make it serve for all times generally, it is best to worke it by fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings $\frac{68}{20}$, and so for three shillings $\frac{3}{20}$, & then will the figure of the question stand thus. Note what a shilling is.



In

In which question because all the Denominators be like, you shall worke only with the Numerators.

Scholar. Then shall I multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will be 45 $\frac{1}{3}$: but how shall I make a fraction of that to stand with the other?

Master. Haue you so soone forgotten what was taught you so lately? this is his forme.

Scholar. I remember it now, & then it signifieth 45 twenty parts, and the third deale of one twenty part. $\frac{45^{1}/_{3}}{20}$

Master. So is it that maketh in shillings 45 s. 4 d. whereby you may note one great errour in the Statute Bookes, which haue constantly 48 s in that Assise. And by this Rule if you examine the Statute, you shall find many summes false. Wherefore for the true vnderstanding of that Statute, and such like as I haue made mention of it, & somewhat recognised it, so doe I wish that all Gentlemen & other Students of the Lawes would not neglect this Art of Arithmetick, as vnneedfull to their studies. Wherefore to encourage them thereto, & to gratifie both them & all other in generall, I will exhibite a Table of that part of the Statute in two columes, & in a third colume I will adde the correction of those errors which haue crept into it. Here followeth the Table.

The

The price of a quarter of Wheate.		The waight of a farthing white loafe by the statute bookes.			The correctiõ by inst Assise.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 1/5	2	14	4 4/5
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/9
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/3
5	0	1	8	2 1/5	1	7	2 2/5
5	6	1	4	8 1/4	1	4	8 8/11
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 4/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	3/4	0	14	3 15/19
10	0	0	13	7 1/3	0	13	1/5
10	6	0	12	11 1/4	0	12	11 3/7
11	0	0	12	4 1/4	0	12	4 4/11
11	6	0	11	10	0	11	9 21/23
12	0	0	11	4	0	11	4

In the common bookes there is no further rate of Assise made, then into 12 s. the quarter of wheat, but in an ancient copy of 200 yeares old (which I haue) there is added the rate of Assise vnto 20 s. the quarter, but yet was that Assise also either wrong cast at the first penning, or else corrupt since that time, for lacke of iust knowledge in the Rule of Proportion, which I wil adde here also to gratifie such as be desirous to vnderstand truth exactly.

The price of a quarter of wheat.		The waight of a farthing white loafe by the Statute bookes.			The correction by iust Assise.		
s.	d.	lb.	s.	d.	l.	s.	d.
12	6	0	11		0	10	$10\frac{14}{25}$
13	0	0	11	$0\frac{1}{2}$	0	10	$5\frac{7}{13}$
13	6	0	10	$1\frac{1}{2}$	0	10	$\frac{8}{9}$
14	0	0	9	7	0	9	$8\frac{4}{7}$
14	6	0	9	$2\frac{1}{2}$	0	9	$4\frac{16}{29}$
15	0	0	9	$1\frac{1}{2}$	0	9	$0\frac{4}{7}$
15	6	0	9	$1\frac{1}{4}$	0	8	$9\frac{9}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	$2\frac{10}{33}$
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	$10\frac{9}{35}$
18	0	0	7	6	0	7	$7\frac{2}{3}$
18	6	0	7	3	0	7	$4\frac{8}{37}$
19	0	0	7	2	0	7	$1\frac{17}{19}$
19	6	0	6	10	0	6	$11\frac{9}{13}$
20	0	0	6	6	0	6	$9\frac{1}{2}$

These two Tables I haue set seuerall, because no man should thinke that I would either adde or take away from any Law those parts which might of right seeme either superfluous, either diminute: but yet I may not bee so curious as to neglect manifest errours, which is not only my part, but euery good subiects duety with sobriety to correct. And for auoiding of offence, I haue rather done it in this priuate Booke, then in any Booke of the Statutes it selfe, trusting that all Men will take it in good part.

Scholar. I would wish so, but I dare not hope so, sith neuer good man that would reforme errour, could escape the venimous tongues of enuious detracters, which because they either cannot, or list not to doe any good themselues, doe delight to barke at the doings of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of iust answers, but I will not seeme so hasty to make the answers before I hear their obiections, but as I trust that men are of a better nature, & more gratefull now then some haue beene in times past, as I haue done in the Statute of Assise for bread in rate of, so will I set forth the like Table in pounds and ounces and the parts thereof, that it may be easily applied to all times: but I meane not by

by this to alter any word of the ſtatute (being ſo good an ordinance, and of ſo great continuance) but only to make it as a kinde of expoſition and declaration of the ſaid Statute, truſting that thereby the Statute may be better underſtood, and conſequently better put in execution. And here you ſhall note that I haue accounted the ſhillings after the rate of 60ſ, to the pound weight, becauſe I eſteeme it the moſt apt for our time. Wherefore in the firſt columme you find the price of Wheat directly againſt it, in the ſecond columme, you may find the weight of a farthing white Loafe, in this our time: and if you double the number (as I haue done in the third columme) then haue you the weight of the halfe penny white Loafe, and ſo in the fourth columme is ſet the weight of a penny white Loafe. It needeth not to tell that, that the ſight doth teſtifie, how that euery columme is parted into three ſmaller pillars, whereof the firſt columnhath theſe three titles, pounds, ounces, and penny weights. And as in the firſt columme 12 pence make a ſhilling, and 20 ſhillings make a pound, ſo in the other three colums 20 pence weight maketh an ounce, and 12 ounces doe make a pound.

A pound weight.

Scholar.

ENTLE READER, t
or 60 shillings to the pound,
ter, till he come to 40. shillin
hard for many to conceiue o
first thing therfore that is so
solde for two shillings, then
fore, for a generall rule, to fi
thus to be wrought. Comm
68. shillings, what weight ought the Farthing-
Table is noted. Then for the second worke, say
which 544. ounces being multiplyed by 3. for 3
12. peny weight, which is halfe an ounce, and 2.
of the Halfe-peny white-loafe is thus. Multiply
ces, and 4 peny weight, as M. Record his Table
shall finde 27. pounds, ½ ounce, and 3. peny we
Halfe-peny white-Loafe, for the weight of the
Thus much for the vnderstanding of this Ta
of Coynage, and the Standard thereof, as appea
altered from the 14. yeare of King Edward the t
time, when this Assise of Bread and Drinke wa
and 20. of these pence made an ounce, & 12. ounc
and proportion: And now finding 60. pence is
meaning not therby to alter any word of the St
ter vnderstood, and so consequently better put
of Knowledge, (according to these prices of a Q
ney, and also reduced the Money into knowne
that a Farthing, Halfe-peny, and Peny-white-

...eth 60. pence to the ounce, and 3. pound
...Table, beginning at 3. shillings the quar-
...mple to continue the worke,) hath beene
...ter understanding of him therein. The
...s this. When the Quarter of wheate is
...by a penny $\frac{1}{30}$ of an ounce. Now there-
...to 40. shillings, 6. pence the quarter, is
...allow the Farthing-white-loafe to weigh
...nce, as before in the correction of the first
...lie and diuide, you shall find 544. ounces,
$\frac{12}{30}$. or rather $\frac{2}{5}$. of an ounce, equall vnto
...continue this Table, to know the weight
...$\frac{1}{12}$, which is equall to 13 pounds, 7. oun-
...by 20. and after by 12. as afore, and you
...fe-peny white loafe, and so doubling the

...tricall proportions, touching the Statute
...nes that the Standard of Coyne is much
...ppeareth that in King Edward the thirds
...rie, & taken out of the middle of the eare,
...ighed 12. ounces, toke Bread his weight
...owns this Table in this priuate Booke;
...ay, that thereby the Statute may be bet-
...fore also, to gratifie such as are desirous
...nto their iust proportion of Sterling mo-
...e Tables, haue I noted the iust weight,

...this Table betweene 340. and 341.

Scholar, Sir, I doe thanke you most hartily for this, not onely in mine owne name, and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in bread, which long time hath bene abused. And if you know any like things more, wherein you would vouchsafe to declare the errors, and set forth the truth, you cannot but obtaine great thanks of all good harted men that loue the Common wealth.

Master. I haue sundry things to declare, but I haue reserued them for a priuate book by it selfe, yet notwithstanding because the statute of the rate of measuring of ground is so common that it toucheth all men, and yet no more common then needfull, but so much corrupt, that is too farre out of all good rate, not only in the English Bookes of statutes, commonly Printed, but also in the Latine, Bookes, and in the French also (for I haue read of each sort, and conferred them diligently) I will giue you a Table for the restitution of those errors, as may suffice for this present time. And first I will propose one question to you touching the vse of that statute, whereby you may perceiue the order how to examine the whole statute, and euery parcell thereof, and the question is this.

When the Acres of ground doth containe foure

A question of measuring of ground.

foure Perches in breadth, then must it contain 40 Perches in length: Then doe I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches, but before you shall answere to this question, I will declare vnto you another Statute, which is the ground of the former Statute. And the Statute is this.

Statute measure.

It is ordained that three Barly-cornes dry, and round, shall make vp the measure of an inch: 12 inches shal make a foote, & 3 foote shall make a yard (the common English bookes haue an Elne) fiue yards and a halfe shall make a Perch, & 40 Perches in length, & 4 in bredth, shal make an Acre. This is that Statute, wherby you may perceiue, that the intent of the Statute is, that one Acre should containe 160 square Perches. Now let me heare you answere to the question.

An Acre.

Scholar. As I perceiue by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, so could I make those numbers all in fractions, and so worke the question, but seing I may doe it also in whole numbers, I take that forme for the most easie, therefore thus I set the question in forme. Then doe I multiply 40 by 4, and it maketh 160, which I diuide by 13 and the quotient is $12\frac{4}{13}$.

4 — 40
13

Master. Now turne that $\frac{4}{13}$ into the com-

mon

mon parts of a Perch, as they be named in the former Statute: howbeit it shal be best to take one of the least parts in Denomination for auoiding of much labour, as Feete, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. Then to returne $\frac{4}{13}$ into Feet, I multiply $16\frac{1}{2}$ by 4, and it maketh 66, which I must diuide by 13, & the quotient is $5\frac{1}{13}$.

Master. So I find that if the Acre hold in breadth 13 Perches, it shall containe in length 12 Perches, 5 foote, and $\frac{1}{13}$ of a Foot, which is not fully an inch, for the inch is $\frac{1}{12}$ of a Foot. But here all the Statute Bookes in Latine and English (that I haue seene) do note it to be 13 Perches, 5 Foot, and one inch, which maketh about 13 Perches, too many in the Acre: so that I would haue thought the errour to haue crept into the Printed Books by the great negligence that Printers in our time doe vse, saue that in written copies of great antiquity, I doe find the same. Yet haue I one French copy which hath 12 Perches $\frac{1}{2}$ and one foote, and that misseth very little of the truth. Note the errour.

Scholar. Then I see it is true that I haue often heard say, that the truest Copies of the Statutes be the French Copies.

Mast. That is often true: but not generally, as I haue by conference tried diuersly: but in this Statute the French Book is most corrupt: in all other places lightly.

But now to performe my promise, I will set

set forth the Table for measuring of an Acre of ground only by such parts as the Statute doth mention, because at this time I doe of purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table following I haue not done as in the other Statute before compared by restitution with the Faults crept into the Statute, but onely haue written that true measure, which the equity of the Statute doth pretend. For it were vile to indge of so noble Princes and worthy Counsellors, as haue authorised and set forth this Statute, that they would make one Acre in any forme greater then another, but euery one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needfull Arithmeticke is to the Students of Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it none obscurity, that I should need to declare.

The

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	7	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an inch.
28	5	11	9	1/7
29	5	8	6	13/29
30	5	5	6	0
31	5	2	7	19/31
32	5	0	0	0
33	4	14	0	0
34	4	11	7	13/17
35	4	9	5	1/7
36	4	7	4	0
37	4	5	4	8/37
38	4	3	5	13/19
39	4	1	8	4/13
40	4	0	0	0
41	3	14	10	28/41
42	3	13	4	2/7
43	3	11	10	33/43
44	3	10	6	0
45	3	9	2	0

Scholar,

Scholar. Indeed Sir. I vnderstand the Table (as I think) by those other which you set forth before. For in the first columne is set the Perches of the breadth of any Acre, and then in the two columnes following appeareth how many Perches, and how many foote that same Acre must haue for his length.

Master. You take it well: howbeit to speak exactly of breadth and length, and the first colum doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limite his length, and the shorter side doth betoken the breadth; yet it is no great abuse in such Tables, where a man cannot well change the title, to let the name remaine, although the proportions of the numbers do change: for still by the first columne is expressed the measure of the one side, & by the two other Pillars in one columne, is set forth the measure of the other side. And this shall be sufficient now for the vse of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certain other rules, which for their seueral names may seem diuers rules, and distinct from this, but indeed they are but

The Rule of Fellowship.

branches of it: yet because they haue seuerall workings in appearance, but also pleasant in vse, I will giue you a taste of each of them. As for the Rule of Fellowship, both single and double, with time and without time, I shall need to say little more then I haue already said in teaching the workes of the whole numbers, yet an example or two will we haue to refresh the remembrance of the same, and to declare certaine proper vses and applications of it, as this for one.

A question of inequall society.

Foure men got a booty or prise in time of warre the prize is in value of mony 8190 pound, and because the men bee not of like degree, therefore their shares may not be equall, but the chiefest person will haue of the booty the third part, and the tenth part ouer: the second wil haue a quarter, and the tenth part ouer, the third will haue the sixt part: and so there is left for the fourth man a very smal portion, but such is his lot (whether he be pleased or wroth) he must be content with one 20 part of the pray: Now I demand of you, what shall euery man haue to his share?

Scholar. You must be faine to answere to your owne question, else it is not like to be answered at this time.

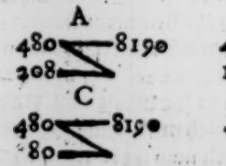
Master. The forme to vnderstand the solution of this question, and all such like is this: Reduce all the Denominators into one number by Multiplication, except that any

any of them be parts of some other of them for all such parts you may ouerpasse, and take for them all those nũbers, whose parts they be: As in this example the shares bee these $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{60}$, if I multiply all the denominators together, beginning with 3, & so goe on vnto 20, it will make 144000: but considering that 3 is a part of 6, I shal omit that 3, & likewise 10, which is a part of 20, I may ouerpasse also, & then is there but 3 denominators to multiply, that is, 4, 6, & 20 which make 480. which summe I take for my worke, because all the denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ & $\frac{1}{10}$, & $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one sum for the first mans share, & it maketh 208. Then for the second mans share, I take $\frac{1}{2}$, which is 120, & $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must haue $\frac{1}{5}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole sum: so that if the whole pray had bene but 480 l, then were the questiõ answered: but because the sum was of greater value, by this meanes now shal I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number that I found by multiplying the Denominators, and in the second place the sum of

the booty, And loke what proporcion is betweene the first number and the second, the same proporcion shall be betwen the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third place, one of the parts or shares, and then worke by the former Rule of proportion or Golden Rule. And because I haue foure seuerall parts of the first number, by which I would find out foure like parts of the second number, therefore must I make foure seuerall figures.

The reason of this rule

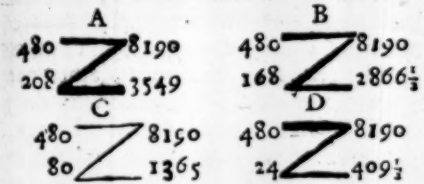
Scholar. Now I trust I can answere to your question, as by your fauour I will proue.



And to trie it, I set the foure figures thus, marked with A, B, C, D, to shew their order, And then in each of them I multiply the second number by the third, and diuide their totall by the first, & so amounteth the fourth summe which I seeke for: For if I doe multiply 8190 by 208, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the first mans portion.

And

And so working with the other three figures, I find for the second man $2866\frac{1}{3}$, & for the third man 1365, and then for the fourth man $409\frac{1}{3}$, and so every mans share is set forth in the figure here annexed.



And thus I thinke I haue done well.

The proofe by Additi-on.

Master. If you misdoubt your working, and list to proue it, adde all the shares together; and if they make the totall, then seemeth it well done.

Scholar. I may set them thus: and then by Addition the iust summe doth amount that is, 8190, and therefore (as you say) it seemeth to be well wrought.

3549	
$2866\frac{1}{3}$	
1365	
$409\frac{1}{3}$	
8190	

But I beseech you, is there any doubt in this triall, that you vse that word, Seemeth?

Master. You may easily coniecture, that if you did assigne the first mans share to the last, & so change all the rest, and one had anothers share, yet would the Addition appeare

all one, and therefore is not the proofe true.

But if you will make a iust proofe for the first mans part, take $\frac{1}{7}$ + $\frac{1}{11}$. of the whole sum, if it agree with the number in the figure, then it is well done. And so doe for the second, third, and fourth summes, and this proofe faileth not. Now will I propound certaine other questions, which haue beene set forth by certaine learned men, albeit not without some ouersight, which questions I protest heartily, I do not repeat to deprane those good men, whose labours and studies I much praise, and greatly delight in. But only according to my profession, to seeke out truth in all things, and to remoue all occasions of errour as much as in me lieth: and for that cause I will onely name the questions without hurting the Authours name. The iust proofe.

The first question is this. A question of building.

Foure men did build an house, which cost them 3000 *crownes, their shares were such that one man should pay* $\frac{1}{2}$ *of the summe, and sixe crownes ouer: the second should pay* $\frac{1}{3}$ *and* 12 *crownes ouer: the third man must lay out* $\frac{3}{4}$ *abating* 8 *crownes, and the fourth man should pay* $\frac{1}{4}$ *and* 20 *crownes more. Can you answere to this question?*

Scholar. No in good soth sir, and that you know best of any man, for I know no more then you haue taught me.

Master. Then I dare say you cannot do it,

it, neither yet the best learned man that ever did propose it, for the question is impossible. For declaration whereof, I will bée bold to vse first the representation of the numbers in their aptest forme, (although I haue not yet taught that manner of worke) because it may appeare plainely that the question is not possible. For here I haue set the parts, and added them, & they make the whole sum, and $\frac{3}{4}$ & 30 more. Now, how is it possible to diuide truly either gains, either charges, so that the particulars $1\frac{3}{4}$ shall bée more then the totall?

An impossible question.

$$\left\{\begin{array}{l} \frac{1}{2} - + \ 6 \\ \frac{1}{3} - + \ 12 \\ \frac{2}{3} - - \ 8 \\ \frac{1}{4} - + \ 20 \end{array}\right\}$$

$$1\frac{3}{4} \ + - 30$$

Scholar. It is against the forme of proofe by addition of parts.

Master. You say truth. And (because you shall perceiue it the better) I will trie it after the vulgar forme, as in this figure you sée, where the $\frac{1}{2}$ with 6 ouer, is 1506; for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe, do make 5280, where the total summe should be but 3000, which summe of 3000 if you diuide by $\frac{4}{3}$, so shall you

1506
1012
1992
770
5280

you haue ⅖ of it, that is 2250, & thereto adde 30 more, then will those 3 sums make 5280: whereby you may see how this forme (as wel as the other) doth declare that the particulars in that question would make more then the whole sum by ⅖, and 30 more and therefore can that question not be accepted as a possible thing: but yet doe certaine Learned men propound such questions, and answere to them. Therfore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anone declare what may be said for their defence: but in the meane season, I will propound the question as it may be wrought by good possibilitie. *As if foer men build a house together, and it cost them 3000 crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 crownes, so often the second man shall pay 4, the third man 8 and the fourth man 3. Or else, thus: that the first man shall pay double so much as the fourth, and the second man shall pay ⅔ of the first mans charge: the third man shall pay double so much as the second: (and these two waies are to one end:) but further for their agreement it is appointed also, that the first shall giue 6 crownes ouerplus, and the second 12, and the fourth shall giue 20: but the third man shall giue no ouerplus, but*

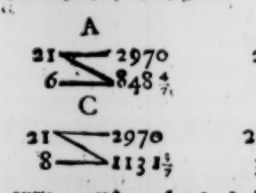
	2000
	2250
	30
	5280

but shall have 8 crownes abated of his charge.

So is the question possible to be allowed, and this is the way to doe it. Make the proportion of the seuerall charges, & set out small numbers in that rate, by which you may reduce the worke to the Golden Rule, as here in the first forme, the numbers are already named, 6, 4, 8, 3: and in the second forme, (although they be but plainely named) yet they may be the same numbers: for 6 is double to 3, and 4 is ⅔ of 6: and againe 4 is double to 4. Now adde these together, & they make 21, which 21 must be set *as the first* number in the Golden Rule: for if it, with the ouerplus of each mans charge would make the totall summe of the charges: then were those seuerall sums the charges of each man, besides his ouerplus: but now it is not so.

But yet this is true, (so excellent are conclusions Atirhmetical,) that loke what proportion each of their seuerall summes doth beare to 21, the same proportion doth the iust charges of euery man (besides his ouerplus) beare to the totall of the charges, the ouerplus being deducted: wherefore this may you note, that before you doe apply the totall of his charges to the Golden rule, you must deduct the ouerplus which is 6, 12, & 20, that is in the whole 38: but then 8 must be restored for the abatement of the third man, The rule.

man, and then remaineth to be deducted 30. Take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule for the second sum: & for the third sum I must put each of the small numbers before mentioned, which although they be not severall charges, yet they represent them in proportion. And so making for every mans charge a severall question, the figures will be 4, which I marke with foure letters, A, B, C, D, thus.



Where I have set for briefenesse the sum of every mans charge in the fourth place, presupposing that you can tell how to trie out that fourth summe by so many examples as ye have had.

Scholar. As I trust that I understand this forme, so I desire much to know what may bee said for them that misstake this question.

Master. You seeme so desirous to know this errour, that you have forgotten to examine, whether this worke be without fault.

Scholar. Ye seemeth this worke to bee well

wel done, because the Addition of the foure seuerall numbers doth make the totall sum of 2970, which was to be diuided into such foure parts.

Master. But then haue you forgotten that the first man must pay fiue Crownes more besides his share, and the second man 12 crownes more, the third man 8 crownes lesse, and the fourth man 10 crownes more, for without these your first totall of 3000 crownes willnot be made.

Scholar. Then must I adde to the first mans sum 6 more, and it will be $854\frac{2}{7}$: and to the seconds sum I must adde 12, and it will be $577\frac{1}{7}$: from the thirds sum I must abate 8, and then will the summe be $1123\frac{3}{7}$: then adding vnto the fourths sum 20, it will be $444\frac{4}{7}$, and these foure summes will make 3000, which is the whole charge, as in this example it may appeare, where first I gather the $\frac{4}{7}$, that maketh 2, and so procede I in the Addition to the end.

	$854\frac{2}{7}$
	$577\frac{1}{7}$
	$1123\frac{3}{7}$
	$444\frac{4}{7}$
	3000

Master. Now haue you wel done, and this worke in the same summes is brought of other learned men for the true solution of the question as it was first proponed, which (as I said) was impossible: and now examin it by these seuerall summes, and see whether it doth agree with the sums in the question proponed.

The

The first man must pay ½ and 6 ouer of the totall summe: how thinke you, is 854 ½ the halfe and more of 3000?

Scholar. No that it is not, for it should be 1505: and for the second man 1012: and for the third man 1992, and for the fourth man 770, whereof not one summe agreeth to this worke. But I marueil, that so wise men could be so much ouer séene.

Master. It is commonly séene that when men will receiue things from elder writers, & will not examine the thing, they séeme rather willing to erre with their Ancients for company, then to be bold to examine their works or writings. Which scrupulosity hath ingendred infinite errors in all kinds of knowledg, & in all ciuill administration, & in euery kind of Art. But these Learned men did not meane any other thing by this question then to find such numbers as should beare the same proportion together, as those numbers in the question proponed did beare one to another: which thing you shall perceiue more plainely by another question of theirs, that is this.

A question of a Testament.

A man lying vpon his death bed bequeached his goods, (which were worth 3600 crownes) this sort. Because his wife was greate with childe, & he yet vncertaine whether the childe were male or female he made his request con- [illegible]

then

then should the wife haue halfe his goods, and the Daughter ⅓, but if she were delinered of a Sonne, then that Sonne should haue ½ of the goods, and his wife but ⅓. Now it chanced her to bring forth both a Sonne and a Daughter, the question is: How shall they part the goods agreeable to the Testator his will?

Scholar, If some cunning Lawiers had this matter in scanning, they would determine this Testament to be quite voide, and so the man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it: and also, it might haue chanced, that he should haue brought forth neither Sonne nor Daughter, as often hath beene seene: so is the Will insufficient in that point also.

Master. Such Scanners should seeme to cunning and yet not so cunning as cruel: for the mind of the Testator is to be taken fauorably, for the aide of the Lygatories, when there ariseth such doubt. But let vs trie this worke, not by force of Law, but by proportion Geometricall, seeing the Testator did minde to prouide for each sort of them.

Scholar. If the Sonne shall haue ½ by force of the Testament, so must the mother haue ⅓ againe, because she hath a Daughter also. therefore ought she to haue ⅓, & the Daughter ⅓, that is both waies ½, ⅓, and ⅙, which

Where-